


DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
MTL101 (LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS)
2018-19 SECOND SEMESTER TUTORIAL SHEET-VIII

1. Which of the following functions are piecewise continuous? Which are of exponential order?

(a) t^2 (b) e^{t^2} (c) e^{3t-2} (d) e^{t^3} (e) $e^{1/t}$ (f) $t \sin t$ (g) $\ln(t+1)$

{Ans: EO (a),(c),(f),(g)}

2. Find the Laplace transform of the following:

(a) $2t^2 - 6t + 8$ (b) $\sinh 3t$ (c) $\sin^2 t$ (d) \sqrt{t} (e) $\frac{\sin 2t}{2}$ (f) $t^3 e^{-3t}$ (g) $e^t \sin^2 t$ (h) $\cosh at \cos at$

(i) $\sinh at \cos at$ (j) $\cos t \cos 2t$ (k) $t^n e^{at}$ (l) $e^{-2t}(\cos t + 6 \sin t)$ (m) $te^{at} \sin at$

{Ans: (a) $4s^3 - 6s^2 + 8s^{-1}$ (b) $3/(s^2 - 9)$ (c) $2/s(s^2 + 4)$

(d) $\sqrt{\pi}/2s^{1.5}$ (e) $1/(s^2 + 4)$ (f) $6/(s + 3)^3$

(g) $\frac{1}{2(s-1)} - \frac{s-1}{2(s-1)^2+8}$ (j) $\frac{s^2+5s}{(s^2+1)(s^2+9)}$ (m) $\frac{2as}{(s^2+a^2)^2}$ }

3. Find $f(t)$ whose Laplace transforms are the following:

(a) $\frac{s}{s^2+\pi}$ (b) $\frac{s+5}{s^2+2s+5}$ (c) $\frac{1}{s(s^2+4)}$ (d) $\frac{1}{s^2(s^2-4)}$ (e) $\frac{2s-\pi}{s(s-\pi)}$ (f) $\frac{\pi^2}{s^4(s^2+\pi^2)}$ (g) $\frac{s^3}{s^4+64}$ (h) $\frac{1}{\sqrt{s}}$

(i) $\ln\left(1 + \frac{1}{s^2}\right)$ (j) $\frac{se^{-4s}}{(3s+2)(s-2)}$ (k) $\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9}$ (l) $\frac{5}{s^2} + \left(\frac{\sqrt{s-1}}{s}\right)^2 - \frac{7}{3s+2}$

{ Ans: (a) $\cos(\sqrt{\pi}t)$ (b) $e^{-t}(\cos 2t + 2 \sin 2t)$ (c) $0.5 \sin^2 t$

(d) $(1/8)(\cosh 2t - 2t)$ (g) $\cosh 2t \cos 2t$

(i) $2t^{-1}(1 - \cos t)$ (j) $(1/12)e^{(-2/3)t} + (1/4)e^{2t}$

(k) $3e^{1.5t} - 0.25 \sinh(4/3)t + (4/9) \cosh(4/3)t + (2/3) \sin(3/4)t - (3/8) \cos(3/4)t$

4. Find the Laplace transform of the following functions:

(a)

$$f(t) = \begin{cases} \sin t & t \in (0, \pi) \\ 0 & t \geq \pi \end{cases}$$

(b)

$$f(t) = \begin{cases} (t-1)^2 & t > 1 \\ 0 & t \in (0, 1) \end{cases}$$

(c)

$$f(t) = \begin{cases} e^t & t \in (0, 5) \\ 3 & t > 5 \end{cases}$$

(d)

$$f(t) = \begin{cases} \cos(t - \frac{2}{3}) & t > \frac{2}{3} \\ 0 & t < \frac{2}{3} \end{cases}$$



{ Ans: (a) $\frac{1+e^{-\pi s}}{1+s^2}$ (c) $\frac{e^{5(1-s)}-1}{1-s} + 3\frac{e^{-5s}}{s}$ }

5. Find (a) $\mathcal{L}\left(\frac{e^{-3t} \sin 2t}{t}\right)$ (b) $\mathcal{L}\left(e^{-3t} \int_0^t \frac{\sin 2t}{t} dt\right)$
 {Ans: (a) $\cot^{-1} \frac{s+3}{2}$ (b) $\frac{1}{s+3} \tan^{-1} \frac{2}{s+3}$ }

6. Using Laplace transform solve the following IVP's:

(a) $4y'' + \pi^2 y = 0, y(0) = 2, y'(0) = 0$

(b) $y'' - 2y' - 3y = 0, y(0) = 2, y'(0) = 0$

(c) $y'' - ky' = 0, y(0) = 2, y'(0) = k$

(d) $y^{(4)} - y = 1, y(0) = y'(0) = y''(0) = y'''(0) = 0$

(e) $y''' + 2y'' - y' - 2y = 0, y(0) = 1, y'(0) = y''(0) = 4$

{Ans: (b) $0.5e^{3x} + 1.5e^{-x}$ (d) $0.25(e^x + e^{-x}) + 0.5 \cos x - 1$
 (e) $3e^x - 3e^{-x} + e^{-2x}$ }

7. Find Laplace transform inverse of the following:

(a) $\frac{s+12}{s^2+4s}$ (b) $\frac{3s^2-2s-1}{(s-2)(s^2+1)}$ (c) $\frac{s^3-2s^2+14s-9}{(s-1)^2(s-2)^3}$

{Ans: (a) $3 - 2e^{-4t}$ (b) $(7/5)e^{2t} + (8/5) \cos t + (6/5) \sin t$
 (c) $-(5 + 4t)e^t + (20 + 19t)\frac{te^{2t}}{2}$ }

8. Using Laplace transforms solve the following differential equations:

(a) $y'' + 4y' + 5y = \delta(t-1); y(0) = 0, y'(0) = 3$

(b) $y'' + 16y = 4\delta(t-\pi), y(0) = 2, y'(0) = 0$

(c) $y'' + 9y = 8 \sin t$, if $0 < t < \pi$ and 0 if $t > \pi, y(0) = 0, y'(0) = 4$

{Ans: (a) $3e^{-2t} \sin t, 0 < t < 1; e^{-2(t-1)} \sin(t-1) + 3e^{-2t} \sin t, t > 1$
 (c) $\sin t + \sin 3t, t < \pi; (4/3) \sin 3t, t > \pi$ }

9. Using the Laplace of integrals, find the Laplace transform inverse of the following functions:

(a) $\frac{1}{s} \left(\frac{s-a}{s+a}\right)$ (b) $\frac{s}{(s^2+4)^2}$ (c) $\ln \frac{s^2+1}{(s-1)^2}$ (d) $\ln \left(\frac{s+a}{s+b}\right)$ (e) $\cot^{-1} \left(\frac{s}{w}\right)$

{Ans: (b) $\frac{t \sin 2t}{4}$ (d) $\frac{e^{-bt} - e^{-at}}{t}$ (e) $\frac{\sin \omega t}{t}$ }