ELL 333 Problem Set 6

1. (a) Design v so that $\ddot{q} = v$ meets the specification $(q, \dot{q}) \to (0,0)$ as $t \to \infty$ for any $(q(0), \dot{q}(0))$.

   (b) Design u so that $M(q)\ddot{q} + h(q,\dot{q}) = u$ meets the specification $(q,\dot{q}) \to (0,0)$ as $t \to \infty$ for any $(q(0), \dot{q}(0))$. $M(q) \neq 0$.

2. For the system $\dot{x} = x - u$, $x(0) = 1$ with feedback control $u = -kx$, find k that minimises the cost $J = \int_0^\infty (x^2 + u^2)\,dt$.

3. Consider the system

$$x_{k+1} = a x_k + b u_k + v_k,$$
$$y_k = c x_k + w_k,$$

where $\{v_k\}$, $\{w_k\}$ are independent zero mean Gaussian white noise processes with variances σ_v^2, σ_w^2 respectively. For the state observer

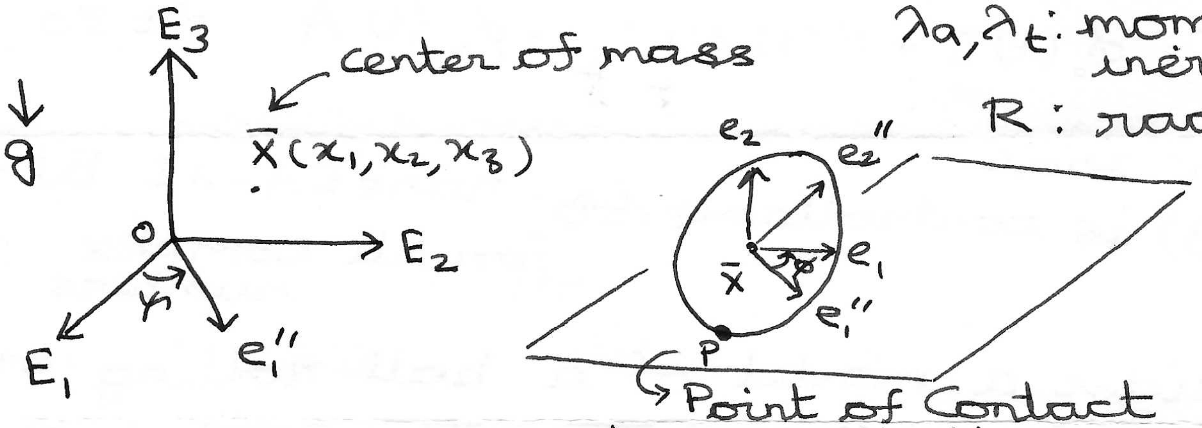
$$\hat{x}_{k+1} = a\hat{x}_k + b u_k + l_k(y_k - c\hat{x}_k)$$

find l_k that minimizes the covariance of the error $e_k = x_k - \hat{x}_k$ at each time instant k. Investigate the behaviour of ~~k~~ l_k as $k \to \infty$.

4. For $\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}}_{B} u$, the Linear Quadratic Regulator (LQR) that minimises the cost $J = \int_0^\infty (x^T Q x + u^T R u)\,dt$ is $u = -R^{-1}B^T P x$ where P is a symmetric matrix that satisfies the Algebraic Ricatti Equation $A^T P + PA - PBR^{-1}B^T P + Q = 0$

For $Q = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$, $R = 1$, find the LQR gain $K_{lqr} = R^{-1} B^T P$. Note: Eigenvalues of $A - BK_{lqr}$ have a negative real part.

5. Consider a wheel / disk on a plane



λ_a, λ_t: moment of inertia

R: radius

$\theta \in (0, \pi)$ represents the inclination angle.

$\theta = \frac{\pi}{2} \Rightarrow$ disk is vertical.

a) Discuss equilibrium points / trajectories and their stability.

b) What inputs are required for the wheel to be controllable?

c) For what set of outputs is the wheel observable?

Following model may be considered in your response,

$$\lambda_t \sin\theta \, \ddot{\psi} = (\lambda_a - 2\lambda_t)\dot{\psi}\dot{\theta}\cos\theta + \lambda_a \dot{\theta}\dot{\phi}$$

$$(\lambda_t + mR^2)\ddot{\theta} = (\lambda_t - \lambda_a - mR^2)\dot{\psi}^2 \sin\theta\cos\theta - (\lambda_a + mR^2)\dot{\psi}\dot{\phi}\sin\theta - mgR\cos\theta$$

$$(\lambda_a + mR^2)\cos\theta\,\ddot{\psi} + (\lambda_a + mR^2)\ddot{\phi} = (\lambda_a + 2mR^2)\dot{\psi}\dot{\theta}\sin\theta$$

$$\dot{x}_1 = -R\dot{\psi}\cos\psi\cos\theta + R\dot{\theta}\sin\psi\sin\theta - R\dot{\phi}\cos\psi$$

$$\dot{x}_2 = -R\dot{\psi}\sin\psi\cos\theta - R\dot{\theta}\cos\psi\sin\theta - R\dot{\phi}\sin\psi$$

$$\dot{x}_3 = R\dot{\theta}\cos\theta$$