$$b - \frac{2a}{RT}$$

$$\int_a^b f(x) = F(b) - F(a)$$

Euler's criterion

$$z(x,y) \qquad \left[\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)_y\right]_x = \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)_x\right]_y$$

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = -1$$

$z(x, y)$ state fns.

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$\cancelto{0}{\left(\frac{\partial z}{\partial y}\right)_z} = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_x \cancelto{1}{\left(\frac{\partial y}{\partial y}\right)_z}$$

$pV = RT$ $\quad C_p - C_v = R$ for Ideal Gases

$$dH = dU + p\,dV$$

$$\left(\frac{\partial H}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

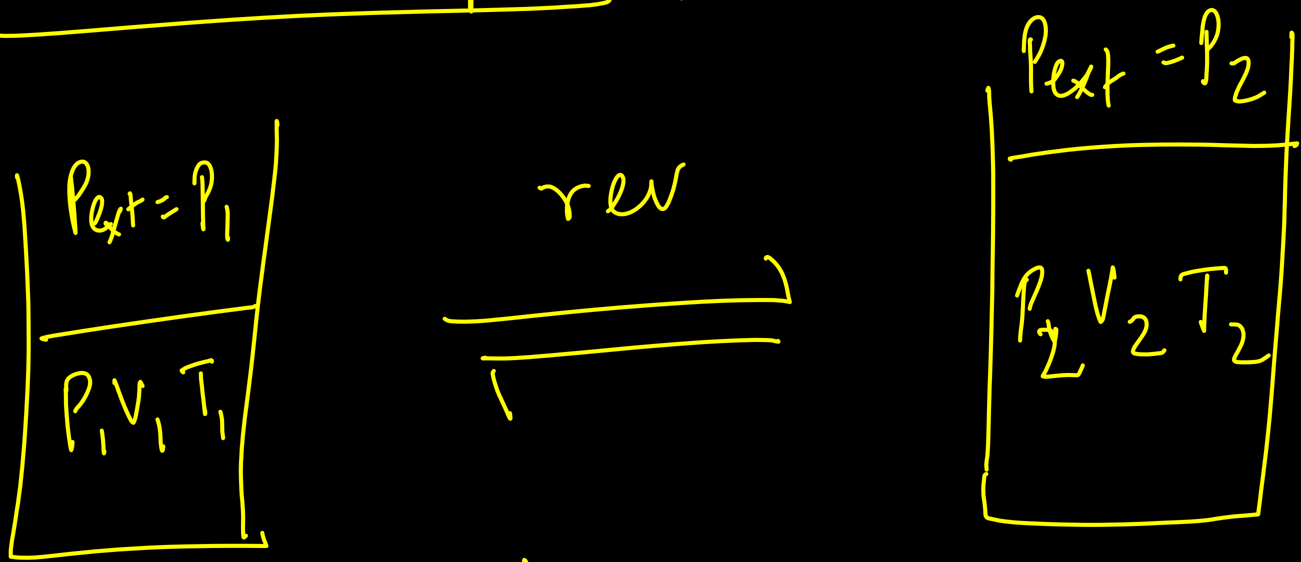
$$\left(\frac{\partial H}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + R$$

$$\left(\frac{\partial U}{\partial T}\right)_p = C_v$$

$$dU = C_v\,dT + \pi_T\,dV \quad 0$$

$$\boxed{C_p = C_v + R}$$

Adiabatic Expansion — Ideal Gases



$PV = \text{const}$

$PV^{\gamma} = \text{const}$

1) Adiabatic $\quad dq = 0$

2) Reversible $\quad dw = -p\,dV$

3) Ideal gas $\quad dU = C_v\,dT\,, \quad pV = RT$

$$C_v\,dT = -p\,dV = -\frac{RT}{V}\,dV$$

$$C_v \frac{dT}{T} = -R \frac{dV}{V} \qquad C_v \, d\ln T = -R \, d\ln V$$

$$C_v \int_{T_1}^{T_2} d\ln T = -R \int_{V_1}^{V_2} d\ln V$$

$$C_v \left(\ln T_2 - \ln T_1\right) = -R\left(\ln V_2 - \ln V_1\right)$$

$$\ln \frac{T_2}{T_1} = R/C_v \ln V_1/V_2$$

$$\ln T_2/T_1 = \ln \left(\frac{V_1}{V_2}\right)^{R/C_v}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_v}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{C_p}{C_v} - 1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

Mono atomic $\gamma = 5/3$

polyatomic $\gamma = 4/3$

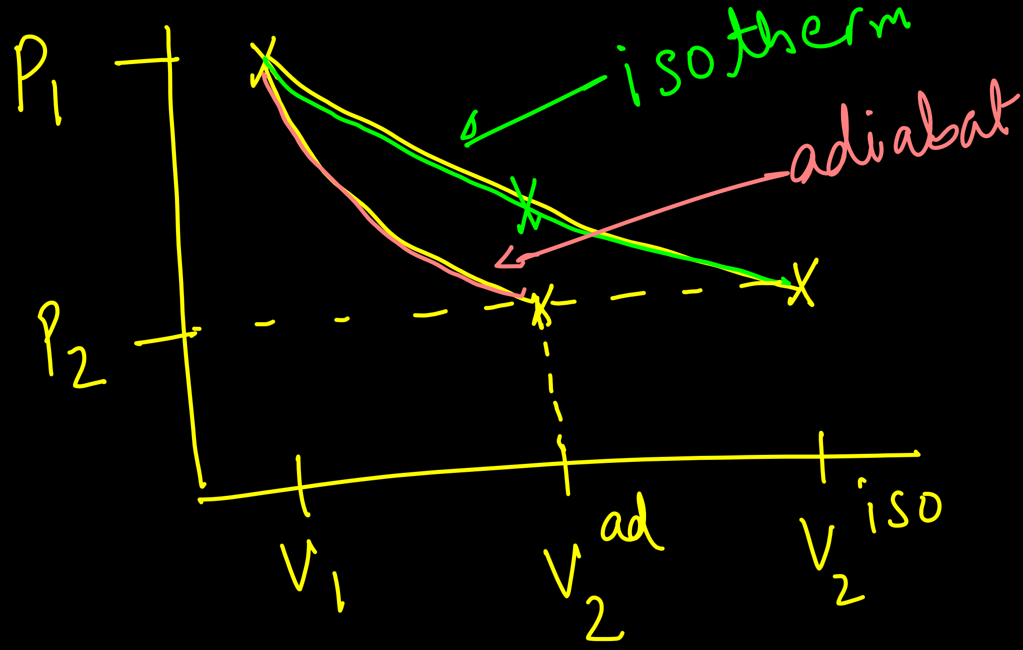
$$\boxed{\gamma > 1}$$

$$\boxed{TV^{\gamma-1} = \text{constant}}$$

$$\frac{P_2 V_2}{P_1 V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\boxed{PV^{\gamma} = \text{constant}}$$



$T_2 < T_1$

not the same as in the rev. case

Irreversible adiabatic Expansion

1) ad $dq = 0$

2) IG $dU = C_V dT$, $PV = RT$

3) $dw = -P_2 dV$

1^{st} Law : $dU = -P_2 dV$

$$C_V dT = -P_2 dV$$

$$C_V (T_2 - T_1) = -P_2 (V_2 - V_1)$$

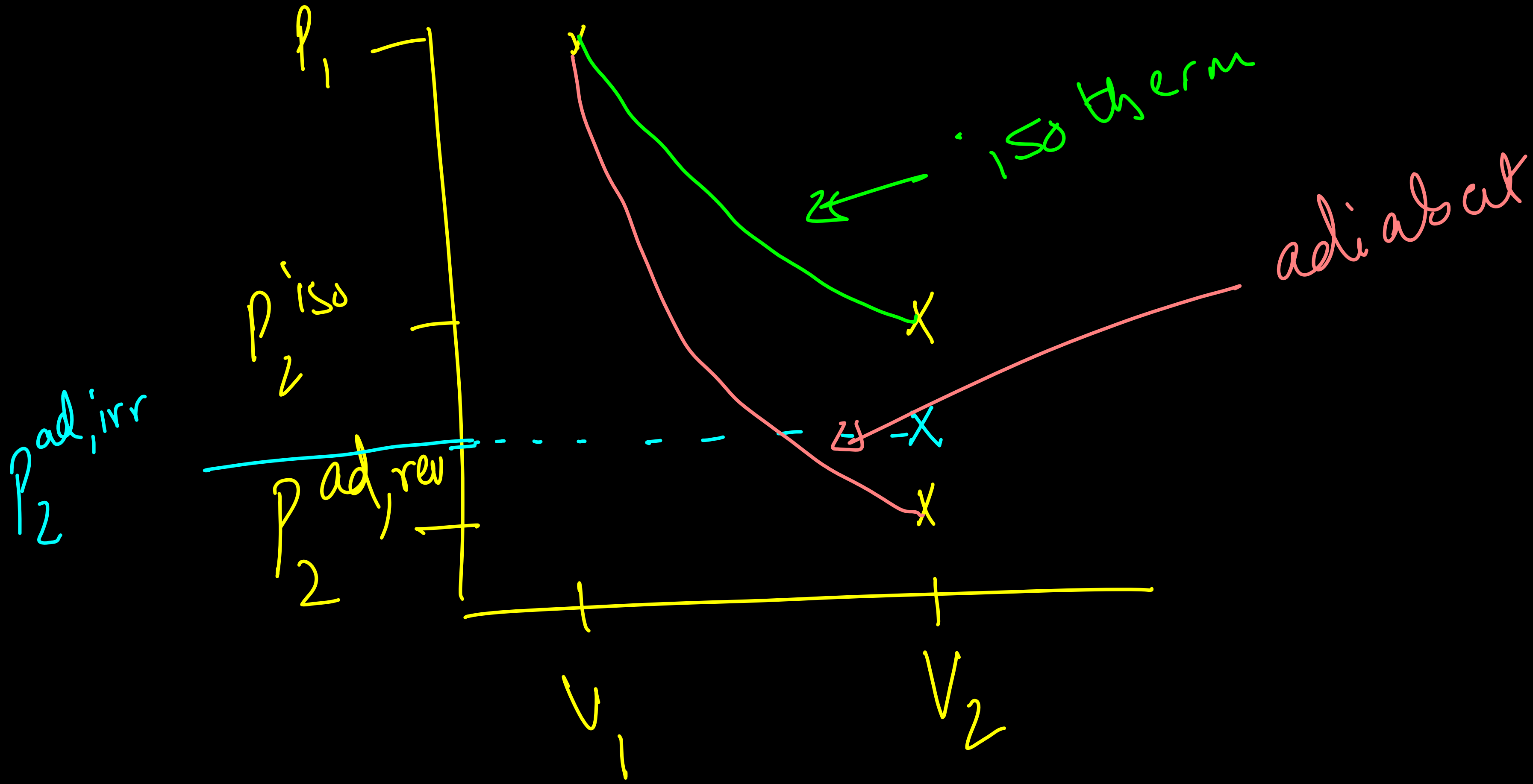
$$T_2 (C_v + R) = T_1 \left(C_v + \frac{P_2}{P_1} R \right)$$

$$\frac{P_2}{P_1} < 1$$

irr. expansion, $\longrightarrow$ gas cools

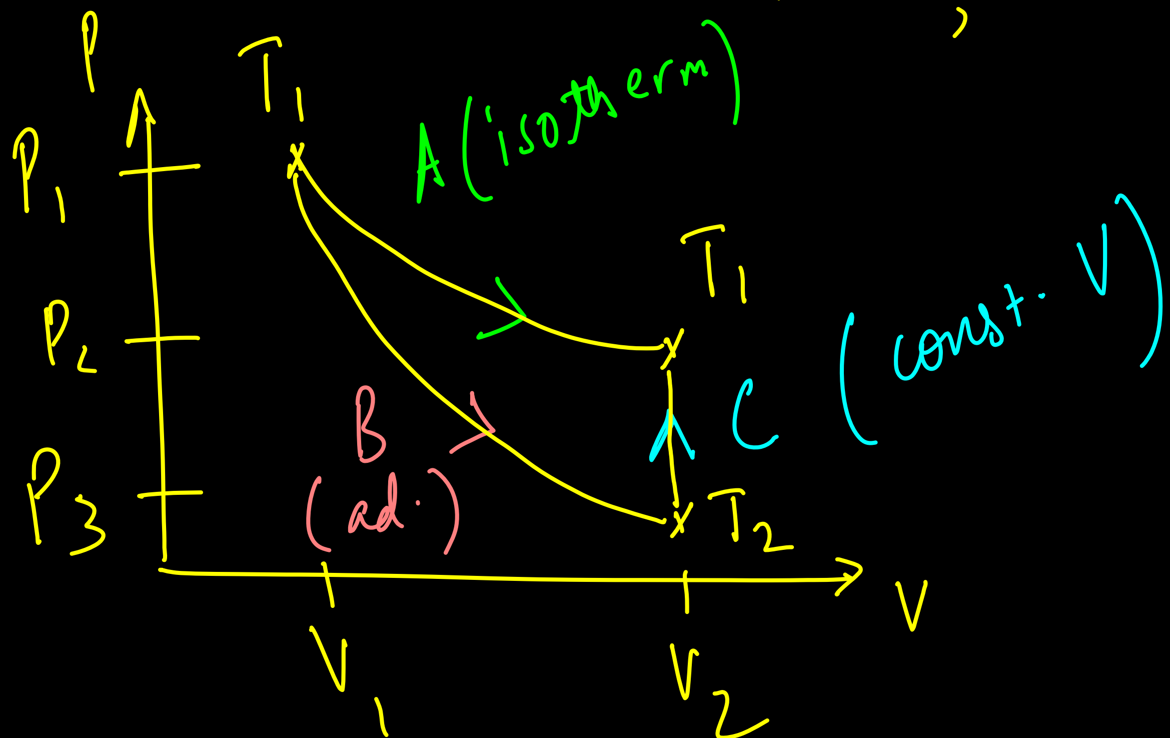
$$T_2^{rev} < T_2^{irr}$$

$$(-w^{rev}) > (-w^{irr})$$



p_1
p_2^{iso}
$p_2^{ad,irr}$
$p_2^{ad,rev}$
isotherm
adiabat
V_1
V_2

Rev. , I.G.



ΔU, ΔH,

q, w $\int \frac{dq}{T}$

<u>A</u>

$\Delta U = 0$, $\Delta H = 0$

$w = -RT_1 \ln \frac{V_2}{V_1}$, $q = RT_1 \ln \frac{V_2}{V_1}$

$\int \frac{dq}{T} = R \ln \frac{V_2}{V_1}$

B

$q = 0 \qquad \Delta U = C_v(T_2 - T_1)$

$$\Delta H = C_p(T_2 - T_1)$$

$$w = C_v(T_2 - T_1)$$

$$\int \frac{dq}{T} = 0$$

C

$w = 0, \qquad \Delta U = C_v(T_1 - T_2)$

$$\Delta H = C_p(T_1 - T_2)$$

$$q = C_v(T_1 - T_2)$$

$$\int \frac{dq}{T} = C_V \ln \frac{T_1}{T_2}$$

$$W_A = -RT_1 \ln \frac{V_2}{V_1}$$

$$W_B + W_C = C_V(T_2 - T_1)$$

$$\Delta U_A = 0$$

$$\Delta U_B + \Delta U_C = C_V(T_2 - T_1) + C_V(T_1 - T_2) = 0$$

$$A \rightarrow \int \frac{dq}{T} = R \ln V_2/V_1$$

$B + C \rightarrow$ $C_v \ln(T_1/T_2) = R \ln \frac{V_2}{V_1}$

$$\frac{dq_{rev}}{T} = dS$$