Physical transformations

Phase diagrams

Number of phases

Gases - one single phase (even gaseous mixtures)

Solids – one solid one phase, two solids – two phases. Mixture of solids could be a single phase if the distribution of the components in uniform on a microscopic scale

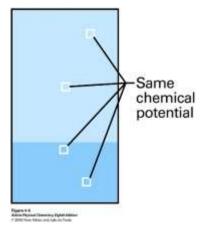
Liquids – miscible liquids form a single phase

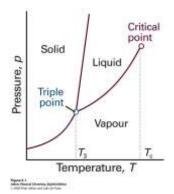
e.g. $CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$ is a 3-phase system.

At equilibrium, the chemical potential of a substance is the same throughout a sample, regardless of how many phases are present.

For a one-component system, $\mu = \overline{G}$. For more components we will define later.

At eq. ΔG must be 0. For two phases, $dG = (\mu_2 - \mu_1)dn$ implies transfer of dn moles of material from one phase to the other. dG = 0 will be true when $\mu_2 = \mu_1$.





The phase rule

F = C - P + 2; F is the degrees of freedom, C the number of components and P the number of phases.

Component: Chemically independent constituent of a system. The # of components is the minimum no. of independent species necessary to define the composition of all the phases present in the system. E.g. NaCl in water has only two components Na+ and H2O as no. of Cl- ions equals the no. of Na+ ions.

Single component, single phase system F = 2. We can change pressure and temperature independently of each other.

For an equilibrium between two phases (e.g. solid-liquid), if we change T, then p will follow on the s-l equilibrium curve. $\therefore F = 1$. $\mu(s, p.T) = \mu(l, p, T)$ gives one equation that relates p and T.

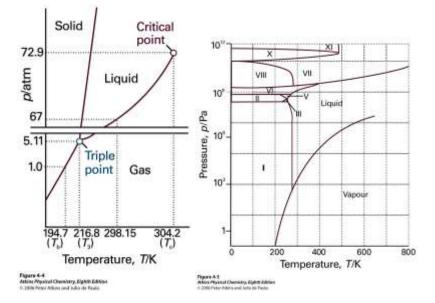
For an equilibrium between the three phases, F = 0. For example, T_3 of water occurs at 273.16 K, 611 mbar. $\mu(s, p, T) = \mu(l, p, T) = \mu(g, p, T)$ gives two independent equations which give a unique solution for p and T.

Derivation

The total no. of intensive variables: p and T, count as 2. In each phase the mole fractions of C - 1 components make it P(C - 1). So total P(C - 1) + 2.

At equilibrium, the chemical potential of a component must be the same in each phase, i.e. $\mu(\alpha, p, T) = \mu(\beta, p, T) = \cdots$ for *P* phases. Implies we need to satisfy *P* - 1 such equations for each component. For all components we need to remove *C*(*P* - 1) degrees of freedom.

$$F = P(C - 1) + 2 - C(P - 1) = C - P + 2$$

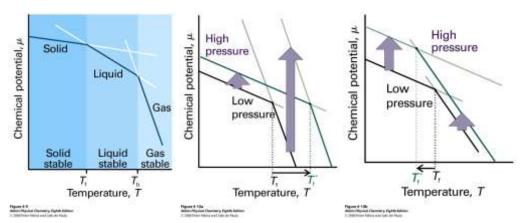


Temperature dependence of phase stability

 $\left(\frac{\partial \mu}{\partial T}\right)_p = -S_m$. Explains the s-l-v phase stability

Response of melting to applied pressure

$$\left(\frac{\partial \mu}{\partial p}\right)_T = V_m$$
. Usually the $V_m(l) > V_m(s)$. So, m.p. inc. on application of pressure.



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Location of phase boundaries

At eq. the changes in chemical potential of the two phases must be equal: $d\mu(\alpha) = d\mu(\beta)$

i.e.
$$-S_m(\alpha)dT + V_m(\alpha)dp = -S_m(\beta)dT + V_m(\beta)dp$$

or $\{V_m(\beta) - V_m(\alpha)\}dp = \{S_m(\beta) - S_m(\alpha)\}dT$
 $\frac{dp}{dT} = \frac{\Delta_{trs}S}{\Delta_{trs}V}$ Clapeyron equation

Solid-liquid boundary

$$\frac{dp}{dT} = \frac{\Delta_{fus}H}{T\Delta_{fus}V}$$

 ΔH_{fus} is +ve and ΔV_{fus} is usually +ve and always small. So, v. steep slope. Assume ΔH and ΔV do not change much with T.

$$\int_{p^*}^p dp = \frac{\Delta_{fus}H}{\Delta_{fus}V} \int_{T^*}^T \frac{dT}{T} \to p = p^* + \frac{\Delta_{fus}H}{\Delta_{fus}V} \ln \frac{T}{T^*}$$

For small x, $\ln(1 + x) = x$, $\ln \frac{T}{T^*} = \ln(1 + \frac{T - T^*}{T^*}) = \frac{T - T^*}{T^*}$

Therefore, $p = p^* + \frac{\Delta_{fus}H}{T^*\Delta_{fus}V}$ $(T - T^*)$, a steep straight line equation.

Liquid-vapor boundary

$$\frac{dp}{dT} = \frac{\Delta_{vap}H}{T\Delta_{vap}V}$$

Enthalpy change is +ve, volume change is large and +ve. dp/dT is +ve but small. Or dT/dp is large. i.e. the boiling temp. is more responsive to pressure than the freezing temperature.

 $V_m(g) \gg V_m(l)$. Therefore, $\Delta_{vap} V \sim V_m(g)$

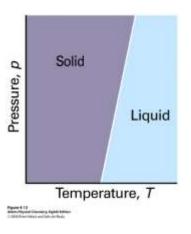
$$\frac{dp}{dT} = \frac{\Delta_{vap}H}{TV_m(g)} = \frac{dp}{dT} = \frac{\Delta_{vap}H}{T(RT/p)} (for I.G.) \rightarrow$$

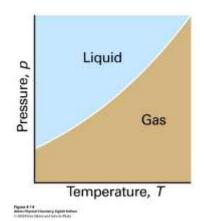
$$\frac{d \ln p}{dT} = \frac{\Delta_{vap}H}{RT^2} \quad Clausius \ Clapeyron \ equation$$

$$\int_{\ln p^*}^{\ln p} d \ln p = \frac{\Delta_{vap}H}{R} \int_{T^*}^{T} \frac{dT}{T^2} = -\frac{\Delta_{vap}H}{R} \left(\frac{1}{T} - \frac{1}{T^*}\right)$$

$$p = p^* \exp{-\chi} \quad where \quad \chi = \frac{\Delta_{vap}H}{R} \left(\frac{1}{T} - \frac{1}{T^*}\right)$$

Similar is the *solid-vapor boundary*





Ehrenfest classification

First order phase transition: the first derivative of the chemical potential with respect to temperature is discontinuous.

Second order phase transition: The first derivative is continuous but not the second derivative

