EEL782 : Solutions to final exam

Objective type questions

1. $g_m r_{ds} \approx 20$

2. $C_{junction}$

3. 2

4. Area , perimeter

5. $\left.\begin{array}{l} W \cdot L = 200 \\ 2(W+L) = 80 \end{array}\right]$ $\left.\begin{array}{l} W + \frac{200}{W} = 40 \\ W^2 - 40W + 200 = 1 \end{array}\right]$ $34\,\mu m \times 6\,\mu m$

6. 2.5 mV

7. Common centroid

8. $V/\sqrt{Hz}$

9. False

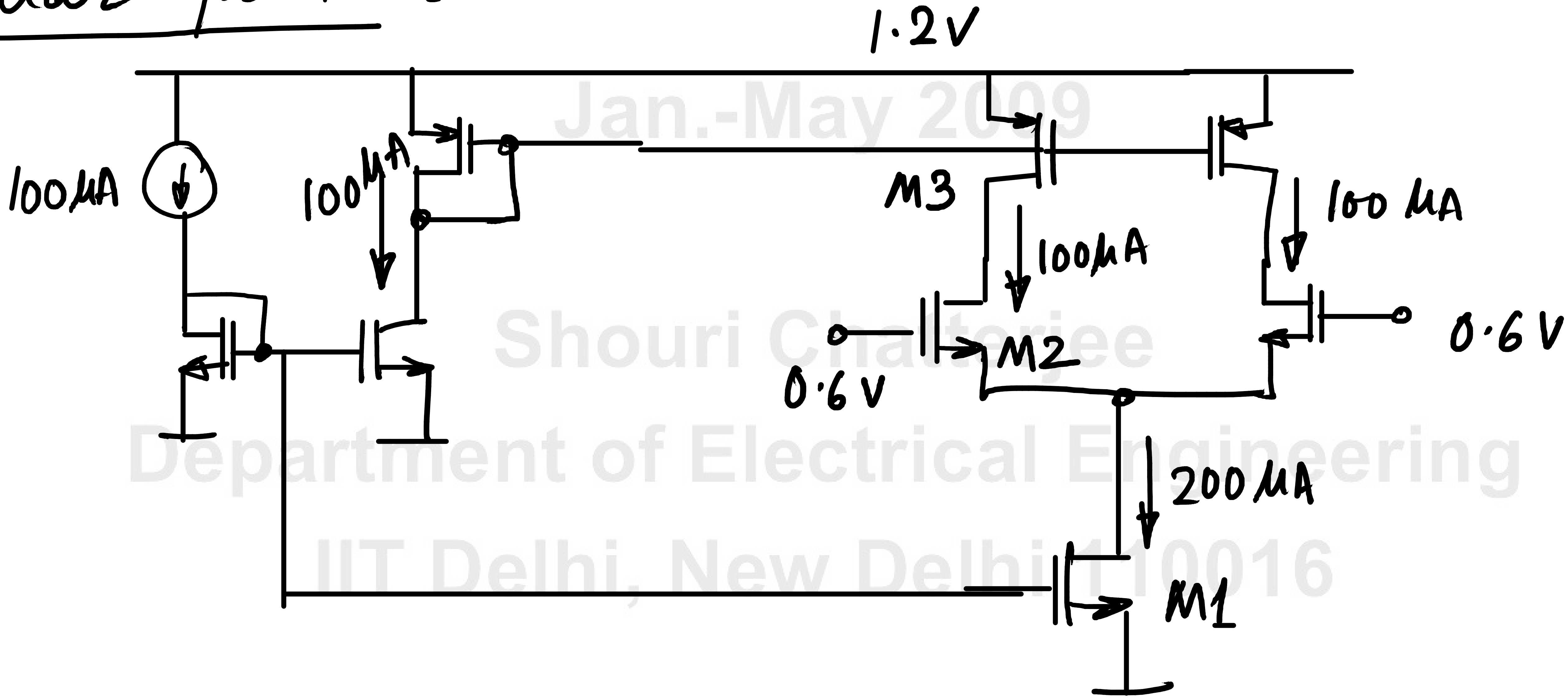
10. True

11. Siemens, Ohms. Increase, increase

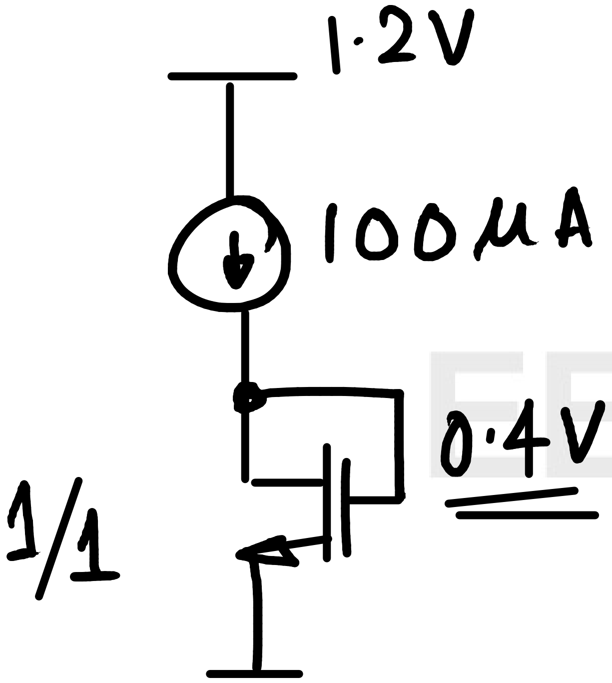
12. DC, even harmonics. Fundamental, odd harmonics.

Regular questions

⑬



Step 1: recognised all the current mirrors.
M1 drawing 200 µA; M2 having 100 µA;
M3, M4, M5



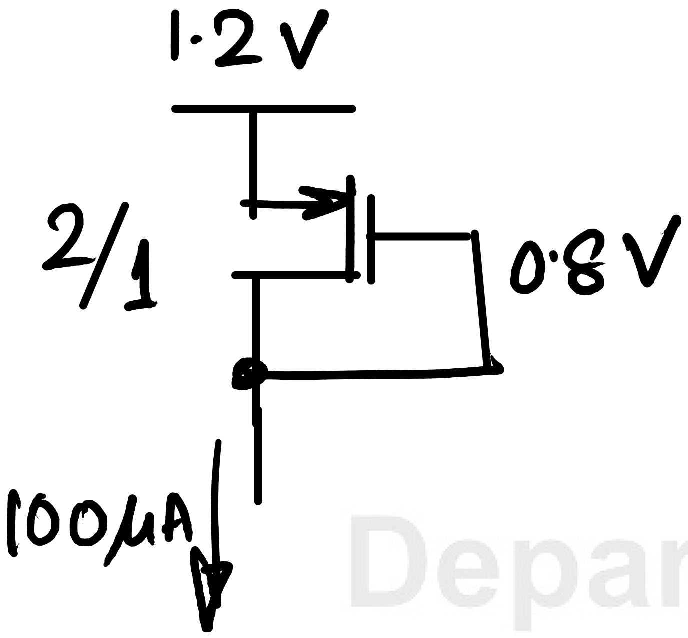
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

Assume the effect of $1 + \lambda V_{DS}$ is small.

$$I_D \approx \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

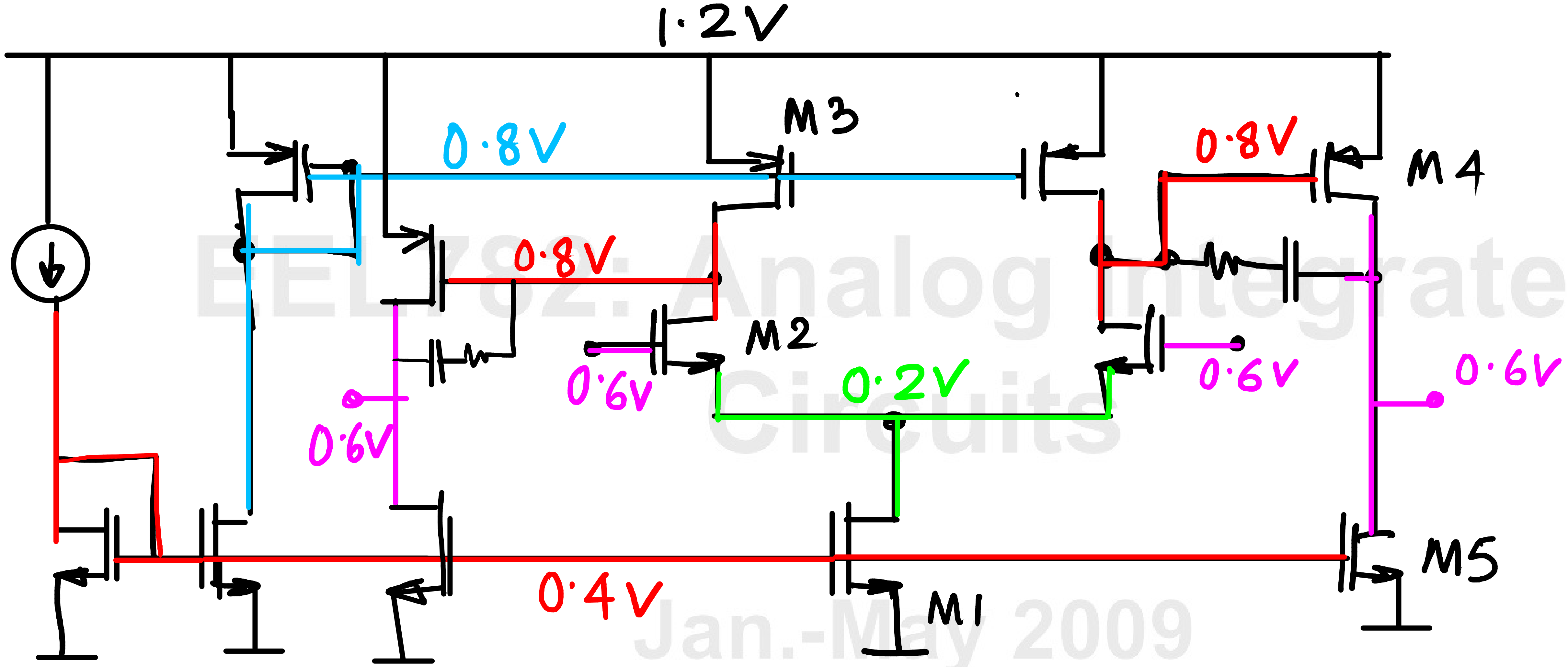
$$100\mu = 10m \cdot 1 \cdot (V_{GS} - 0.3)^2$$

$$\therefore \; V_{GS} = 0.4\,V$$



$$V_{SG} = 0.4\,V$$

Also, for M4, V_{SG} must be 0.4 V for it to conduct 100 µA.



Yes, all devices are in saturation.

$V_{GS} - V_T = 0.1\,V$ ⟶ this is borderline between moderate and strong inversion

for all devices

	V_{GS}	V_{DS}	I_D	g_m	r_{ds}
M1	0·4V	0·2V	200μA	4mS	10kΩ
M2	0·4V	0·6V	100μA	2mS	20kΩ
M3	-0·4V	-0·4V	100μA	2mS	20kΩ
M4	-0·4V	-0·6V	100μA	2mS	20kΩ
M5	0·4V	0·6V	100μA	2mS	20kΩ

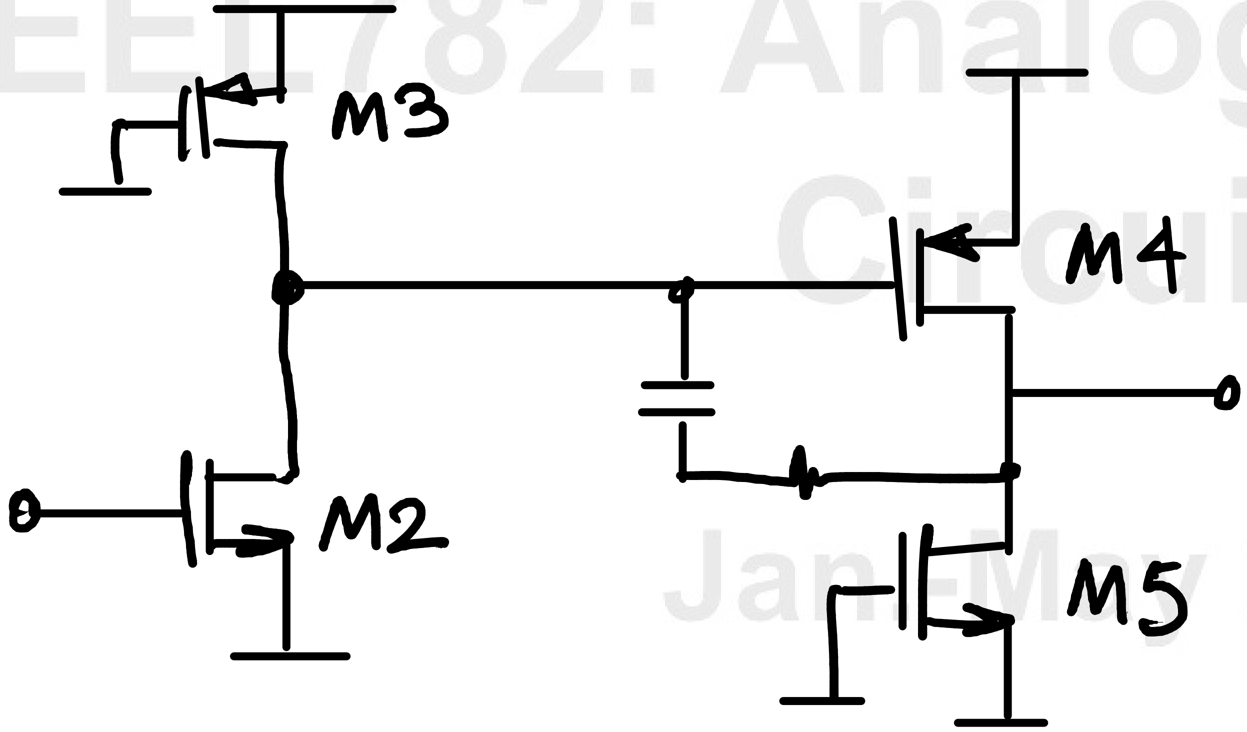
$$g_m = \frac{\partial I_D}{\partial V_{GS}} \approx \sqrt{4 \cdot \frac{\mu C_{ox}}{2} \cdot \frac{W}{L} \cdot I_D}$$

$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} \approx \lambda \cdot I_D$$

$$r_{ds} = \frac{1}{\lambda I_D}$$

⑭

Small signal differential half-circuit



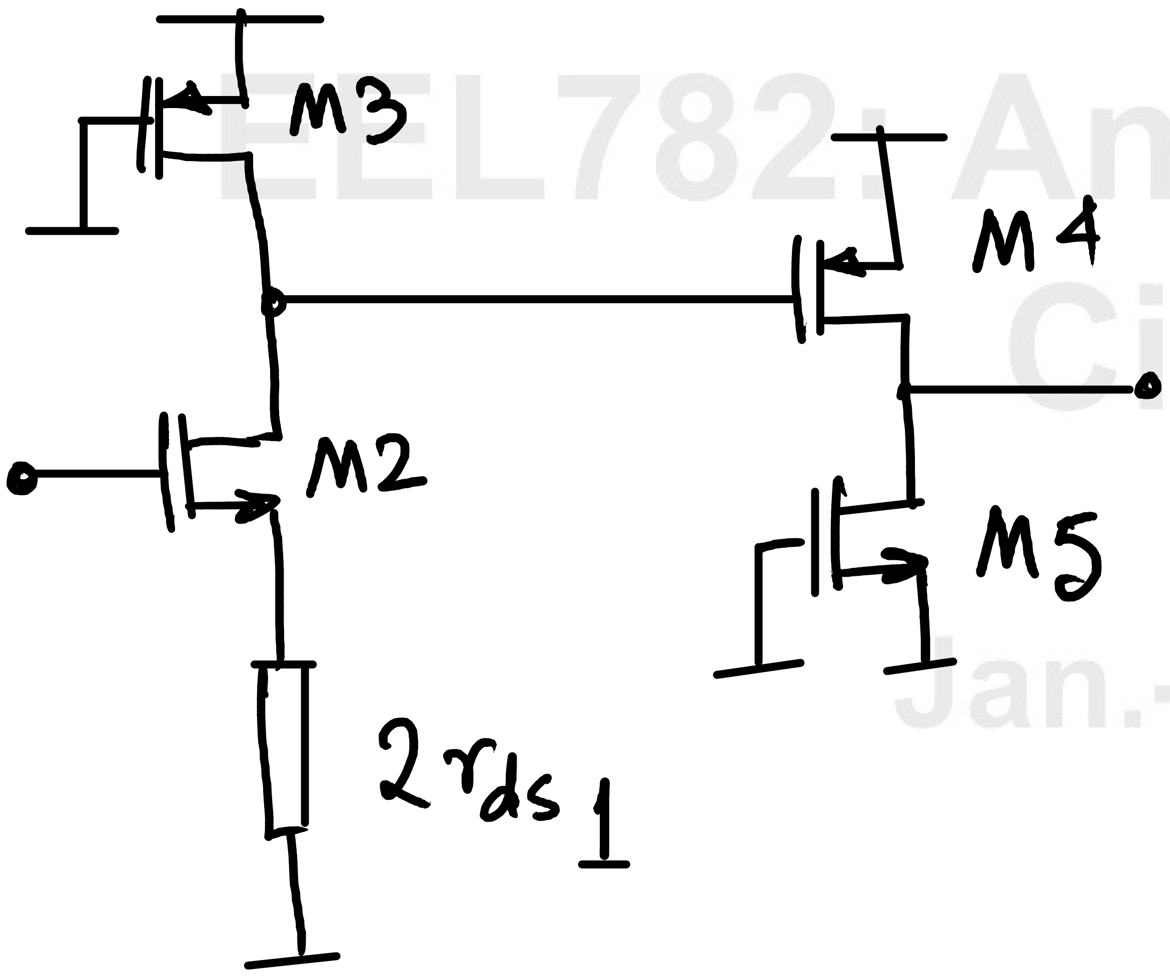
Differential mode gain :

$$g_{m_2}(r_{ds_2} \| r_{ds_3}) \times g_{m_4}(r_{ds_4} \| r_{ds_5})$$

$$= 2mS \cdot (20k \| 20k) \cdot 2mS \cdot (20k \| 20k)$$

$$= 400 .$$

⑮ Common mode half circuit



Common-mode gain

$$\approx \frac{r_{ds3}}{2r_{ds1}} \times g_{m4}\left(r_{ds4} \| r_{ds5}\right)$$

$$= \frac{20\,k\Omega}{20\,k\Omega} \times 2\,mS\left(20k \| 20k\right)$$

$$= 20.$$

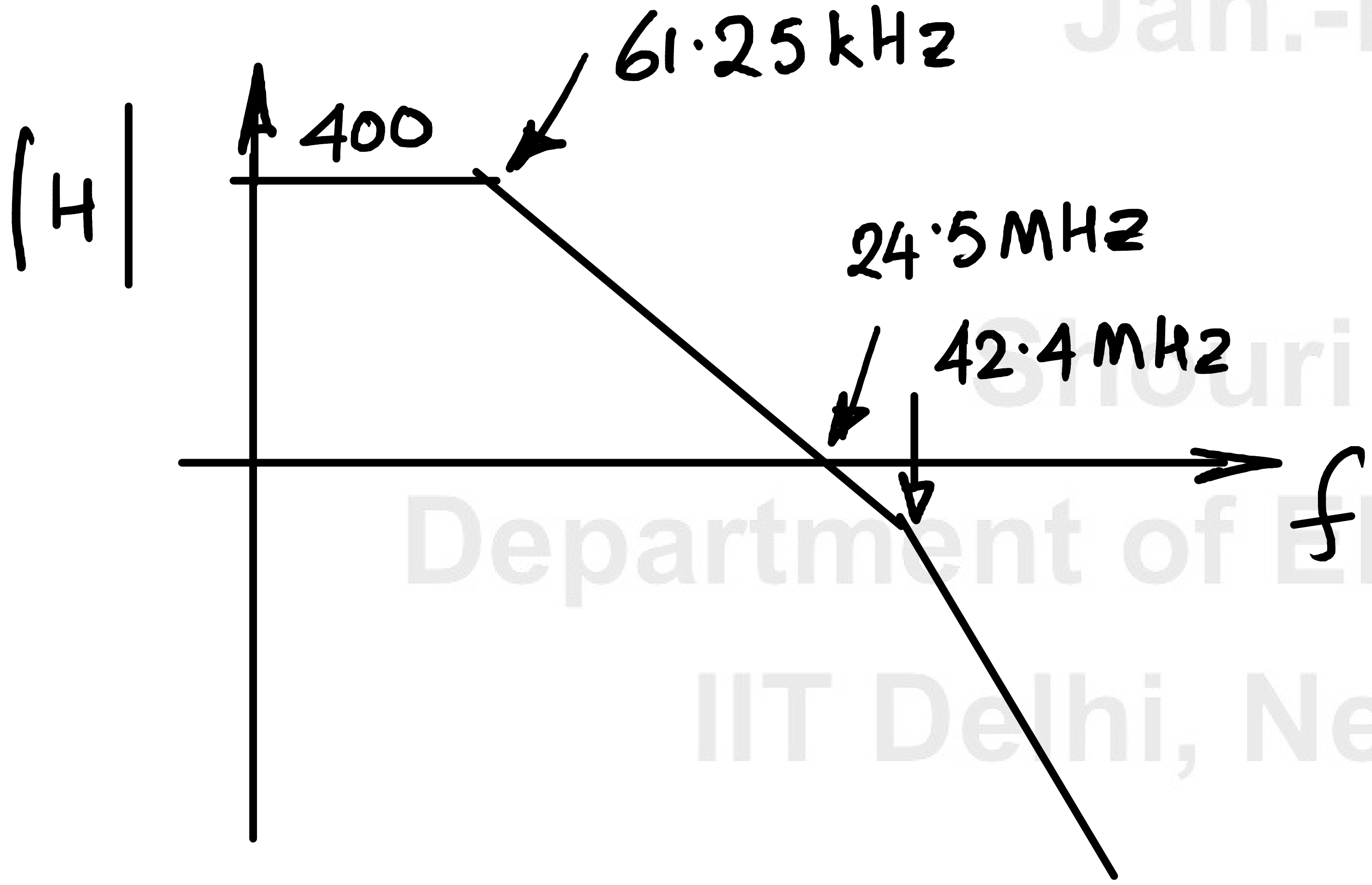
$$A_c = \frac{v_o^+ + v_o^-}{v_i^+ + v_i^-} \approx 20$$

⑯ Unity-gain bandwidth $\simeq \dfrac{g_{m_2}}{C} = \dfrac{2\,mS}{13\,pF} = 153.85\ \text{Mrad/sec}$

(first-pole at $24.5\,MHz/400 = 61.25\,kHz$) $= 24.5\ MHz$

Second pole $\simeq \dfrac{g_{m_4}}{C_L} = \dfrac{2\,mS}{7.5\,pF} = 266.7\ \text{Mrad/sec}$

$= 42.4\ MHz$



$$H(j\omega) = \frac{A_d}{(1 + j\omega/p_1)(1 + j\omega/p_2)}$$

$$\angle H(j\omega) = -\tan^{-1}\omega/p_1 - \tan^{-1}\omega/p_2$$

$p_1 = g_{m_2}/C \cdot A_d$; $p_2 = g_{m_4}/C_L$

At $\omega_0 = g_{m2}/C$ (or, at the unity gain bandwidth frequency)

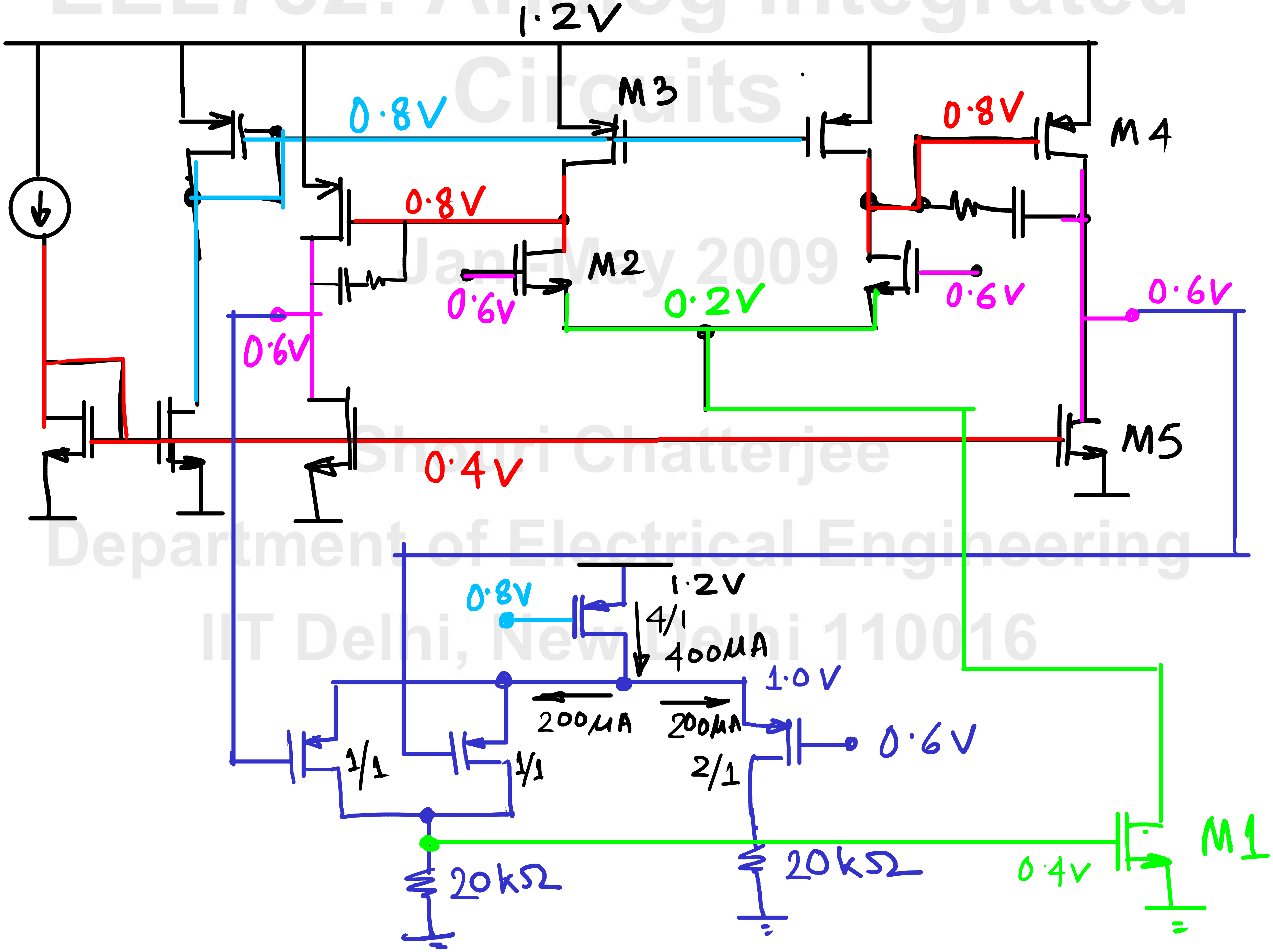
$$\angle H(j\omega_0) = -\tan^{-1}\left(\frac{\omega_0}{P_1}\right) - \tan^{-1}\left(\frac{\omega_0}{P_2}\right)$$

$$= -\tan^{-1}\left(\frac{g_{m2}/C_c}{g_{m2}/C_c/A_d}\right) - \tan^{-1}\left(\frac{g_{m2}/C_c}{g_{m4}/C_L}\right)$$

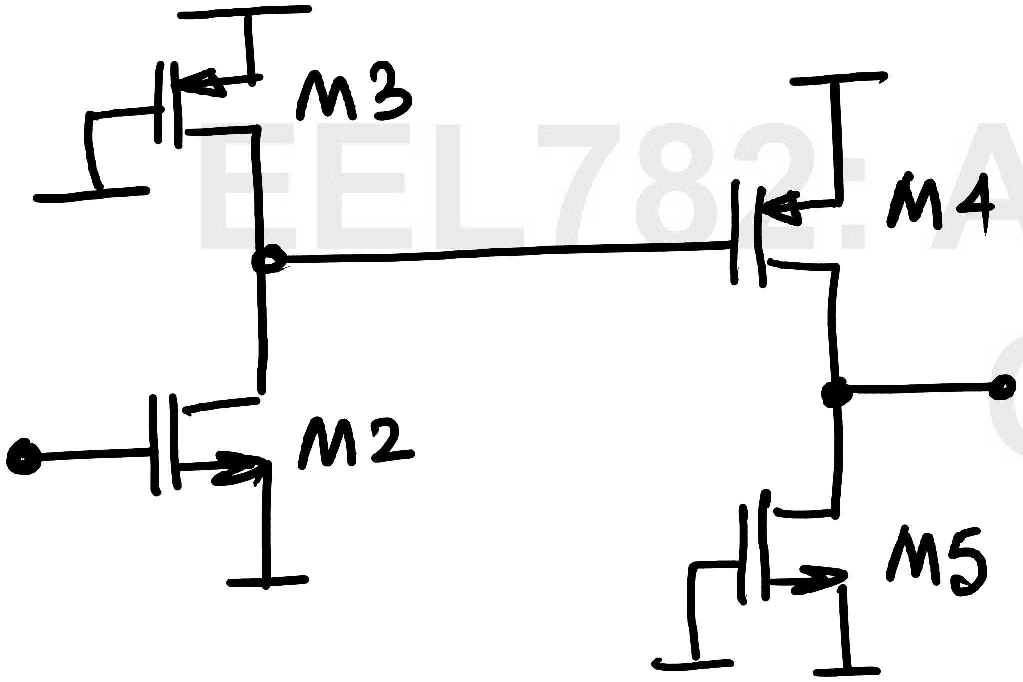
$$= -\tan^{-1}(A_d) - \tan^{-1}(C_L/C_c)$$

$$= -90^\circ - 30^\circ = -120^\circ$$

$\therefore$ phase margin $= -120^\circ - (-180^\circ) = 60^\circ$


1.2V
M3
0.8V
0.8V
M4
0.8V
M2
0.6V
0.2V
0.6V
0.6V
0.6V
M5
0.4V
1.2V
0.8V
4/1
400µA
1.0V
200µA
200µA
0.6V
1/1
1/1
2/1
20kΩ
20kΩ
0.4V
M1

⑱ Noise computation



M2 contribution at input:

$$\overline{i_{n_2}^2}/g_{m_2}^2$$

M3 contribution at input:

$$\overline{i_{n_3}^2}/g_{m_2}^2$$

(Noise current i_{n_3} gets referred back to the input.)

M4 contribution at input:

$$\frac{\overline{i_{n_4}^2}\cdot(r_{ds_4}\|r_{ds_5})^2}{g_{m_2}^2(r_{ds_2}\|r_{ds_3})^2\,g_{m_4}^2(r_{ds_4}\|r_{ds_5})^2} = \frac{\overline{i_{n_4}^2}}{g_{m_2}^2(r_{ds_2}\|r_{ds_3})^2\,g_{m_4}^2}$$

M5 contribution at input: $\overline{i_{n_5}^2}/g_{m_2}^2(r_{ds_2}\|r_{ds_3})^2\,g_{m_4}^2$

Total input referred noise voltage squared :

$$\frac{1}{g_{m_2}^2}\left(\overline{i_{n_2}^2}+\overline{i_{n_3}^2}+\frac{\overline{i_{n_4}^2}+\overline{i_{n_5}^2}}{g_{m_4}^2\left(r_{ds_2}\|r_{ds_3}\right)^2}\right)$$

$$\overline{i_{n_2}^2}=\overline{i_{n_3}^2}=\overline{i_{n_4}^2}=\overline{i_{n_5}^2}$$

$$=\frac{8}{3}\cdot 0.025\times1.6\times10^{-19}\times2\times10^{-3}$$

$$=21.33\times10^{-24}\ \mathrm{A^2/Hz}$$

$$=\frac{21.33\times10^{-24}}{4\times10^{-6}}\times\left(1+1+\frac{2}{4\times10^{-6}\times\left(10\times10^{3}\right)^2}\right)$$

$$=\frac{21.33\times10^{-24}}{4\times10^{-6}}\times2.005\quad=10.7\times10^{-18}\ \mathrm{V^2/Hz}$$

Root-mean-squared noise voltage per unit frequency

$$=3.3\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$$