

EEL 301 Homework #1 Solution

Shaunak Sen

Question 1. (Ogata, Table A-1 on Pg. 864 #20) Find the Laplace transform of $f(t) = e^{-at} \sin \omega t$.

Solution. We note that $\mathcal{L}(\sin \omega t) = \omega / (s^2 + \omega^2)$ and that $\mathcal{L}(f(t)) = F(s) \implies \mathcal{L}(e^{-at} f(t)) = F(s + a)$. Combining these two together, the required Laplace transform is $\omega / ((s + a)^2 + \omega^2)$.

Question 2. Using the Laplace transform method, solve the differential equation,

$$\ddot{x} + 2\dot{x} + 2x = 0, x(0) = 0, \dot{x}(0) = 10.$$

Solution. Laplace transform of this equation is,

$$\begin{aligned} s^2 X(s) - sx(0) - \dot{x}(0) + 2sX(s) + 2X(s) &= 0, \\ \implies (s^2 + 2s + 2)X(s) &= 10, \\ \implies X(s) &= \frac{10}{s^2 + 2s + 2}, \\ \implies X(s) &= \frac{10}{(s + 1)^2 + 1}. \end{aligned}$$

From Solution to Question 1, the required solution is, $x(t) = 10e^{-t} \sin t$.