## Physical transformations

Phase diagrams

## Number of phases

Gases - one single phase (even gaseous mixtures)
Solids - one solid one phase, two solids - two phases. Mixture of solids could be a single phase if the distribution of the components in uniform on a microscopic scale

Liquids - miscible liquids form a single phase
e.g. $\mathrm{CaCO}_{3}(\mathrm{~s}) \rightarrow \mathrm{CaO}(\mathrm{s})+\mathrm{CO}_{2}(\mathrm{~g})$ is a 3-phase system.

At equilibrium, the chemical potential of a substance is the same throughout a sample, regardless of how many phases are present.

For a one-component system, $\mu=\bar{G}$. For more components we will define later.

At eq. $\Delta G$ must be 0 . For two phases, $d G=\left(\mu_{2}-\mu_{1}\right) d n$ implies transfer of $d n$ moles of material from one phase to the other. $d G=0$ will be true when $\mu_{2}=\mu_{1}$.



## The phase rule

$F=C-P+2 ; F$ is the degrees of freedom, $C$ the number of components and $P$ the number of phases.

Component: Chemically independent constituent of a system. The \# of components is the minimum no. of independent species necessary to define the composition of all the phases present in the system. E.g. NaCl in water has only two components $\mathrm{Na}+$ and H 2 O as no. of Cl - ions equals the no. of Na+ions.

Single component, single phase system $F=2$. We can change pressure and temperature independently of each other.

For an equilibrium between two phases (e.g. solid-liquid), if we change $T$, then $p$ will follow on the $s-l$ equilibrium curve. $\therefore F=1 . \mu(s, p . T)=\mu(l, p, T)$ gives one equation that relates p and T .

For an equilibrium between the three phases, $F=0$. For example, $T_{3}$ of water occurs at 273.16 K , 611 mbar. $\mu(s, p . T)=\mu(l, p, T)=\mu(g, p, T)$ gives two independent equations which give a unique solution for $p$ and $T$.

## Derivation

The total no. of intensive variables: p and T , count as 2 . In each phase the mole fractions of $C-1$ components make it $P(C-1)$. So total $P(C-1)+2$.

At equilibrium, the chemical potential of a component must be the same in each phase, i.e.
$\mu(\alpha, p, T)=\mu(\beta, p, T)=\cdots$ for $P$ phases. Implies we need to satisfy $P-1$ such equations for each component. For all components we need to remove $C(P-1)$ degrees of freedom.

$$
F=P(C-1)+2-C(P-1)=C-P+2
$$



## Temperature dependence of phase stability

$\left(\frac{\partial \mu}{\partial T}\right)_{p}=-S_{m}$. Explains the s-l-v phase stability

## Response of melting to applied pressure

$\left(\frac{\partial \mu}{\partial p}\right)_{T}=V_{m}$. Usually the $V_{m}(l)>V_{m}(s)$. So, m.p. inc. on application of pressure.





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## Location of phase boundaries

At eq. the changes in chemical potential of the two phases must be equal: $d \mu(\alpha)=d \mu(\beta)$
i.e. $-S_{m}(\alpha) d T+V_{m}(\alpha) d p=-S_{m}(\beta) d T+V_{m}(\beta) d p$
or $\left\{V_{m}(\beta)-V_{m}(\alpha)\right\} d p=\left\{S_{m}(\beta)-S_{m}(\alpha)\right\} d T$ $\frac{d p}{d T}=\frac{\Delta_{\text {trs }} S}{\Delta_{t r s} V} \quad$ Clapeyron equation
Solid-liquid boundary

$$
\frac{d p}{d T}=\frac{\Delta_{f u s} H}{T \Delta_{f u s} V}
$$

$\Delta H_{f u s}$ is +ve and $\Delta V_{f u s}$ is usually +ve and always small. So, v. steep slope. Assume $\Delta H$ and $\Delta V$ do not change much with $T$.

$$
\int_{p^{*}}^{p} d p=\frac{\Delta_{f u s} H}{\Delta_{f u s} V} \int_{T^{*}}^{T} \frac{d T}{T} \rightarrow p=p^{*}+\frac{\Delta_{f u s} H}{\Delta_{f u s} V} \ln \frac{T}{T^{*}}
$$

For small $x, \ln (1+x)=x, \quad \ln \frac{T}{T^{*}}=\ln \left(1+\frac{T-T^{*}}{T^{*}}\right)=\frac{T-T^{*}}{T^{*}}$


Therefore, $p=p^{*}+\frac{\Delta_{f u s} H}{T^{*} \Delta_{f u s} V}\left(T-T^{*}\right)$, a steep straight line equation.

## Liquid-vapor boundary

$$
\frac{d p}{d T}=\frac{\Delta_{v a p} H}{T \Delta_{v a p} V}
$$

Enthalpy change is +ve, volume change is large and +ve. $d p / d T$ is +ve but small. Or $d T / d p$ is large. i.e. the boiling temp. is more responsive to pressure than the freezing temperature.

$$
\begin{aligned}
& V_{m}(g) \gg V_{m}(l) . \text { Therefore, } \Delta_{v a p} V \sim V_{m}(g) \\
& \frac{d p}{d T}=\frac{\Delta_{v a p} H}{T V_{m}(g)}= \frac{d p}{d T}=\frac{\Delta_{v a p} H}{T(R T / p)}(\text { for I.G.) } \rightarrow \\
& \frac{d \ln p}{d T}=\frac{\Delta_{v a p} H}{R T^{2}} \quad \text { Clausius Clapeyron equation } \\
& \int_{\ln p^{*}}^{\ln p} d \ln p=\frac{\Delta_{v a p} H}{R} \int_{T^{*}}^{T} \frac{d T}{T^{2}}=-\frac{\Delta_{v a p} H}{R}\left(\frac{1}{T}-\frac{1}{T^{*}}\right) \\
& p=p^{*} \exp -\chi \quad \text { where } \quad \chi=\frac{\Delta_{v a p} H}{R}\left(\frac{1}{T}-\frac{1}{T^{*}}\right)
\end{aligned}
$$



Temperature, $T$

Similar is the solid-vapor boundary

## Ehrenfest classification

First order phase transition: the first derivative of the chemical potential with respect to temperature is discontinuous.

Second order phase transition: The first derivative is continuous but not the second derivative


Figure 4-16
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