Instructions: Read carefully

1. Write the appropriate question paper set, A or B, in the OMR sheet and darken the appropriate letter.

2. Please write your application number in the OMR sheet as follows. Write the last 5 digits (or, 4 digits as applicable) of your application number in the box and darken the correct number in the OMR sheet. For example, if your application number is PG-242501-0015, then write 0015 in the box starting from the first box and leave the last box blank. Also darken the appropriate digits below the boxes. If your application number is PG-242501-00150, then write 00150 in the boxes starting from the first box and darken the digits below the boxes.

3. Exam duration is 3 hours. You are not allowed to leave the exam hall before the end of the exam.

4. There are two sections. Section I has 20 one-mark questions. For each question in Section I, a correct answer fetches one mark, an incorrect answer penalizes negative $\frac{1}{3}$, and no answer gives no marks. Section II has 40 questions, with a correct answer getting two marks, an incorrect answer getting negative $\frac{2}{3}$, and no answer to a question getting zero.

5. Maximum possible marks is 100.

6. You should use a black/blue ballpoint pen to darken the correct responses on the OMR sheet. See the OMR sheet for instructions on how to fill a bubble correctly.

7. A separate booklet with ruled pages is provided for rough work.

8. You will need to submit the entire question paper along with the OMR sheet. But take the rough work booklet with you when you leave the hall.

9. There should be 16 pages in the exam booklet. Check your copy and request a new booklet if you do not have 16 pages.

10. If any candidate is caught using unfair means during the exam, he/she will be immediately disqualified and will be asked to leave the exam hall.

11. Mobile phones, smart phones, smart watches, calculators, or any other electronic gadgets are NOT allowed inside the exam hall.
I Multiple Choice Questions (One mark each)

1. For $X$ and $Y$, two jointly distributed random variables, which of the following statements is false?
   A. If $X$ and $Y$ are independently distributed, then the conditional distribution of $Y$ given $X$ equals the marginal distribution of $Y$.
   B. If the covariance between $X$ and $Y$ is 0, then $X$ and $Y$ are independent.
   C. If covariance between $X$ and $Y$ is 0, then $X$ and $Y$ are not correlated.
   D. If $X$ and $Y$ are independently distributed, then the joint density of $X$ and $Y$ is the product of their marginal densities.

2. In testing a hypothesis $H_0$, with an alternative $H_A$, Type-I and Type-II error corresponds to
   A. wrongly rejecting $H_0$ and falsely accepting $H_A$, respectively.
   B. wrongly rejecting $H_A$ and falsely accepting $H_0$, respectively.
   C. wrongly rejecting $H_0$ and falsely accepting $H_0$, respectively.
   D. wrongly rejecting $H_A$ and falsely accepting $H_A$, respectively.

3. The value of a correlation between $x$ and $y$ is $r = -0.5$. Which of the following statements is correct?
   A. The x-variable explains 25% of the variation in the y-variable.
   B. The x-variable explains -25% of the variation in the y-variable.
   C. The x-variable explains 50% of the variation in the y-variable.
   D. The x-variable explains -50% of the variation in the y-variable.

4. Which statement is correct about a $p$-value?
   A. The smaller the $p$-value the stronger the evidence in favor of the alternative hypothesis.
   B. The smaller the $p$-value the stronger the evidence in favor the null hypothesis.
   C. The larger the $p$-value the stronger the evidence in favor of the alternative hypothesis.
   D. Whether a small $p$-value provides evidence in favor of the null hypothesis depends on whether the test is one-sided or two-sided.

5. The level of significance (usually, 0.05) associated with testing a hypothesis is the probability
   A. that the null hypothesis is true.
   B. that the alternative hypothesis is true.
   C. of not rejecting a true null hypothesis.
D. of rejecting a true null hypothesis.

6. Let \( u, v \) be two distinct vectors in \( \mathbb{R}^k \) such that \( |u| = |v| = 1 \), where \( |u| \) is length of vector \( u \). The vectors \( \frac{u + v}{2} \) and \( \frac{u - v}{2} \) are
   A. linearly dependent.
   B. are independent, but not orthogonal to each other.
   C. are orthogonal to each other.
   D. None of the above.

7. Let \( A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \) and \( A^\top A = I \) (\( A^\top \) is transpose of \( A \) and \( I \) identity matrix), then the value of \( x^2 + y^2 + z^2 \) is
   A. \( \frac{1}{2} \).
   B. 2.
   C. 1.
   D. None of the above.

8. Let \( \beta \) be a real number. Consider the matrix
   \[
   A = \begin{pmatrix}
   \beta & 0 & 1 \\
   2 & 1 & -2 \\
   3 & 1 & -2
   \end{pmatrix}.
   \]
   then \( |\det(A)| \)
   A. \( \beta \).
   B. 2.
   C. 1.
   D. \( 1+\beta \).

9. Let \( x \) and \( y \) be the only two commodities a person consumes. There is a rise in \( P_x \) and the quantity demanded of \( y \) rises. Here:
   A. the income effect dominates the substitution effect.
   B. the substitution and income effects cancel each other.
   C. none of the answers is correct.
   D. the substitution effect dominates the income effect.

10. An expected utility function is:
    A. quadratic in probabilities.
    B. non-linear in probabilities.
C. linear in probabilities.
D. ordinal because the utility numbers have no meaning.

11. Consumer surplus is:
A. the area above the market price and below the demand curve.
B. a measure of the net welfare gained by consumers buying a particular good.
C. the difference between the amounts people are willing to pay for specific quantities of goods and the amounts they pay at market prices.
D. all of the above.

12. Consider a Walrasian equilibrium with production.
A. There is no excess supply.
B. The firms choose production to maximise profits.
C. The Walras law holds if and only if production technologies exhibit constant returns to scale.
D. there is no excess demand.

13. Which of the following statements is false?
A. An externality is an indirect effect that a firm’s production bears on a consumer’s utility function.
B. In a Walrasian equilibrium, a firm exhibiting a negative externality produces too much.
C. In a Walrasian equilibrium, a firm exhibiting a positive externality produces too little.
D. A Walrasian equilibrium of an economy with externalities need not be Pareto efficient.

14. Nash equilibrium in a normal form game is:
A. a strategy profile that maximises the sum of utilities of all players.
B. a strategy profile that maximises the utility of one of the players.
C. a strategy profile that is randomly selected by the players together.
D. none of the above.

15. The Lagrange multiplier in a constrained optimization problem, say in utility maximization problem subject to budget constraint, can be interpreted as:
A. the change in the objective function by relaxing the constraint by one unit.
B. marginal utility of money in utility maximization problem.
C. shadow price.
16. If the government of country Z increases spending by Rs 12 million and raises tax collections by the same amount, then what will be the overall impact of these moves on real GDP in country Z?
   A. Real GDP will increase by Rs 6 million
   B. Real GDP will remain unchanged
   C. Real GDP will increase by Rs 12 million
   D. Real GDP will decrease by Rs 12 million

17. Suppose the Central Bank makes an open market purchase of Rs 3 million. Assume that the money multiplier equals 2. What is the change in the money supply?
   A. The money supply has increased by Rs 1.5 million
   B. The money supply has increased by Rs 6 million
   C. The money supply had decreased by Rs 1.5 million
   D. The money supply has decreased by Rs 6 million

18. Consider an economy with a total population of 10,000 people. The size of the labour force is 8,000, while the number of people employed is 7,000. What is the unemployment rate in this economy?
   A. 10 per cent
   B. 12.5 per cent
   C. 20 per cent
   D. 30.33 per cent

19. Assume a simple Keynesian model of a closed economy with no government sector and autonomous investment. The marginal propensity to save (MPS) is 0.2. There is a sudden drop in investment spending by 100 million. By how much will output eventually fall?
   A. 20 million
   B. 100 million
   C. 500 million
   D. 125 million

20. In an AD-AS diagram, an increase in nominal money supply is represented by:
   A. A shift of the AS-curve to the right
   B. A shift of the AD-curve to the right
   C. Movement along the AD-curve from right to left
   D. Movement along the AD-curve from left to right
21. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A: x + y = 7\}$ is
   A. symmetric, but neither reflexive nor transitive.
   B. reflexive, but neither symmetric nor transitive.
   C. transitive, but neither symmetric nor reflexive.
   D. an equivalence relation.

22. Totrep has constant absolute risk aversion and initial wealth $y$ and does not prefer to accept a gamble such that with probability 0.5 she would win $a$ and with probability 0.5 she would lose $b$. Is it possible that if Totrep’s wealth were much higher, she would like to accept the gamble:
   A. There is not enough information to be able to answer.
   B. No.
   C. Yes, for some parameter values.
   D. It depends on the functional form of the utility function.

23. Which of the following statements is false?
   A. the convexity of the production possibility set implies that the production function is weakly concave.
   B. the convexity of the production possibility set implies the convexity of the input requirement set.
   C. the convexity of the production possibility set implies constant returns to scale.
   D. the convexity of the input requirement set implies that the production function is quasi-concave.

24. Consider the following utility function: $u(x, y) = \sqrt{xy}$. Here $x$ and $y$ are
   A. net and gross substitutes.
   B. net substitutes and gross complements.
   C. net substitutes and neither gross substitutes nor complements.
   D. net and gross complements.

25. Totrep owns a cargo flight worth 5 crores. With a probability of $p = 0.1$ the flight crashes and its worth becomes 0. Totrep is risk loving and no further information on her risk attitudes is available. Would she sell the ship for 4.75 crores?
   A. Yes.
   B. No.
C. She would be indifferent between selling at this price and not selling.
D. The information provided does not suffice to answer the question.

26. A firm produces one commodity with the production function \( y = f(x) = x^{1/3} \). The factor price \( w \) and the price \( p \) for the commodity are fixed. Pick the true statement:
   A. The production function exhibits increasing returns to scale.
   B. The production function exhibits decreasing returns to scale.
   C. The production function exhibits constant returns to scale.
   D. It is not possible to find the answer based on the information provided.

27. Consider the same production function: \( y = f(x) = x^{1/3} \) and factor and commodity prices \( w \) and \( p \), respectively. The cost function is:
   A. \( C(y) = w.y^{3/2} \).
   B. \( C(y) = w.y^3 \).
   C. \( C(y) = y.w^{1/3} \).
   D. \( C(y) = w + y^{3/2} \).

28. Again consider the same production environment as in the last two questions. The input demand function is:
   A. \( x = (\frac{p}{3w})^{\frac{2}{3}} \).
   B. \( x = (\frac{3}{p.w})^{\frac{2}{3}} \).
   C. \( x = (\frac{p}{3w})^{\frac{1}{3}} \).
   D. \( x = (\frac{3p}{w})^{\frac{2}{3}} \).

29. Again consider the same production environment as in the last three questions. The supply function is:
   A. \( y = (\frac{p}{3w})^{\frac{1}{3}} \).
   B. \( x = (\frac{3}{p.w})^{\frac{1}{3}} \).
   C. \( x = (\frac{p}{3w})^{\frac{1}{2}} \).
   D. \( x = (\frac{3p}{w})^{\frac{1}{3}} \).

30. Consider the following normal form game and pick the correct statement:
A. \((C, A)\) is the only Nash equilibrium in this game.

B. \((C, A)\) and \((D, B)\) are pure strategy Nash equilibria and there is a mixed strategy Nash equilibrium where the row player chooses \(C\) with probability \(\frac{1}{2}\) and the column player chooses \(A\) with probability \(\frac{1}{3}\).

C. \((C, A)\) and \((D, B)\) are pure strategy Nash equilibria and there is no mixed strategy Nash equilibrium in this game.

D. \((C, A)\) and \((D, B)\) are pure strategy Nash equilibria and there is a mixed strategy Nash equilibrium where the row player chooses \(C\) with probability \(\frac{1}{3}\) and the column player chooses \(A\) with probability \(\frac{1}{4}\).

31. Consider the following figures and pick the correct statement:

A. Choices in figure A satisfy WARP, choices in figure B do not satisfy WARP.

B. Choices in figure B satisfy WARP, choices in figure A do not satisfy WARP

C. Choices in figure A and B satisfy WARP

D. Choices in figure A and B do not satisfy WARP
32. A family has two children. The probability of a child being a boy is 0.4 and it is independent of the other. We know that one of them is a boy. What is the probability that the other one is also a boy?
   A. 0.16
   B. 0.24
   C. 0.25
   D. 0.33

33. X and Y are two jointly distributed random variables. The conditional mean and variance of Y given X are given by 3X and 9X. The unconditional mean and variance of X are 3 and 9, respectively. What is the unconditional variance of Y?
   A. 90
   B. 108
   C. 12
   D. 144

34. A box contains eight balls. Three of them are blue, three are green, and the remaining two are red. Balls of same color are identical to each other. Five balls are chosen randomly, one after the other, without replacement. What is the probability that two of will be blue, two green, and one red?
   A. \( \frac{2}{9} \)
   B. \( \frac{9}{28} \)
   C. \( \frac{3}{26} \)
   D. \( \frac{3}{16} \)

35. Let X be a discrete random variable with a probability mass function.
   \[
P(X = n) = \begin{cases} 
   -\frac{2c}{n} & \text{if } n = -1, -2 \\
   d & \text{if } n = 0 \\
   cn & \text{if } n = 1, 2 \\
   0 & \text{Otherwise}
   \end{cases}
   \]
   where c and d are positive real numbers. If \( P(|X| \leq 1) = 0.75 \), then \( \mathbb{E}[X] \) equals
   A. 1/12
   B. 1/6
   C. 1/3
   D. 1/2

36. Consider the following two simple linear regression models where the dependent and independent variables are flipped
   \[
y_i = \alpha_1 + \beta_1 x_i + \epsilon_i
   \] (1)
\[ x_i = \alpha_2 + \beta_2 y_i + \nu_i \]  

(2)

If \( \hat{\alpha}_i \) and \( \hat{\beta}_i \) are the Ordinary Least Square estimators of \( \alpha_i \) and \( \beta_i \), \( R^2_i \) denote the R-square from equation (i) and \( \rho_{xy} \) denote the correlation coefficient between \( x \) and \( y \), then

A. \( \hat{\alpha}_1 \times \hat{\alpha}_2 = 1 \)
B. \( \hat{\beta}_1 \times \hat{\beta}_2 = R^2_1 \)
C. \( \hat{\alpha}_1 \times \hat{\beta}_1 = \rho_{xy} \)
D. \( \hat{\alpha}_1 \times \hat{\beta}_2 = \sqrt{R^2_2} \)

37. Four balls of different colors are randomly placed in four identical boxes. After the allocation, some boxes may remain empty, while others may have multiple balls. Find the probability that exactly one box will remain empty.

A. \( \frac{3}{32} \)
B. \( \frac{9}{16} \)
C. \( \frac{96}{256} \)
D. \( \frac{16}{64} \)

38. Professor Calculus has 10 keys on his key ring. He forgot which one opens the door to his office and started trying them at random, discarding those that do not work. What is the probability that the 8th key will open the door?

A. 0.100  
B. 0.150  
C. 0.133  
D. 0.033

39. A masters student is using data from a representative sample of \( n = 20 \) male IIT Delhi students to estimate the parameters of a regression equation

\[ y = \beta_0 + \beta_1 x + \epsilon, \]

where \( y = \text{weight (kg)} \), \( x = \text{height (cm)} \), and \( \epsilon \) is the error. The estimated least squares regression with standard errors in parenthesis below the point estimates is reported below

\[ \hat{y} = 30.34 + 2.00 \times x. \]

The student is testing the following hypothesis

\[ H_0 : \beta_1 = 0 \]
\[ H_a : \beta_1 \neq 0. \]

What is the value of the appropriate test statistic for testing \( H_0 \)?

A. 2
40. Let \( f : \mathbb{R} \to (0, \infty) \) be an increasing function such that \( \lim_{x \to \infty} \frac{f(7x)}{f(x)} = 1 \). Then
\[
\lim_{x \to \infty} \left[ \frac{f(5x)}{f(x)} - 1 \right] =
\]
A. 0.
B. 2.
C. 4.
D. 1.

41. If the equation system \( Ax = b \) has no solution (where \( A \) is a matrix and \( b \) is a vector), then
A. \( b \) belongs to the null space of \( A \).
B. \( b \) belongs to the null space of \( A^\top \).
C. \( b \) belongs to the column space of \( A^\top \).
D. \( b \) belongs to the column space of \( A \).

42. If \( \binom{2n}{3} / \binom{n}{3} = 10 \), then the ratio \( \frac{n^2 + 3n}{n^2 - 3n + 1} \) is equal to:
A. \( 27/11 \).
B. \( 35/16 \).
C. \( 65/37 \).
D. 2.

43. Let \( 5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0 \). Then \( 18 \int_{1}^{2} f(x) \, dx \) is equal to:
A. \( 5 \log_e 2 - 3 \).
B. \( 5 \log_e 2 + 3 \).
C. \( 10 \log_e 2 + 6 \).
D. \( 10 \log_e 2 - 6 \).

44. If coefficient of \( x^7 \) in \( (ax^2 + \frac{1}{26x})^{11} \) and \( x^{-7} \) in \( (ax - \frac{1}{36ax})^{11} \) are equal, then
A. \( 32ab = 729 \).
B. \( 64ab = 243 \).
C. \( 243ab = 64 \).
D. \( 729ab = 32 \).

45. If \( S_K = \sum_{j=1}^{K} j \), then \( \lim_{n \to \infty} \frac{\sum_{j=1}^{n} S_j}{n^4} \) is
46. For any set $A \subset \mathbb{R}^k$, lets define
\[ \delta(q) = \sup_{x \in A} q^\top x. \]

A. $\delta(\cdot)$ is a convex function.
B. $\delta(\cdot)$ is a linear function.
C. $\delta(\cdot)$ is a concave function.
D. None of the above.

47. If $f(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Statement I: $f(x)f(y) = f(x+y)$.
Statement II: $f(x)$ is invertible for all $x$.

A. Statement I is True, Statement II is False.
B. Statement I is False, Statement II is True.
C. Statement I is True, Statement II is True.
D. Statement I is False, Statement II is False.

48. Suppose $A_1, A_2, \ldots$, is a countably infinite family of subsets of a vector space. Suppose all of these sets are linearly independent, and that $A_1 \subseteq A_2 \subseteq \ldots$. Then $\bigcup_{i=1}^\infty A_i$ is

A. a linearly dependent set of vectors.
B. a linearly independent set of vectors.
C. linearly independent provided the vectors are orthogonal.
D. not necessarily either dependent or independent.

49. If $M = \begin{pmatrix} 5 & -2 \\ 2 & -3 \end{pmatrix}$, then which of the following matrices is equal to $M^{2022}$?

A. $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$.
B. $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$.
C. $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$. 
50. Let \( f : [0, 2] \rightarrow \mathbb{R} \) be a function which is continuous on \([0, 2]\) and its differentiable on \((0, 2)\) with \( f(0) = 1 \). Let
\[
F(x) = \int_0^x f(\sqrt{t})dt
\]
for \( x \in [0, 2] \). If \( F'(x) = f'(x) \) for all \( x \) belongs to \((0, 2)\), then \( F(2) \) equals
\[\text{A. } e^2 - 1.\]
\[\text{B. } e^4 - 1.\]
\[\text{C. } e - 1.\]
\[\text{D. } e^4.\]

51. For positive integer \( n \), define
\[
f(n) = n + \frac{16 + 5n - 3n^2}{4n + 3n^2} + \frac{32 + n - 3n^2}{8n + 3n^2} + \frac{48 - 3n - 3n^2}{12n + 3n^2} + \ldots + \frac{25n - 7n^2}{7n^2}.
\]
Then, the value of \( \lim_{n \to \infty} f(n) \) is equal to
\[\text{A. } 3 + \frac{4}{3} \log_e 7.\]
\[\text{B. } 4 - \frac{3}{4} \log_e \left(\frac{7}{3}\right).\]
\[\text{C. } 4 - \frac{4}{3} \log_e \left(\frac{7}{3}\right).\]
\[\text{D. } 3 + \frac{3}{4} \log_e 7.\]

52. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be defined by \( f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4} \). Then which of the following statements is TRUE?
\[\text{A. } f \text{ is decreasing in the interval } (-2, -1).\]
\[\text{B. } f \text{ is decreasing in the interval } (1, 2).\]
\[\text{C. } f \text{ is onto.}\]
\[\text{D. } f \text{ is increasing in the interval } (-2, -1).\]

53. The function \( f(x) = \log(1 + \exp(x)) \) is
\[\text{A. Concave.}\]
\[\text{B. Convex.}\]
\[\text{C. Convex for } x < 0 \text{ and concave for } x > 0.\]
\[\text{D. Neither concave nor convex.}\]

54. Suppose year 1 is taken as the base period. The nominal GDP in year 1 is Rs 500 billion. The nominal GDP has reached Rs 700 billion in year 2, and the GDP deflator is 125. What will the real GDP be each year?
55. An important assumption underlying the Classical monetary theory is the idea that an increase in the money supply causes a proportional increase in the price level because the:
   A. Marginal propensity to consume is constant
   B. Money supply is continuous
   C. Exchange rate is fixed
   D. Velocity of money is constant

56. A liquidity trap refers to a situation when:
   A. The economy reaches the full employment level
   B. A rise in interest rates causes people to want to hold less money
   C. Households’ wealth becomes trapped in assets that cannot be easily exchanged into money
   D. The general public has a strong preference for holding the most liquid asset, money

57. Suppose the investment (I) in the goods market is not responsive to the interest rate. Then:
   A. The IS curve is a vertical line and monetary policy is very effective in raising output
   B. The IS curve is a horizontal line and monetary policy is very effective in raising output
   C. The IS curve is a vertical line and monetary policy does not affect output
   D. The IS curve is a horizontal line and monetary policy does not affect output

58. Assume a Cobb-Douglas production function where the output is $Y$, the share of capital ($K$) is 1/4 and the share of labor ($L$) is 3/4. If the growth in total factor productivity is zero and labor and capital each grow by 1%, then
   A. Output growth is 1% and the marginal product of capital is $Y/4K$
   B. Output growth is 1% and the marginal product of capital is $Y/K$
   C. Output growth is 1% and the marginal product of labor is $Y/4N$
   D. Output growth is 2% and the marginal product of labor is $3Y/4N$

59. In an IS-LM framework, an increase in autonomous saving will result in
A. A decrease in income but an increase in the interest rate
B. A decrease in both income and the interest rate
C. An increase in both income and the interest rate
D. An increase in income but a decrease in interest rates

60. Assume an aggregate production function with a constant marginal product of capital and with capital as the only factor of production, such that $Y = aK$. If the rate of population growth is $n = 0.03$, the rate of depreciation is $d = 0.05$, the savings rate is $s = 0.08$ and $a = 1.5$, then the growth rate of per capita output is

A. 0.04
B. 0.004
C. 0.40
D. 0.20