CML 100: 2017-2018
Quantum Tutorial 2

1. What are the results of operating on the following functions with the operator d/dx and d^2/dx^2 : (a) $exp(-ax^2)$, (b) $cos(bx)$, (c) $exp(ikx)$? Which functions are eigen functions of these operators? What are the corresponding eigen values?

2. Which of the following operators are linear? (a) d/dx; (b) √; (c) exponentiation; (d) integration.

3. Determine $\psi^*\psi$ for the following wave functions: (a) $\cos\theta + i\sin\theta$ and (b) $\exp -x^2$.

4. Show that $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$ (Hint: Use the operator in Cartesian coordinates).
5. Show that $\left[\hat{L}_x, y\right] = i\hbar z$
6. Show by direct operation that the functions $\sin\theta \exp i\phi$, $\sin\theta \exp(-i\phi)$, and $\cos\theta$ are eigenfunctions of $\hat{L}_z$. What are the eigenvalues?
7. Use the operator for $\hat{L}^2$ in polar coordinates to show that the function $(3\cos^2\theta - 1)$ is an eigenfunction of this operator. What is the eigenvalue? What is the quantum number l for this function?
8. Show that $Y_1^{-1}(\theta, \phi)$ is normalized and it is orthogonal to $Y_2^1(\theta, \phi)$.
9. Calculate the moment of inertia of $H^{35}Cl$, $H^{37}Cl$, and $D^{35}Cl$ all of which have an equilibrium bond length of 1.275 Å. Calculate the positions of the first three rotational transitions for $H^{35}Cl$ and $D^{35}Cl$.
10. In the far infrared spectrum of $H^{79}Br$, there is a series of lines separated by $16.72\ cm^{-1}$. Calculate the values of the moment of inertia and the internuclear separation in $H^{79}Br$.
11. For a hydrogen atom in the ground state find the classically forbidden region and calculate the probability of finding the electron in this region.[Hint: KE + PE = total E]
12. Compute the average value of r, the most probable value of r, and the root-mean-square value of r for the 1s and 2p levels of the hydrogen atom. Compare the three kinds of values and explain the origin of their differences.
13. Show that the hydrogenlike atomic wave function ψ_{210} is normalized and that it is orthogonal to ψ_{200}.
14. Calculate the probability that an electron described by a hydrogen 1s wave function will be found within one Bohr radius of the nucleus.
15. Where do the maxima in $r^2\psi_{2s}^2(r)$ occur?