JRL302: Robotics Technologies

Lecture 5: Potential functions

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Potential fields

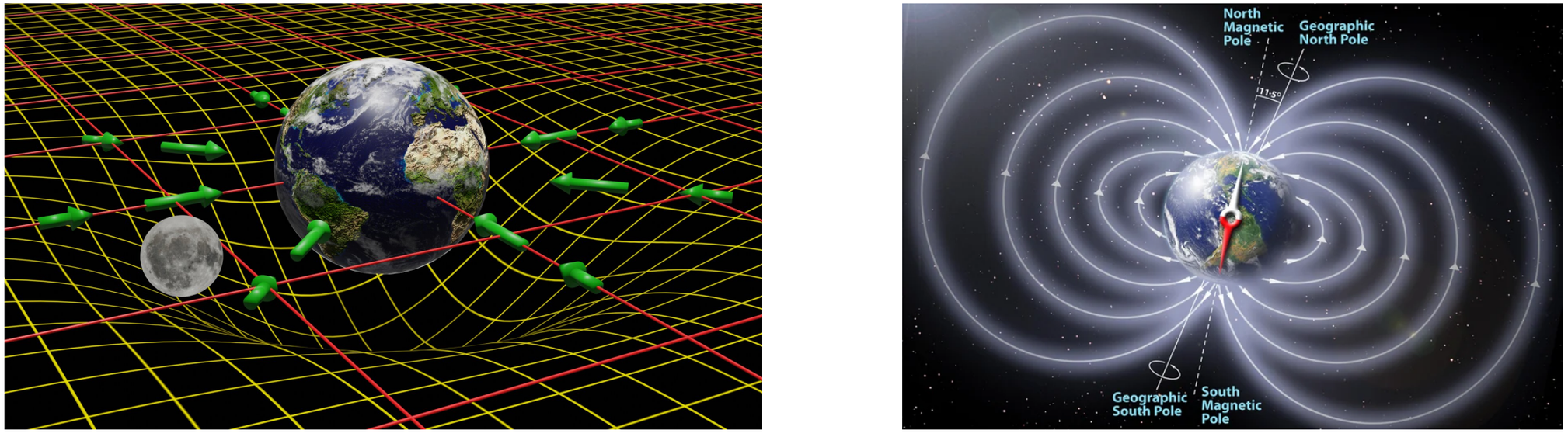


Fig: Potential energy fields: gravitational and magnetic fields

Potential field: $P(q)$

Induced force: $F = -\dfrac{\partial P}{\partial q}$

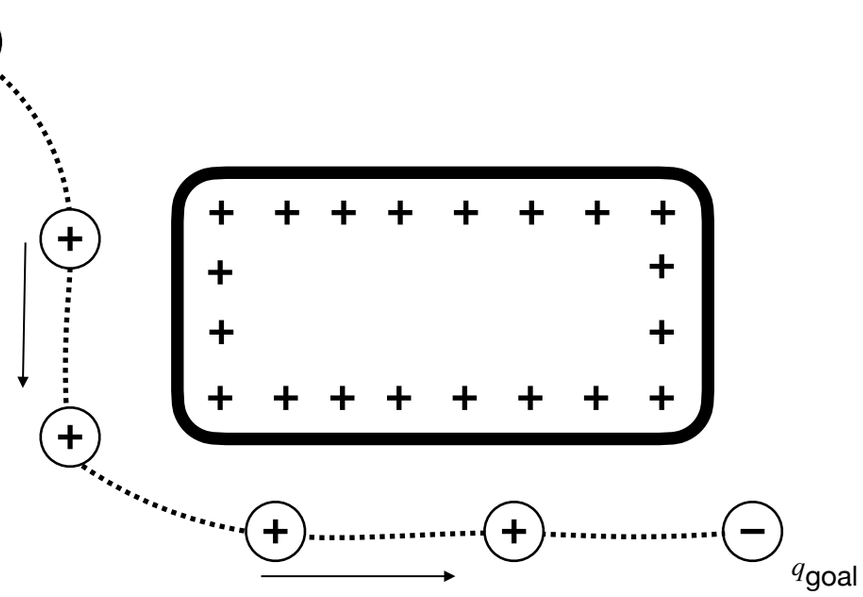
Eg: h height above earth, m mass, potential energy is $P(h) = mgh$

Force acting $F = -\dfrac{\partial P}{\partial h} = -mg$

Virtual Potential fields

Suppose the goal is q_{goal}
The robot is at $q(t)$

Create a potential field that the q_{goal} is attractive from every other point in $\mathscr{C}$



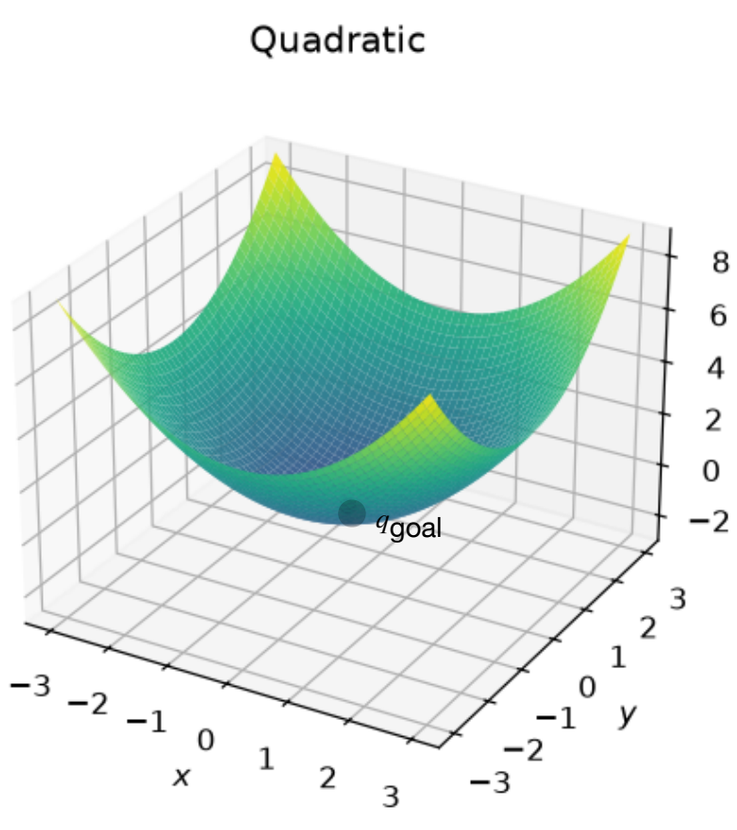
Virtual Potential fields

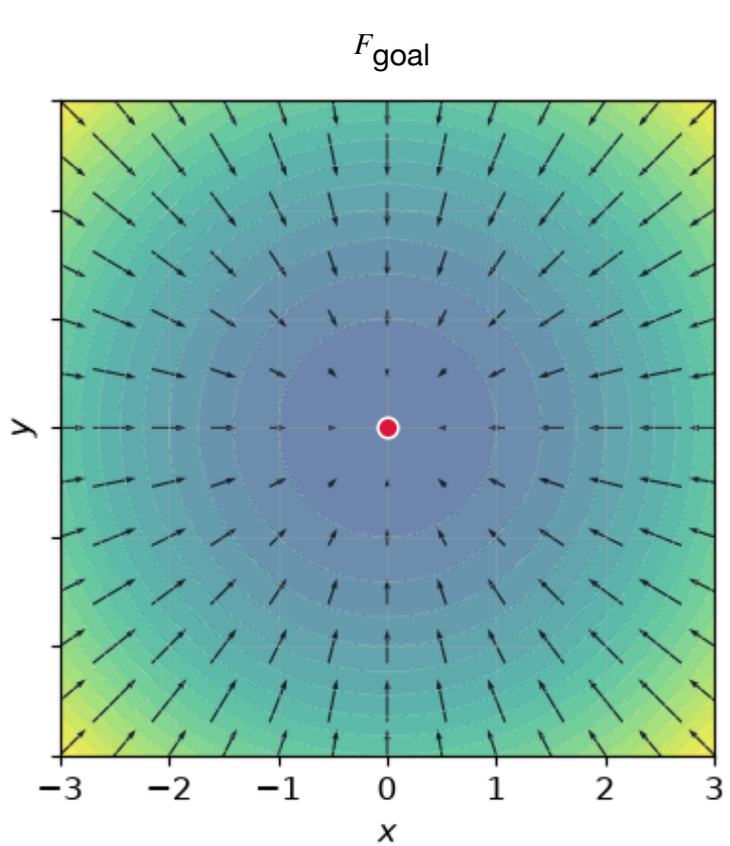
Quadratic potential energy with zero energy at the goal

$$P_{\text{goal}}(q) = \frac{1}{2}(q - q_{\text{goal}})^{\mathrm{T}} K (q - q_{\text{goal}}),$$

The force induced by this potential is

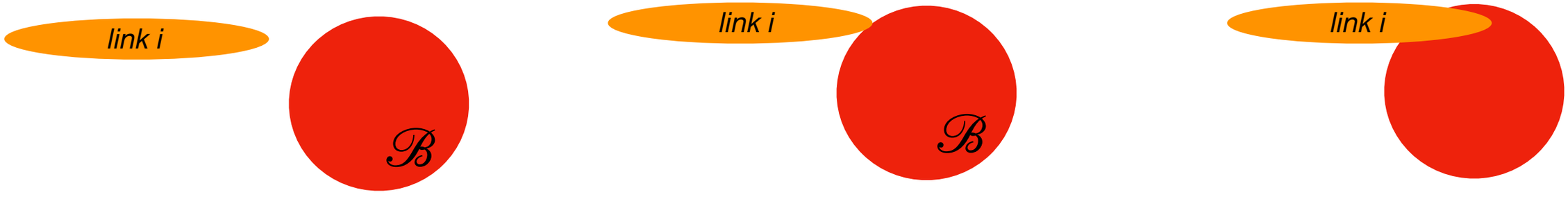
$$F_{\text{goal}}(q) = -\frac{\partial P_{\text{goal}}}{\partial q} = K(q_{\text{goal}} - q),$$



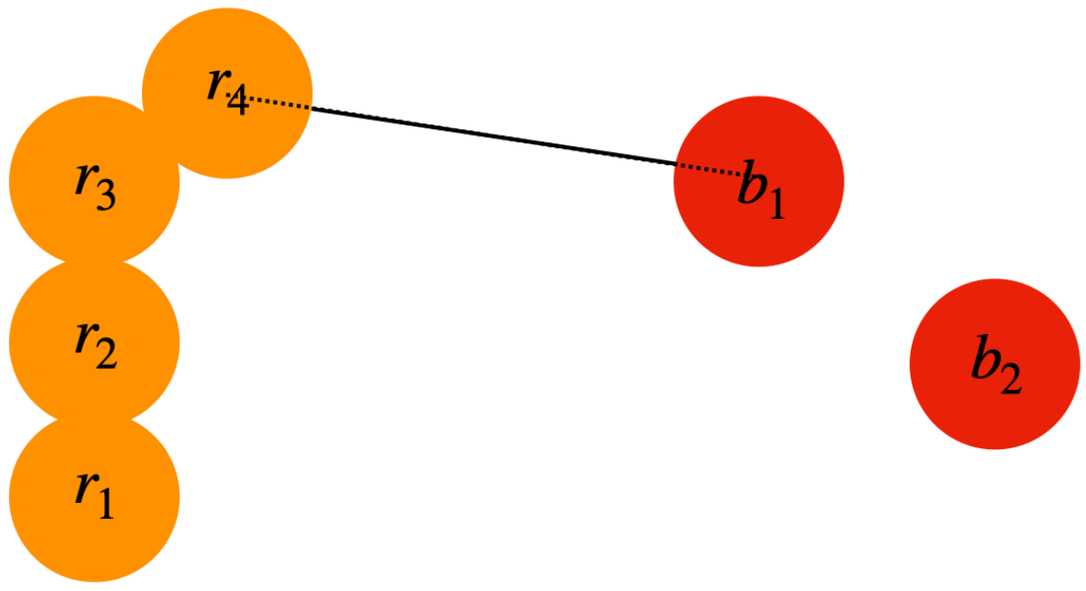


Virtual Potential fields

$d(q, \mathscr{B}) > 0$ $d(q, \mathscr{B}) = 0$ $d(q, \mathscr{B}) < 0$



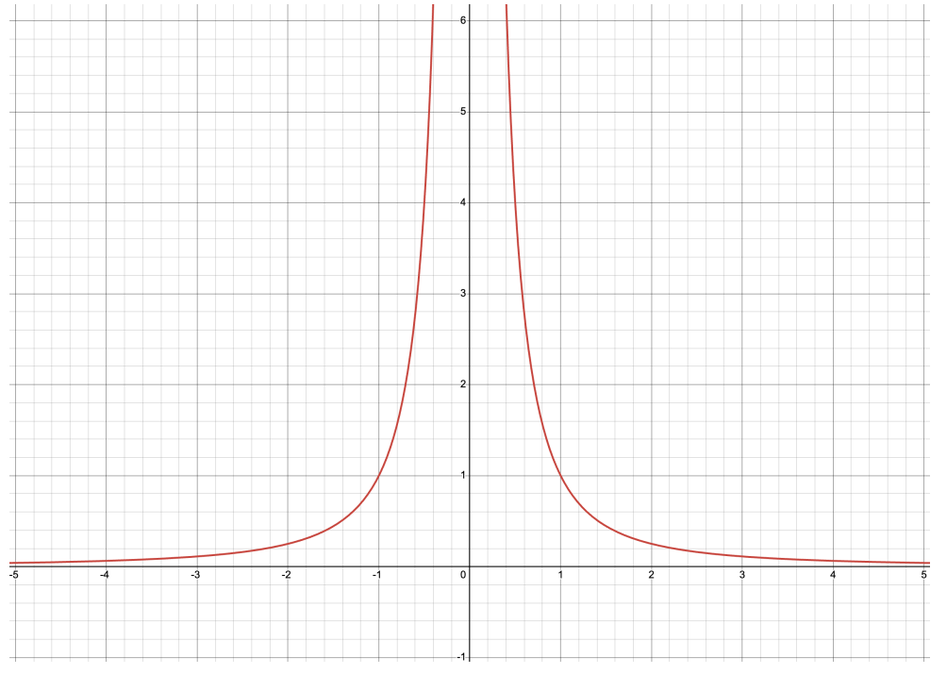
Minimum separation distance $d(q, \mathscr{B}) = \min_{i,j} \left(\|r_i(q) - b_j\| - R_i - B_j \right)$



Virtual Potential fields

Repulsive potential energy calculated from the distance to obstacle

$$P_{\mathcal{B}}(q) = \frac{k}{2d^2(q, \mathcal{B})},$$



Force induced by the obstacle potential is

$$F_{\mathcal{B}}(q) = -\frac{\partial P_{\mathcal{B}}}{\partial q} = \frac{k}{d^3(q, \mathcal{B})}\frac{\partial d}{\partial q}.$$

The total potential is obtained by summing the attractive goal potential and repulsive obstacle potential

$$P(q) = P_{\text{goal}}(q) + \sum_i P_{\mathcal{B}_i}(q)$$

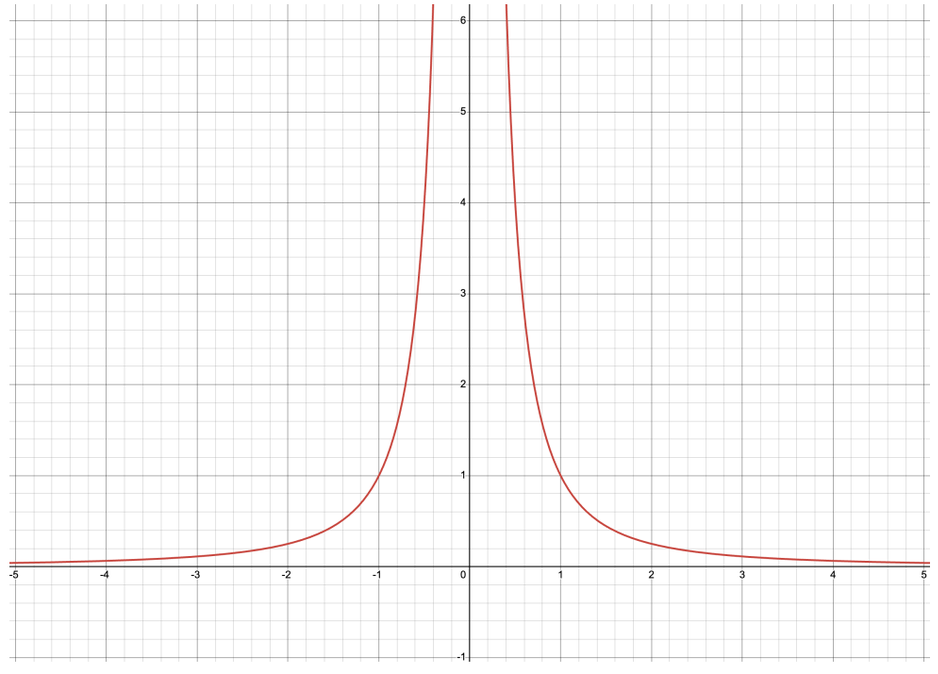
yielding a total force

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Virtual Potential fields

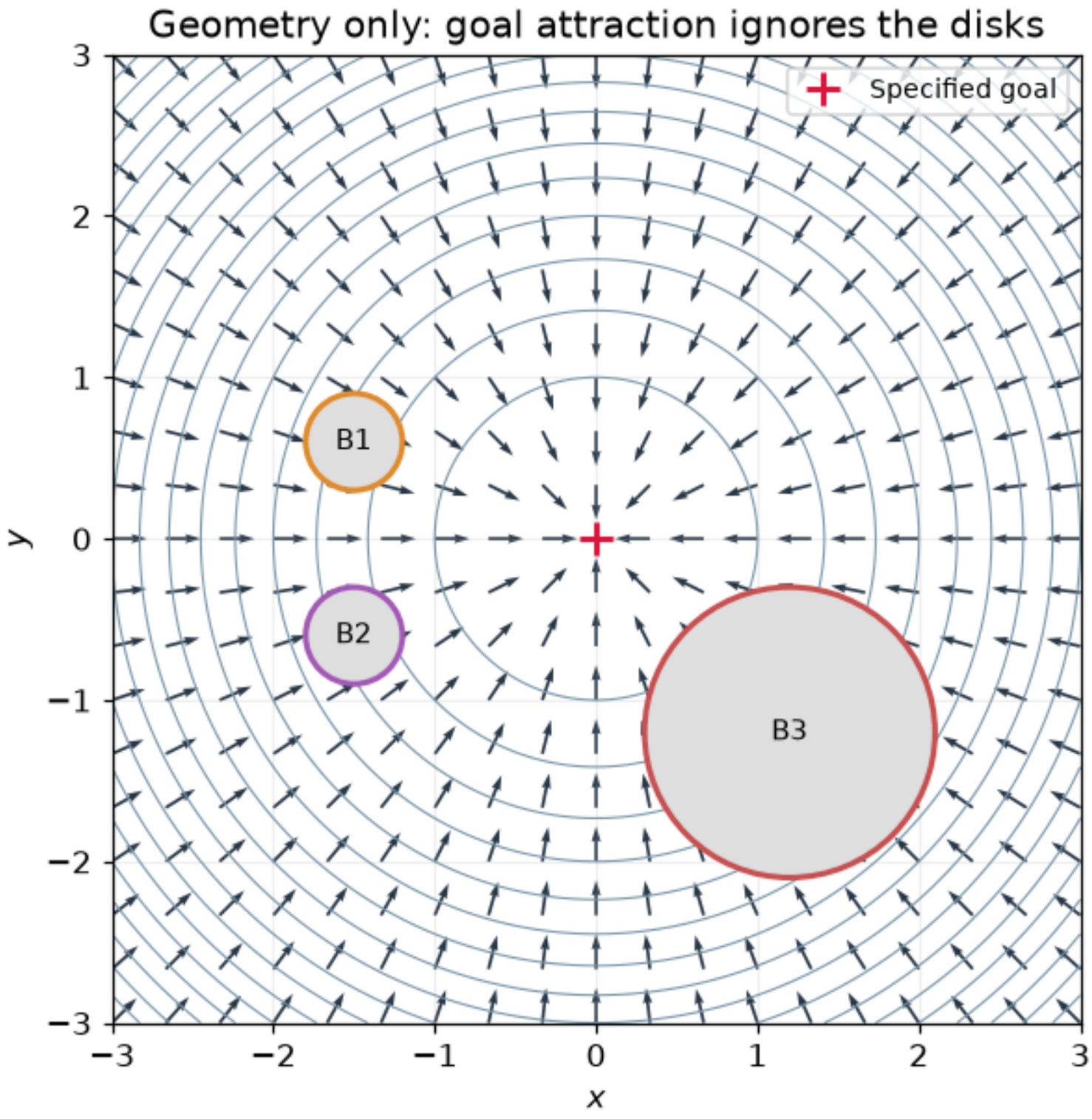
Control input $u = F(q) + B\dot{q}$

Command velocity $\dot{q} = F(q)$

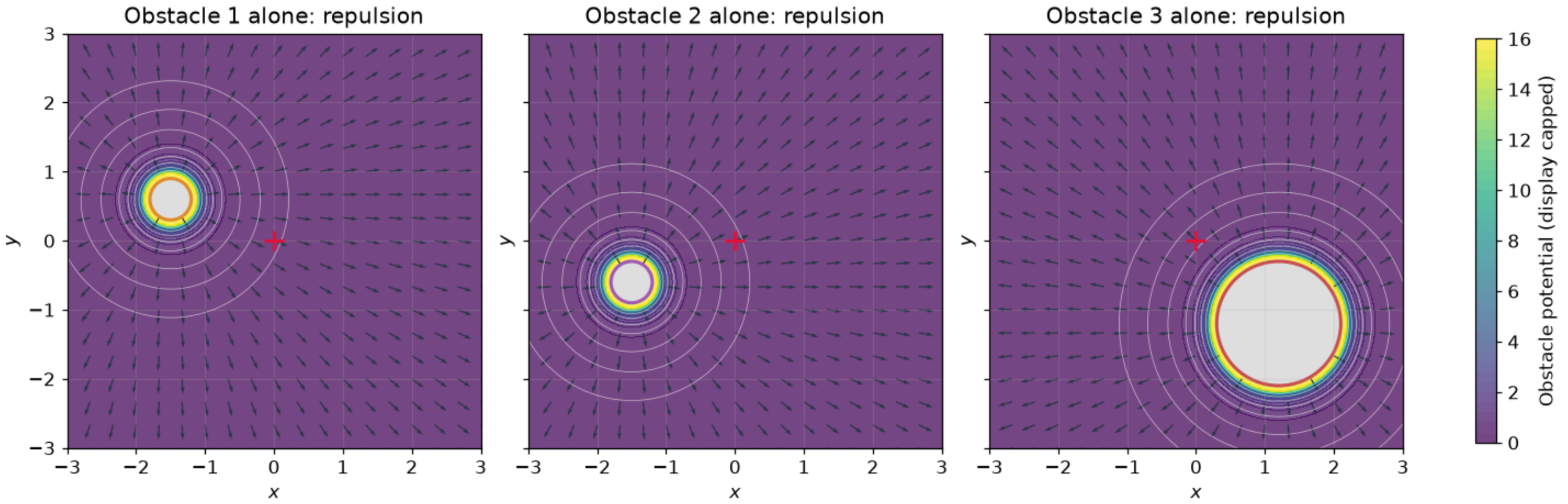
Range of influence of the obstacle

$$U_{\mathscr{B}}(q) = \begin{cases} \frac{k}{2}\left(\frac{d_{\text{range}} - d(q,\mathscr{B})}{d_{\text{range}} d(q,\mathscr{B})}\right)^2 & \text{if } d(q,\mathscr{B}) < d_{\text{range}} \\ 0 & \text{otherwise.} \end{cases}$$

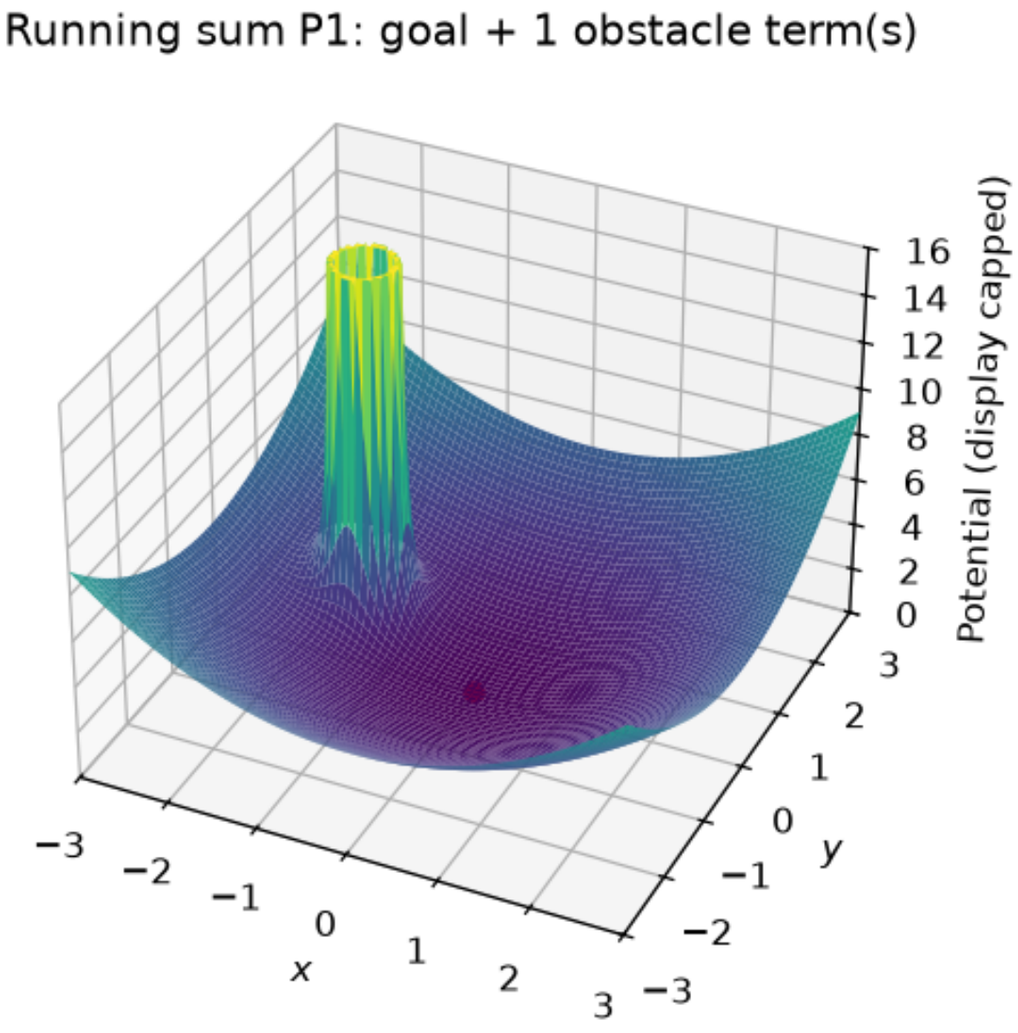
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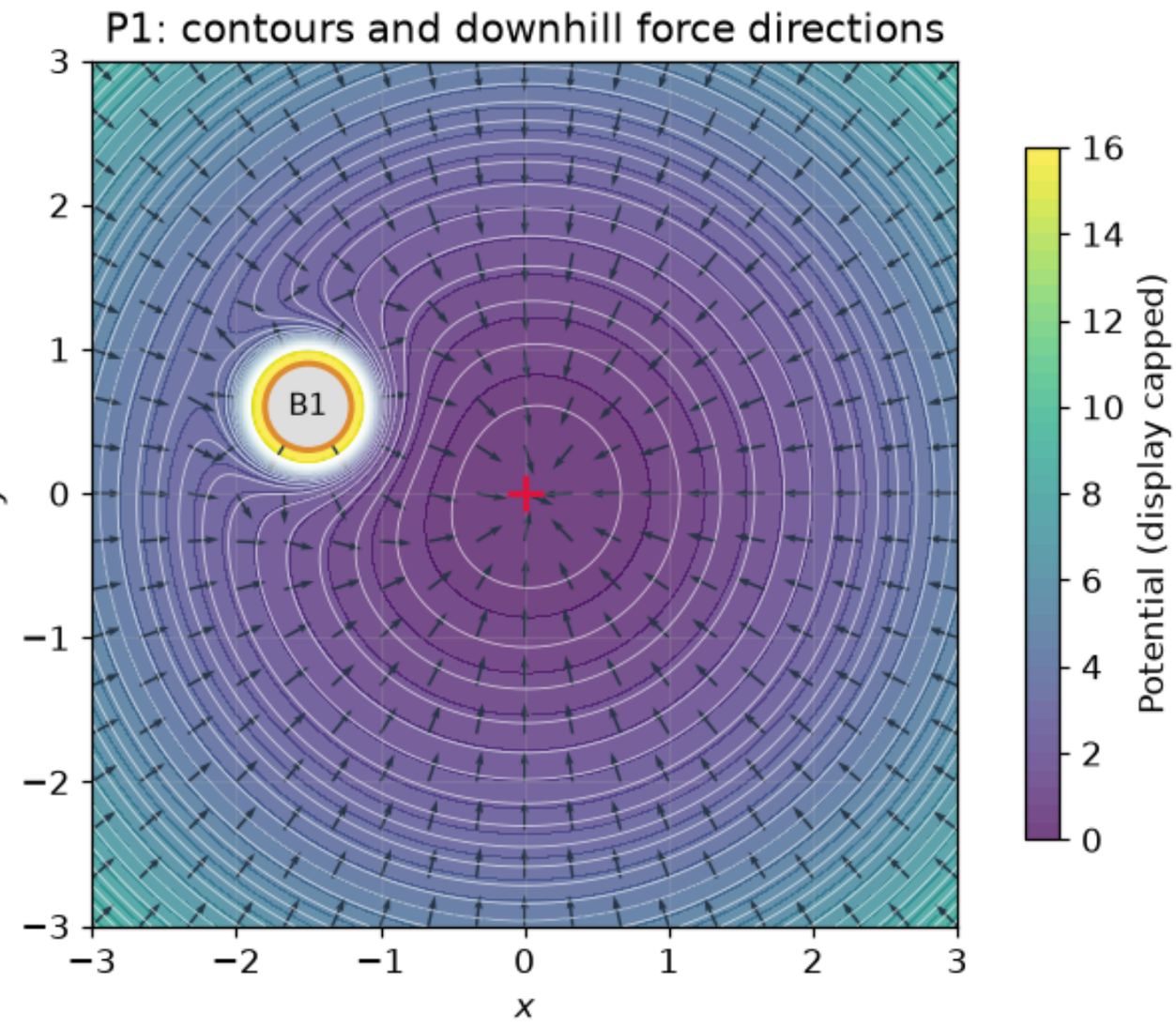


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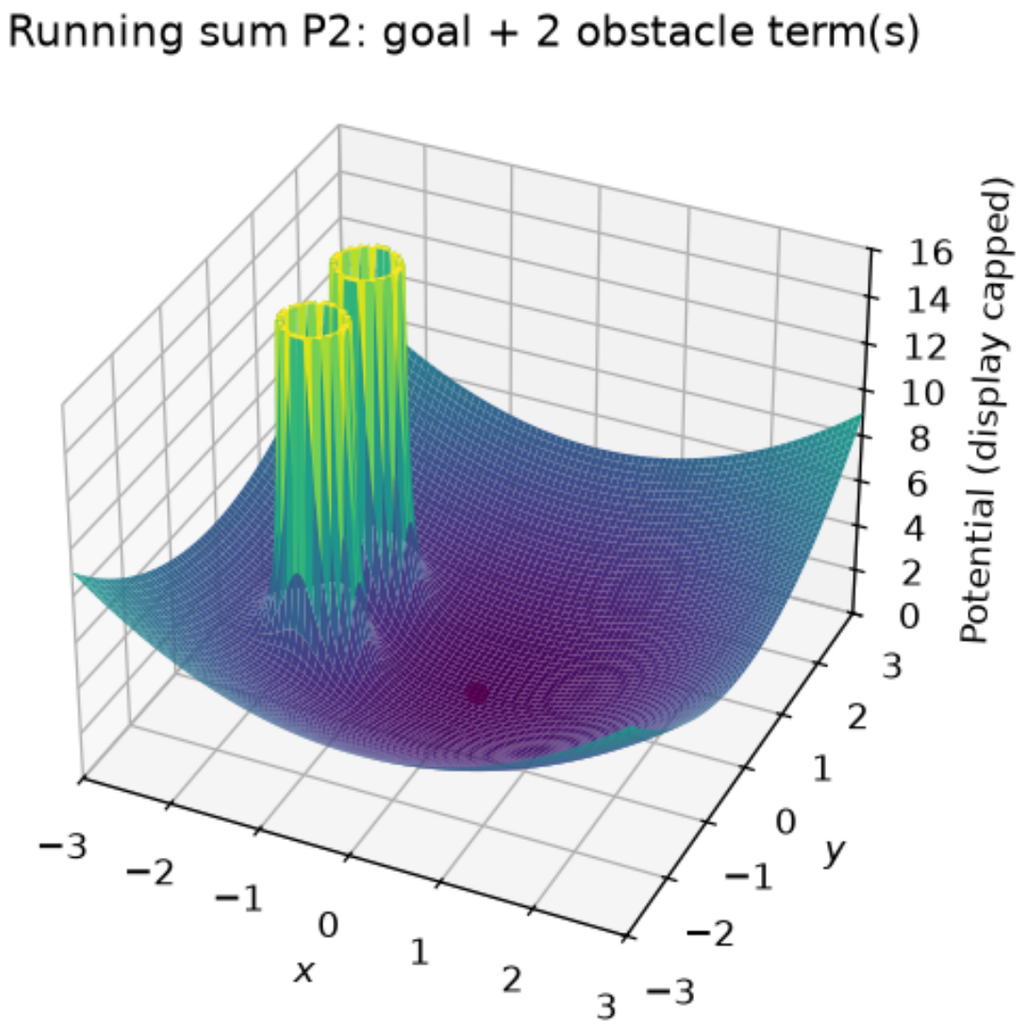


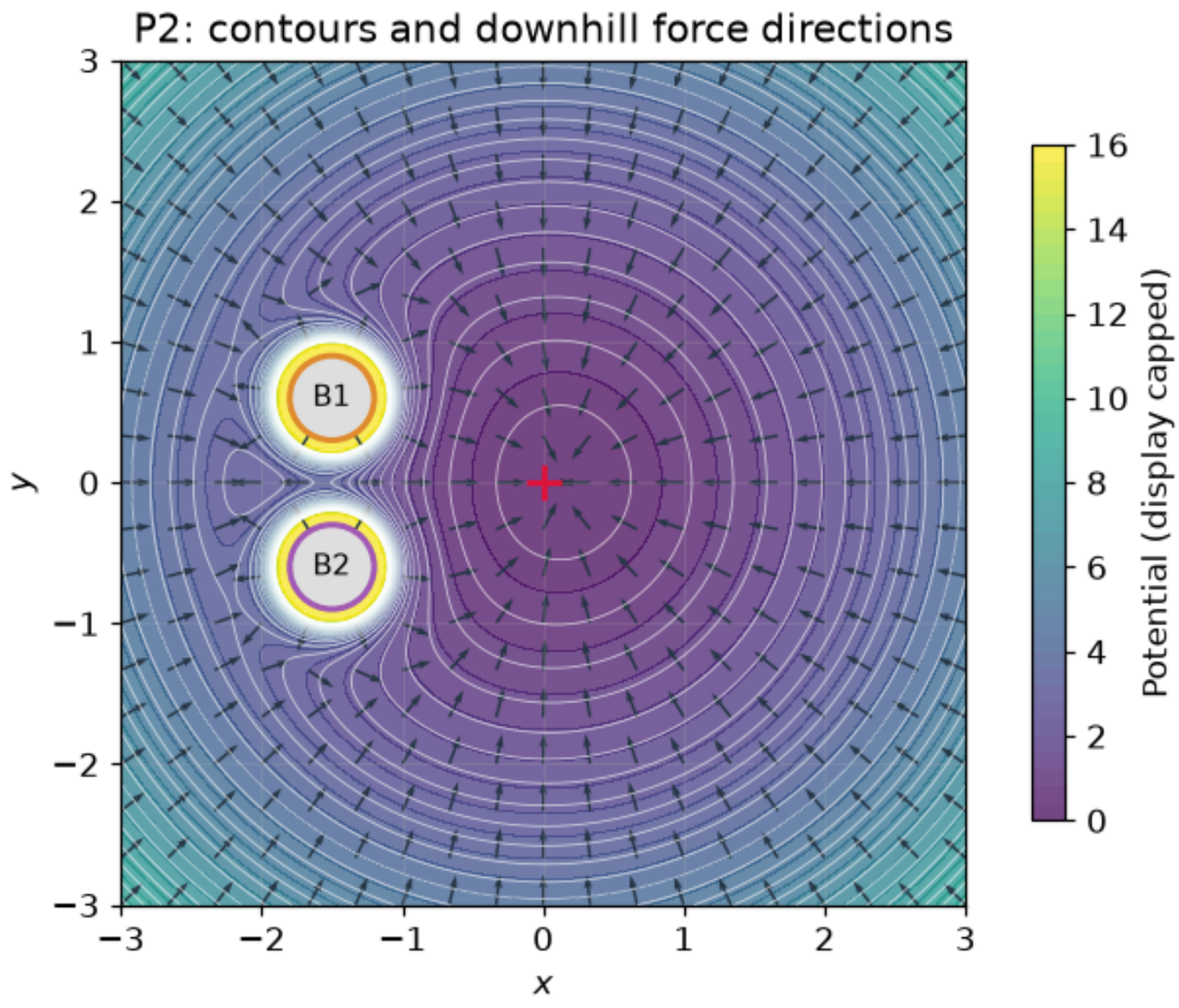
Virtual Potential fields



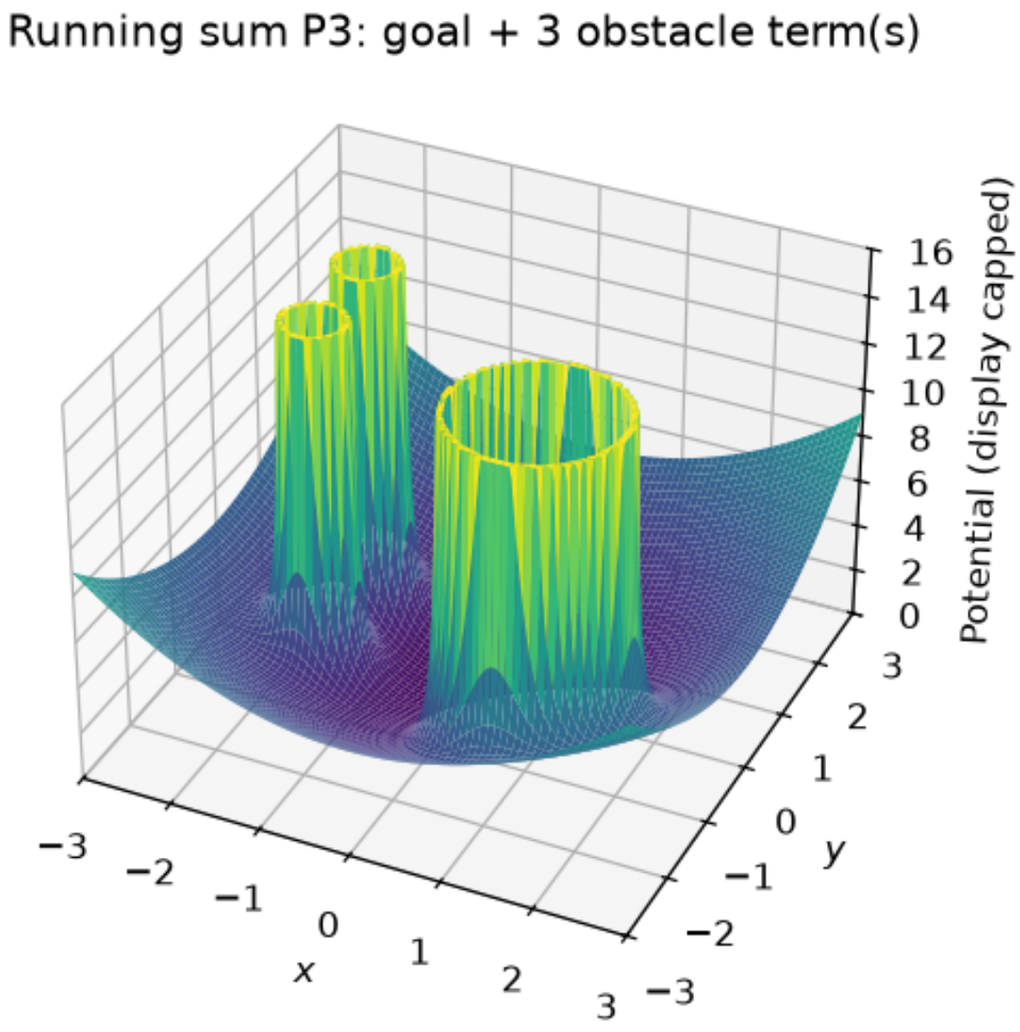


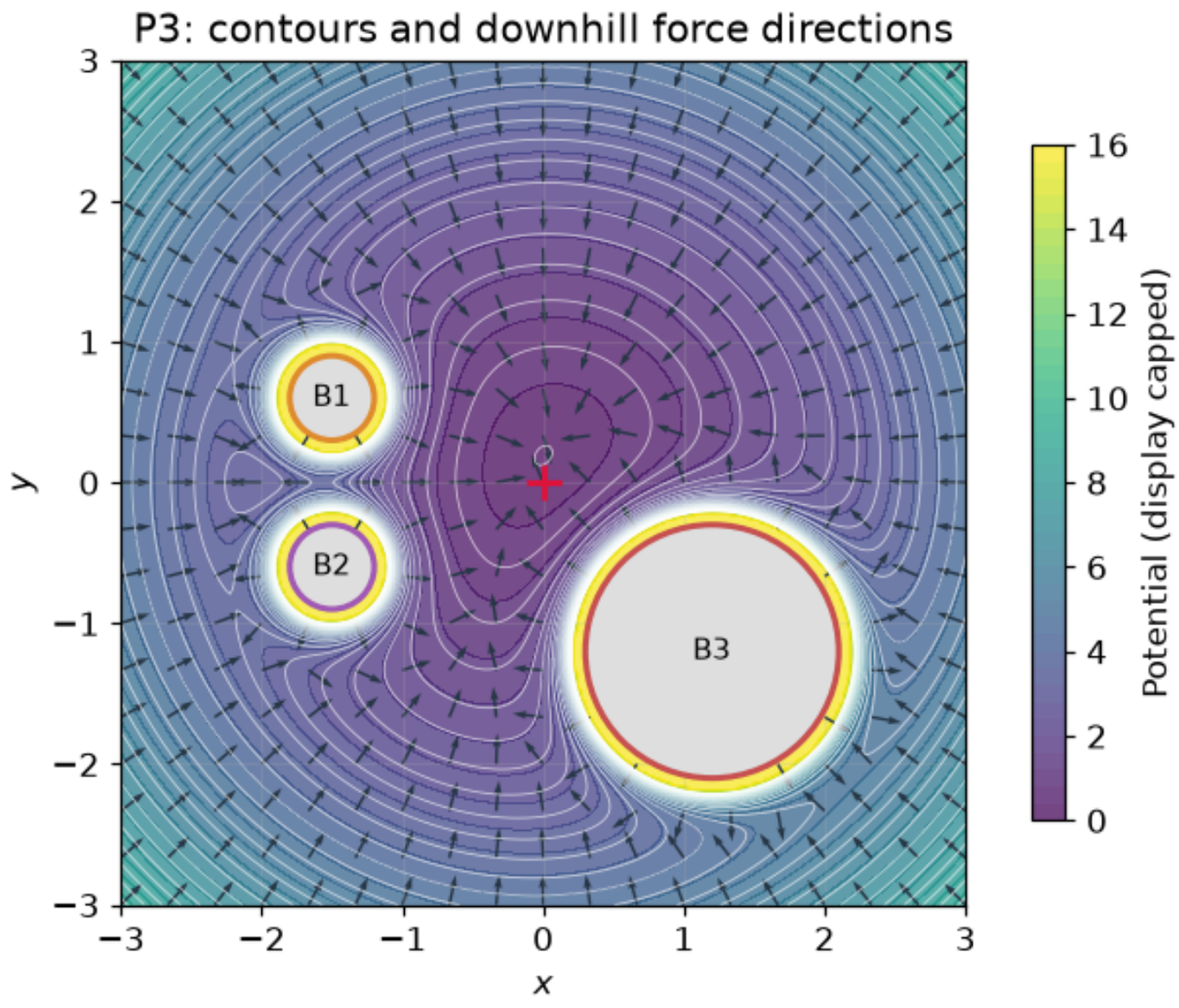
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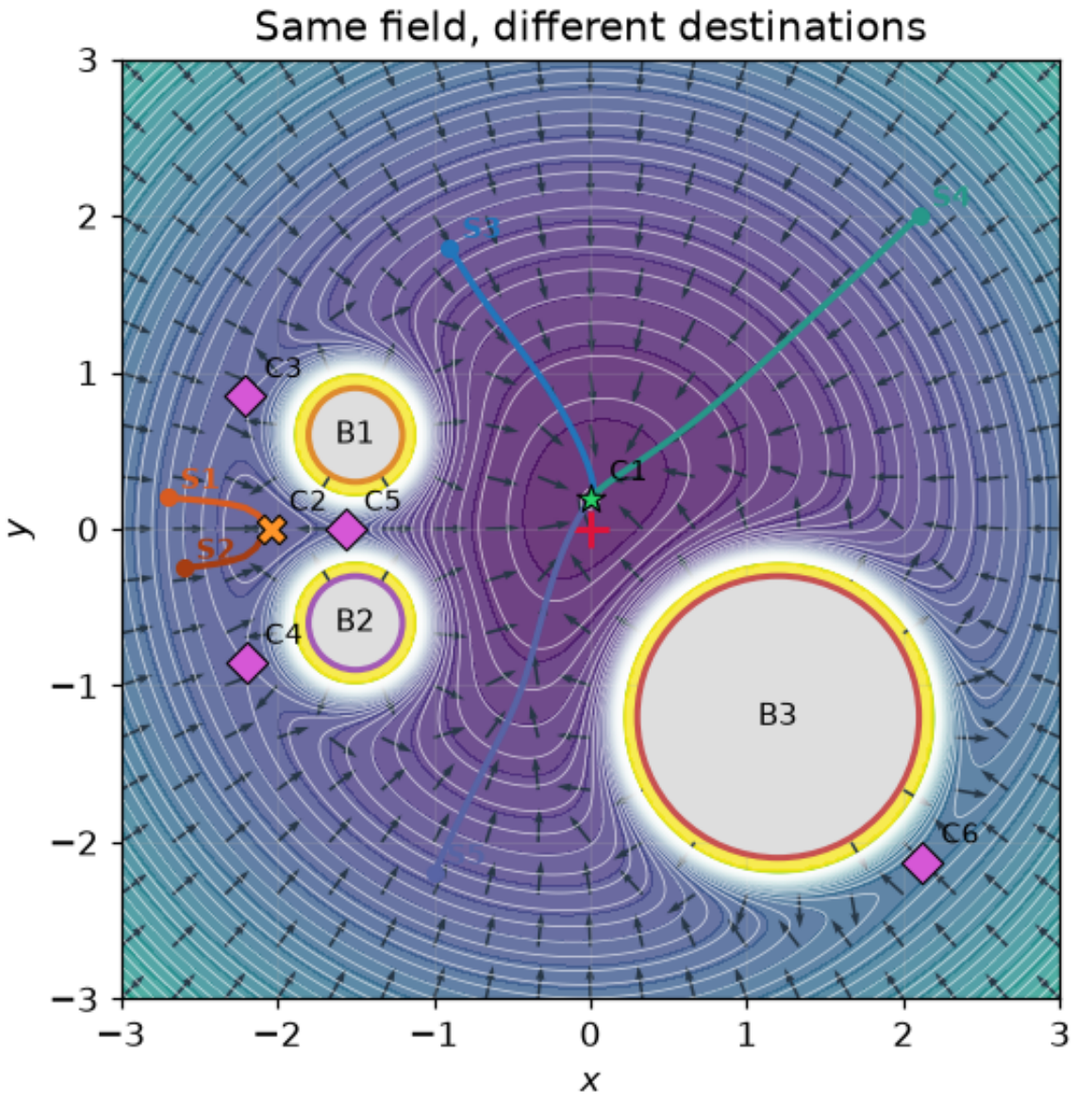


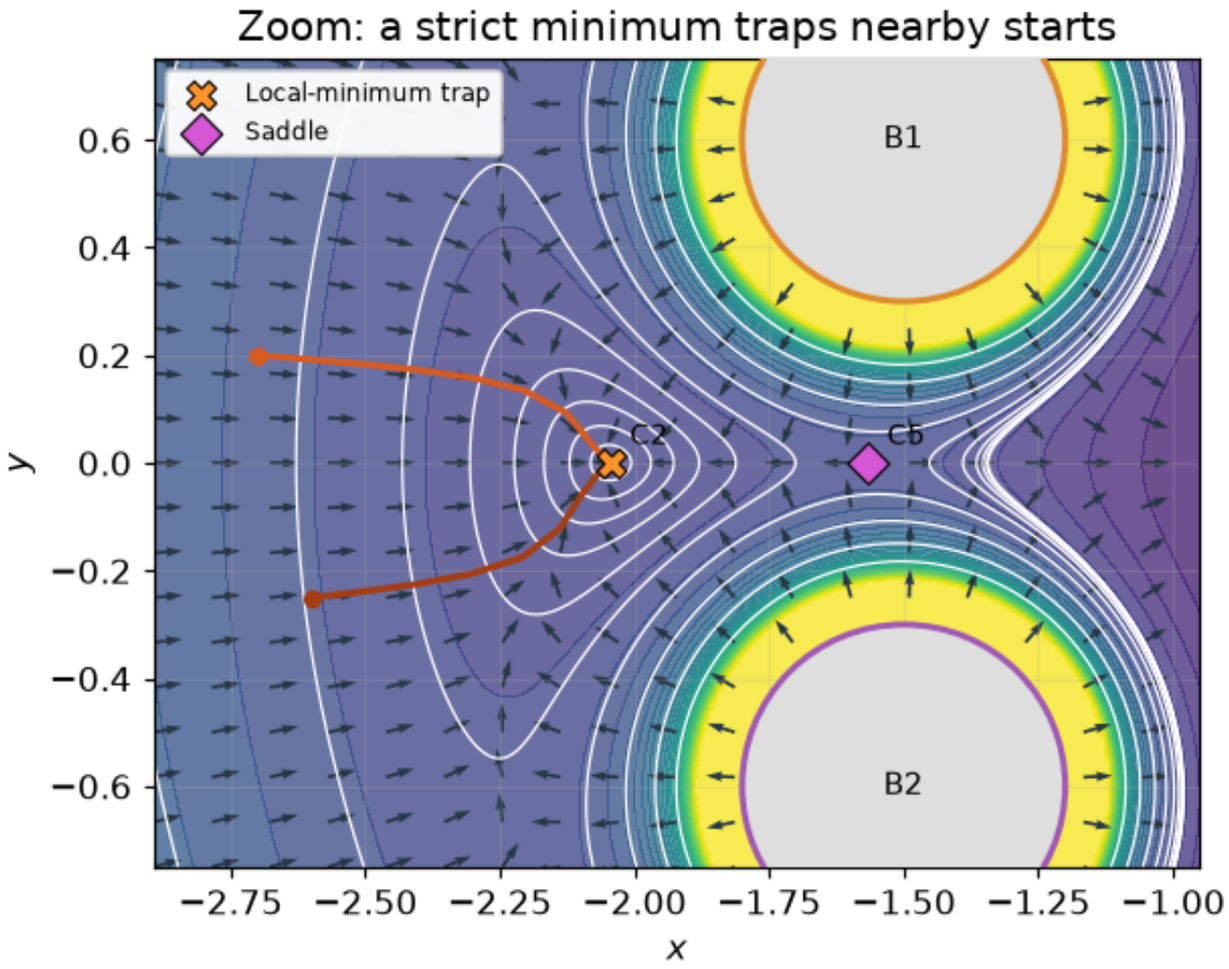
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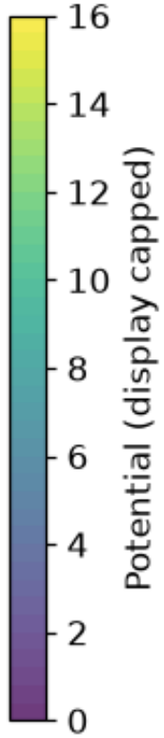




Virtual Potential fields









Navigation functions

A navigation function $\varphi(q)$ is a type of virtual potential function with properties,

1. is smooth (or at least twice differentiable) on q;
2. has a bounded maximum value (e.g., 1) on the boundaries of all obstacles;
3. has a single minimum at q_{goal}; and
4. has a full-rank Hessian $\partial^2\varphi/\partial q^2$ at all critical points q where $\partial\varphi/\partial q = 0$ (i.e., $\varphi(q)$ is a **Morse** function).

Navigation functions

Step 1: Normalise the geometry and define the goal function

$$\gamma(q) = \|q - q_g\|^2$$

Step 2: Define the obstacle and workspace functions

$$\beta_i(q) = \|q - c_i\|^2 - r_i^2, \qquad \beta_0(q) = R^2 - \|q\|^2$$

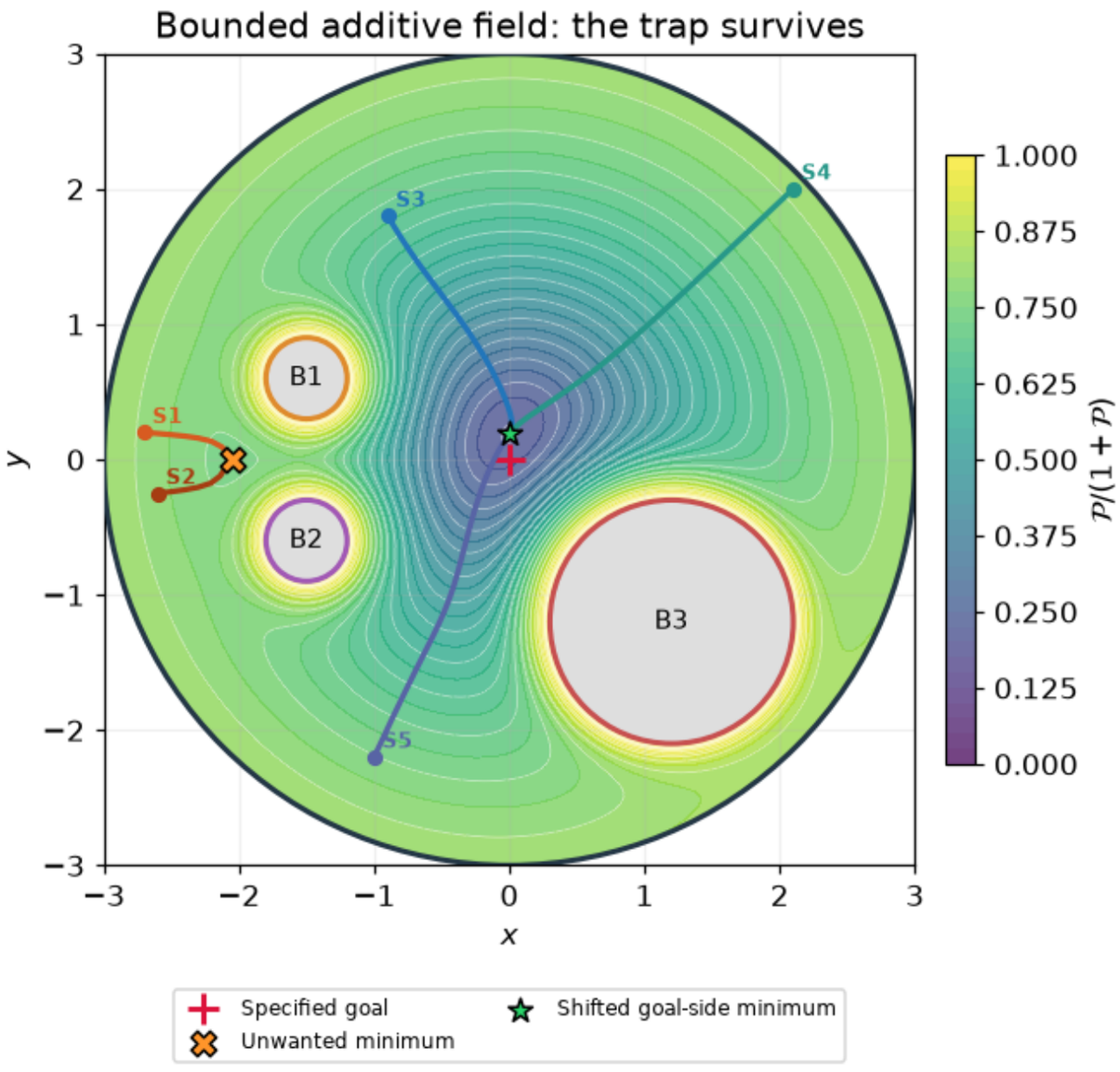
Step 3: Construct the navigation function

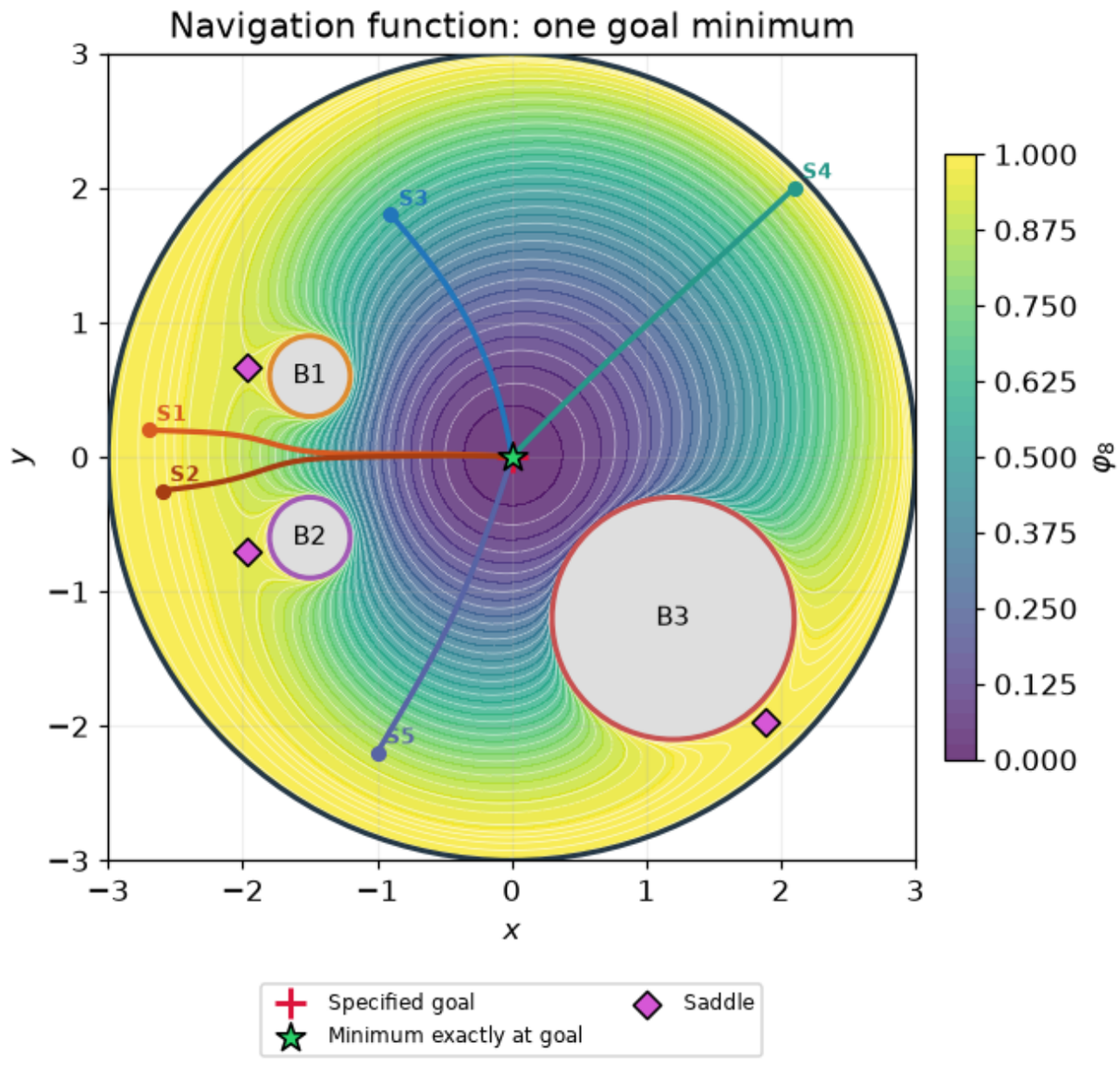
$$\boxed{\varphi_\kappa(q) = \frac{\gamma(q)}{\left(\gamma(q)^\kappa + \beta(q)\right)^{1/\kappa}}}$$

Step 4: Apply gradient based control

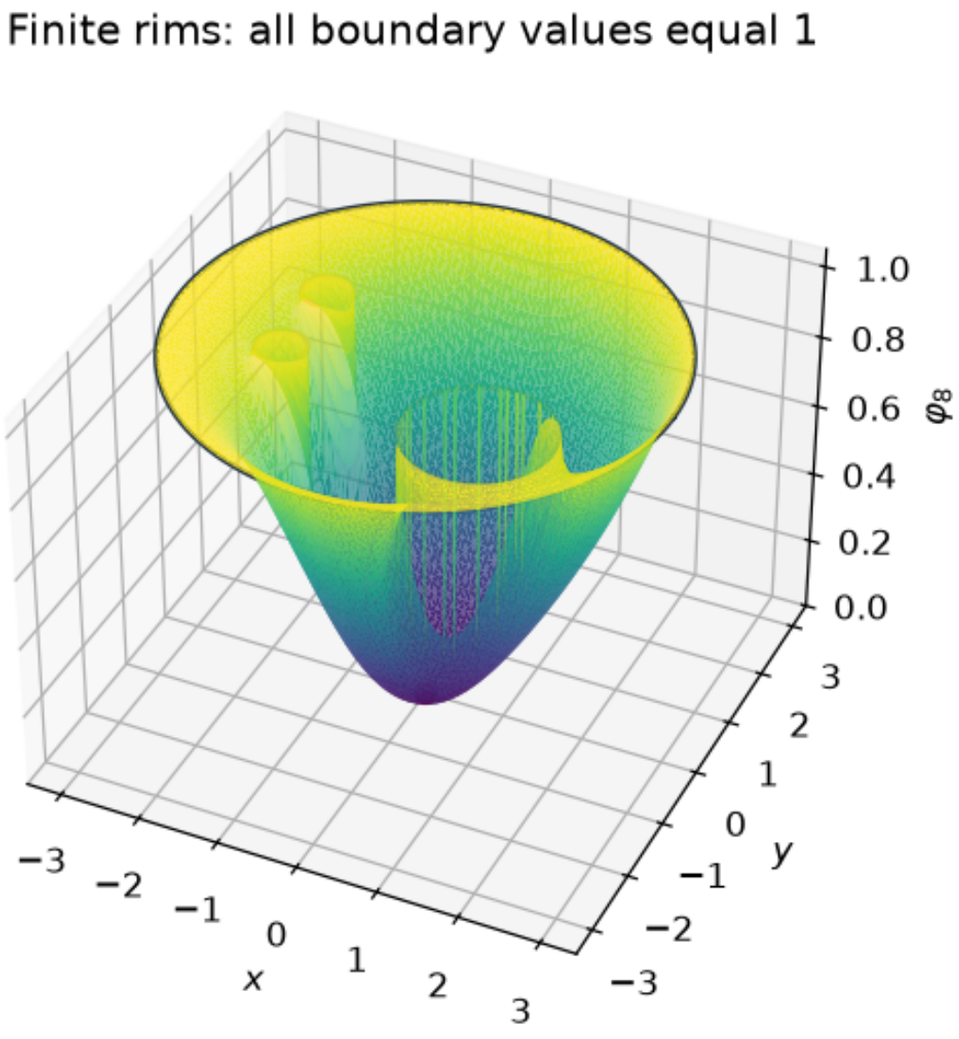
$$\boxed{\dot{q} = -\nabla\varphi_\kappa(q) = -\frac{\beta\nabla\gamma - \frac{\gamma}{\kappa}\nabla\beta}{(\gamma^\kappa + \beta)^{1+1/\kappa}}}$$

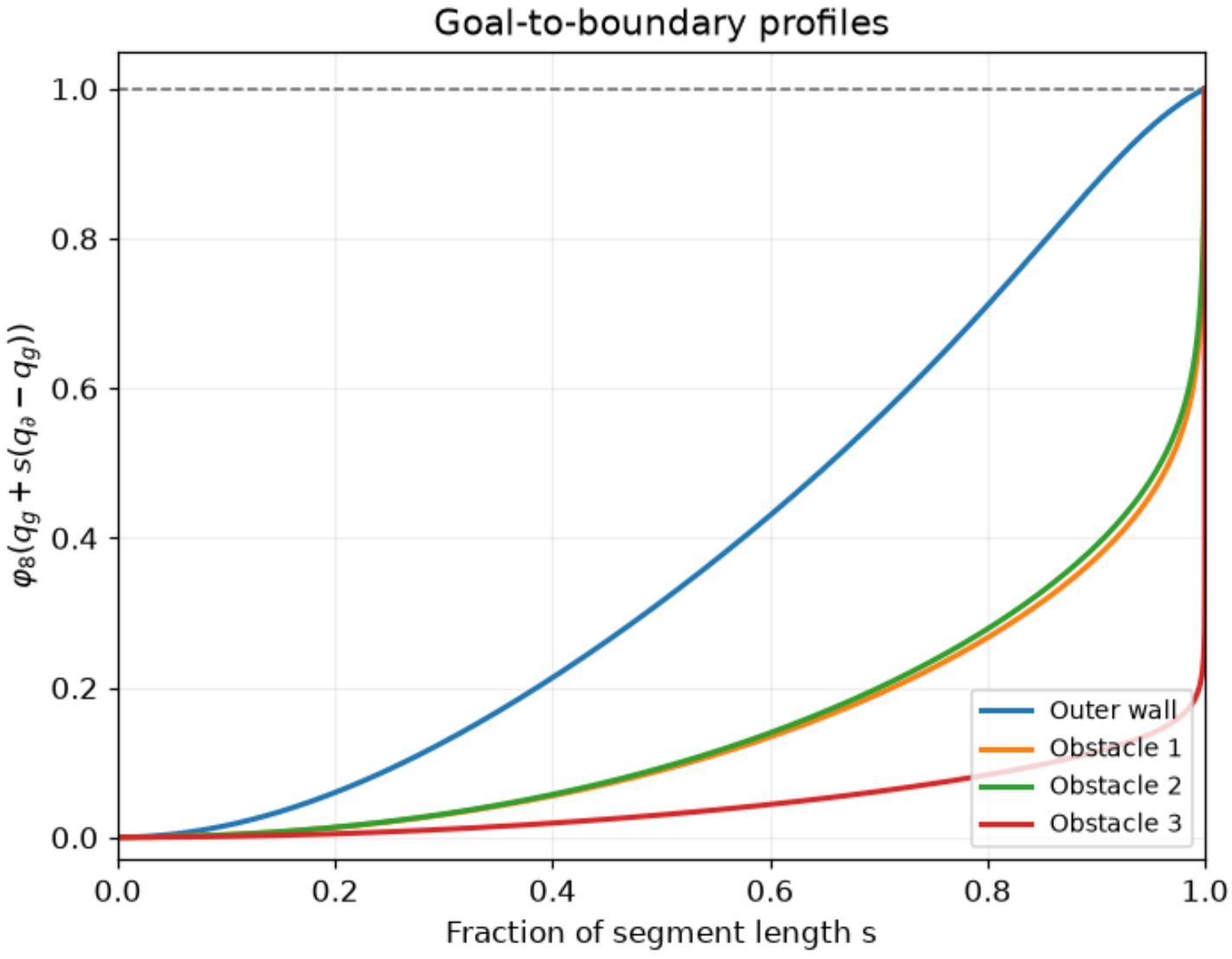
Navigation functions



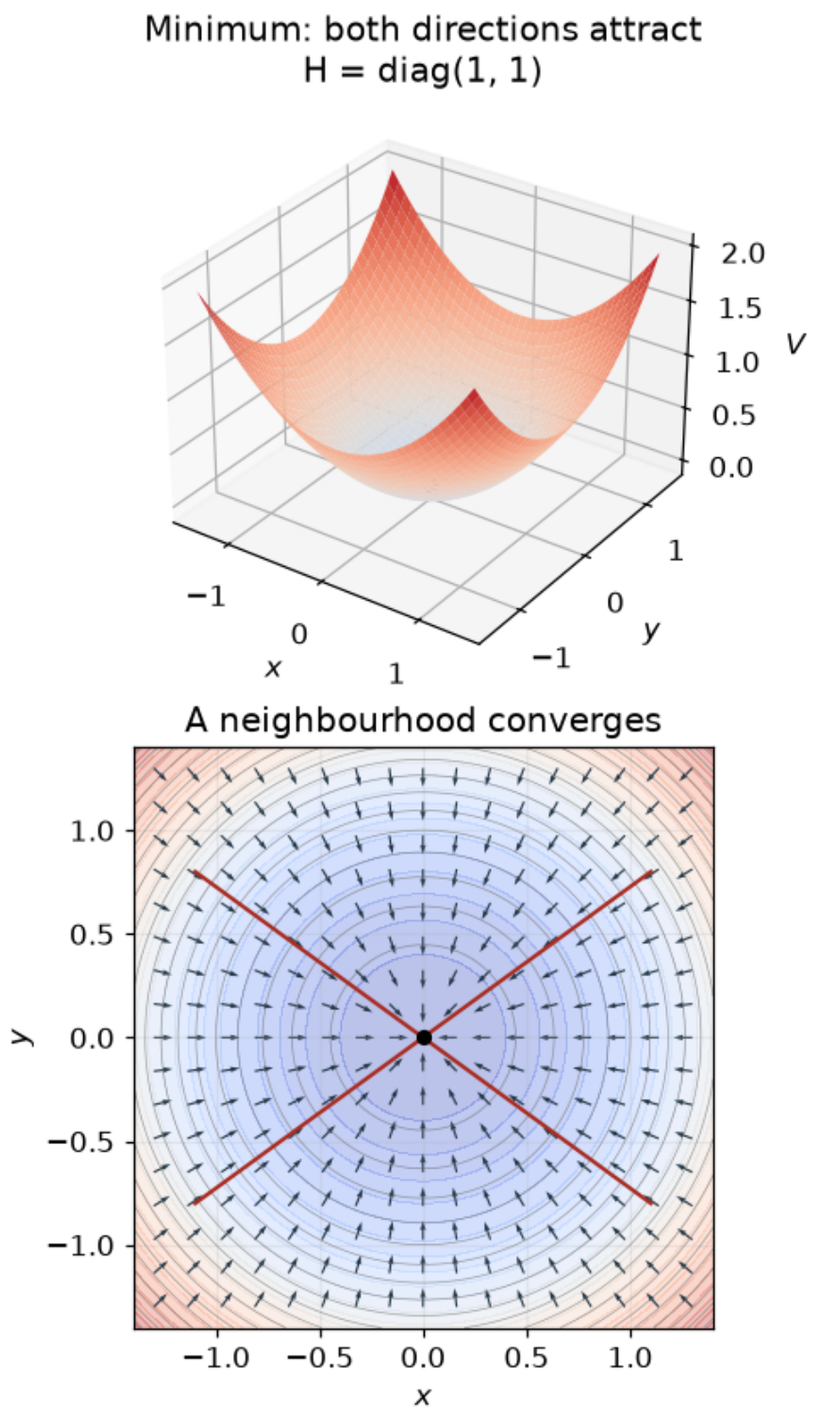


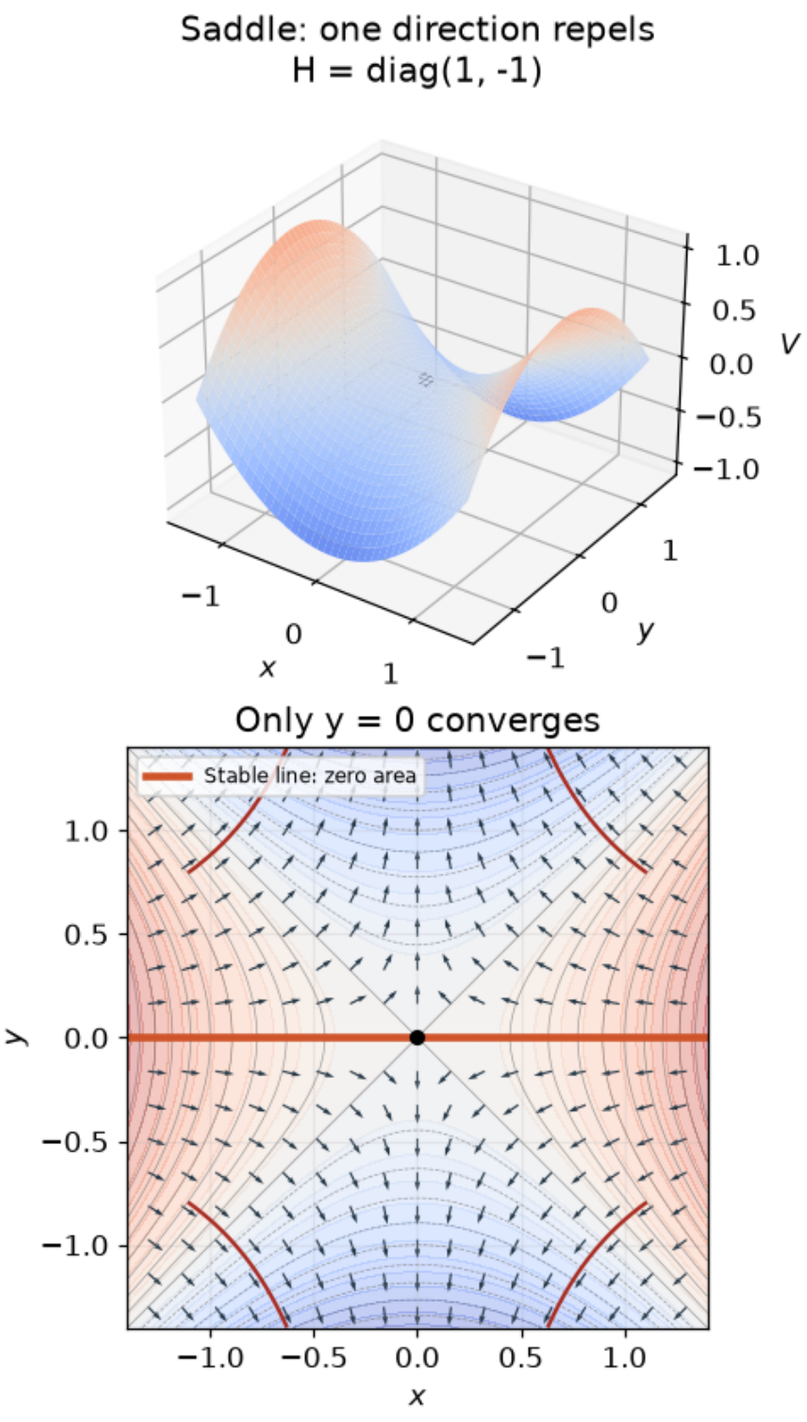
Navigation functions

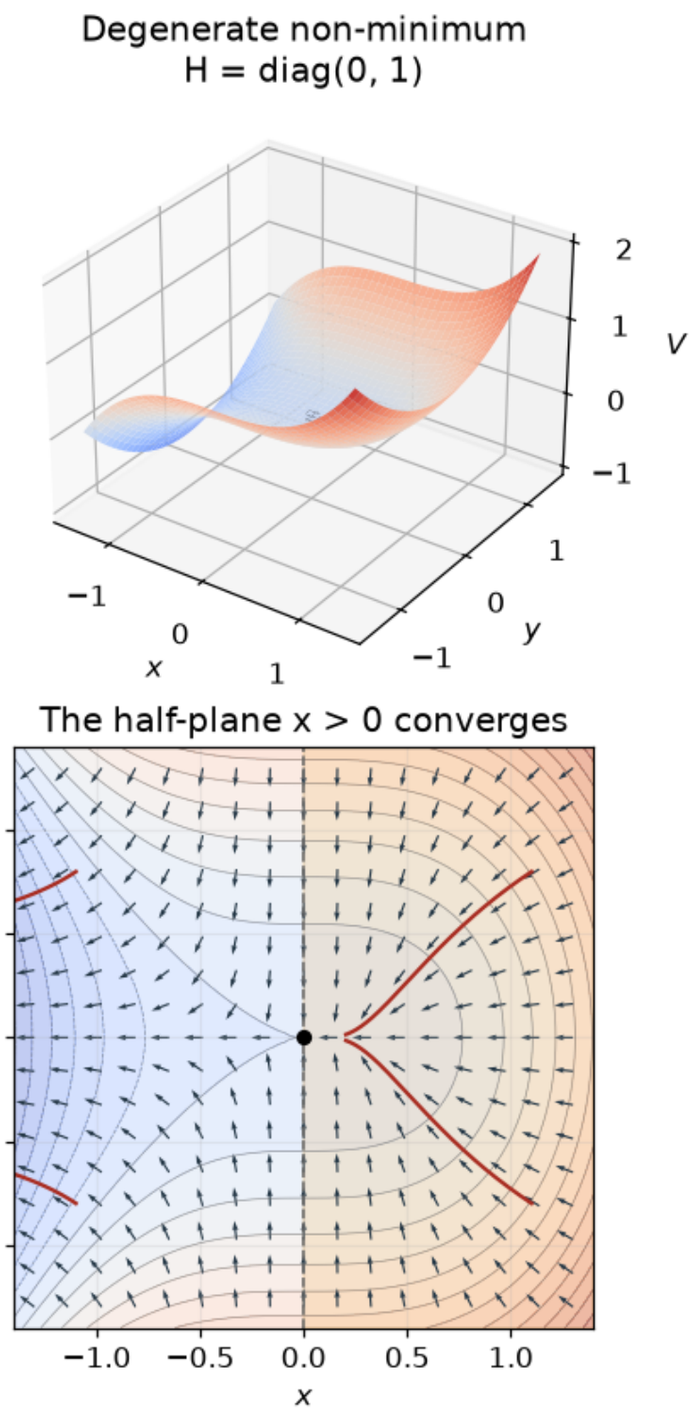




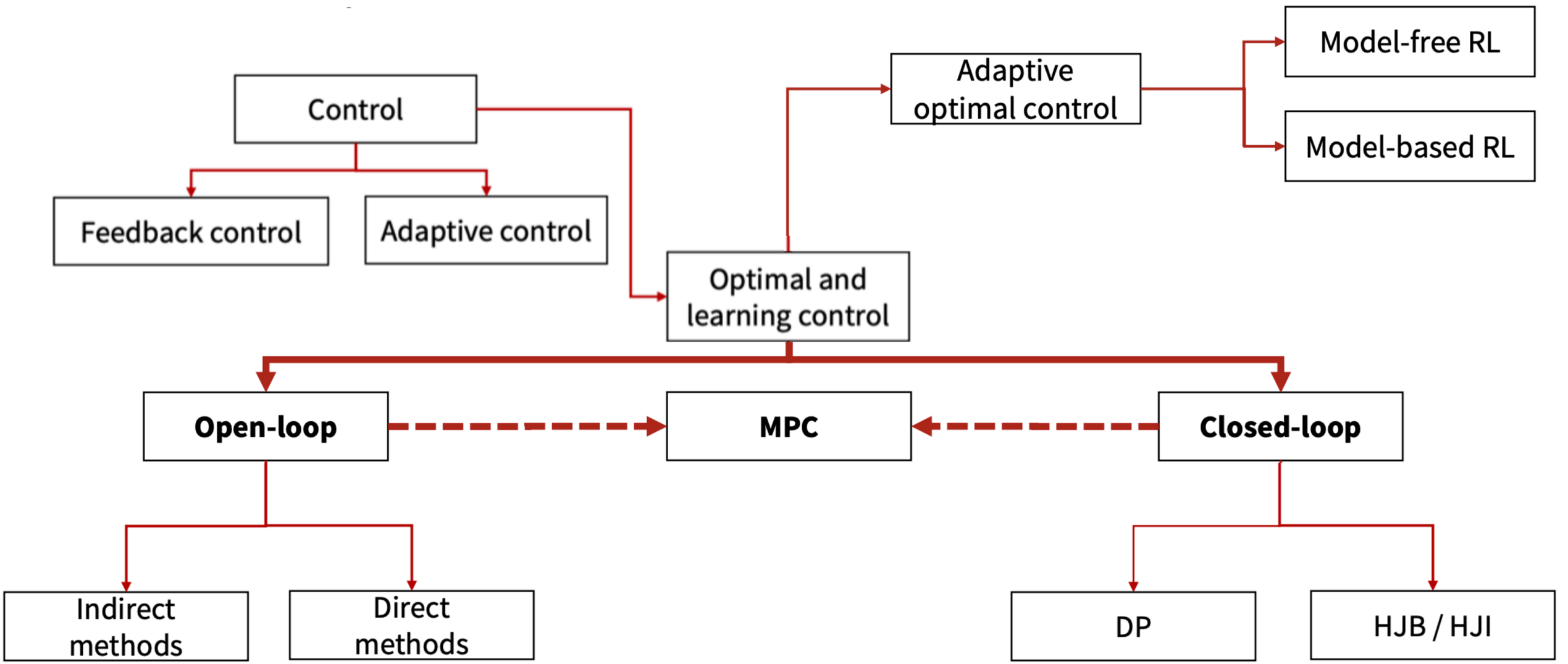
Navigation functions







Navigation functions



References: Modern Robotics by Kevin Lynch