ELL 333 Problem Set 1

1. A model of an inverted pendulum (length: l) is $\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\dot{\theta} \\ \frac{2l}{1+l^3}\theta + \frac{1}{2}u\end{bmatrix}$ u: input, θ: output.
Using $x = \begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix}$, represent the model in the state space form $\dot{x} = Ax + Bu$, $y = Cx + Du$.

2. Find the eigenvalues and eigenvectors of
(a) $A = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$, (b) $A = \begin{bmatrix}1 & 1 \\ 0 & 1\end{bmatrix}$.

3. For $A, T \in \mathbb{R}^{2\times 2}$, $b, v_1, v_2 \in \mathbb{R}^{2\times 1}$, verify that
(a) $A[v_1 \,\vdots\, v_2] = [Av_1 \; Av_2]$
(b) $[\lambda_1 v_1 \,\vdots\, \lambda_2 v_2] = \begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}[v_1 \,\vdots\, v_2]$, $\lambda_1, \lambda_2 \in \mathbb{R}$ or $\mathbb{C}$
(c) $T[b \,\vdots\, Ab \,\vdots\, \ldots \,\vdots\, A^{n-1}b] = [Tb \,\vdots\, TAb \,\vdots\, \ldots \,\vdots\, TA^{n-1}b]$.

4. If $A \in \mathbb{R}^{n\times n}$ and $f(\lambda) = \det(\lambda I - A)$, show that $f(A) = 0_{n\times n}$. [This is the Cayley-Hamilton Theorem]

5. Show that any square matrix with real entries can be expressed as a sum of a symmetric matrix ($A^T = A$) and a skew-symmetric matrix.