Minor 1 Solutions

EEL 782 Analog ICs

(1)

$$I_{DS} = I_0 \exp\left(\frac{|V_{GS}|}{\varphi_s v_T}\right)\left(1 - \exp\left(-\frac{|V_{DS}|}{\eta v_T}\right)\right)$$

For nMOS, $V_{GS}, V_{DS} > 0$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{I_0}{\varphi_s v_T}\left(1 - \exp\left(-\frac{V_{DS}}{\eta v_T}\right)\right) \cdot \exp\left(\frac{V_{GS}}{\varphi_s v_T}\right) = \frac{I_{DS}}{\varphi_s v_T}$$

For pMOS, $V_{GS}, V_{DS} < 0$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{I_{DS}}{\varphi_s v_T}$$

Similarly, for nMOS

$$g_{ds} = \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{I_0}{\eta v_T} \exp\left(\frac{V_{GS}}{\varphi_s v_T}\right) \exp\left(-\frac{V_{DS}}{\eta v_T}\right)$$

$$\approx \frac{I_{DS}}{\eta v_T} \cdot \exp\left(-\frac{V_{DS}}{\eta v_T}\right)$$

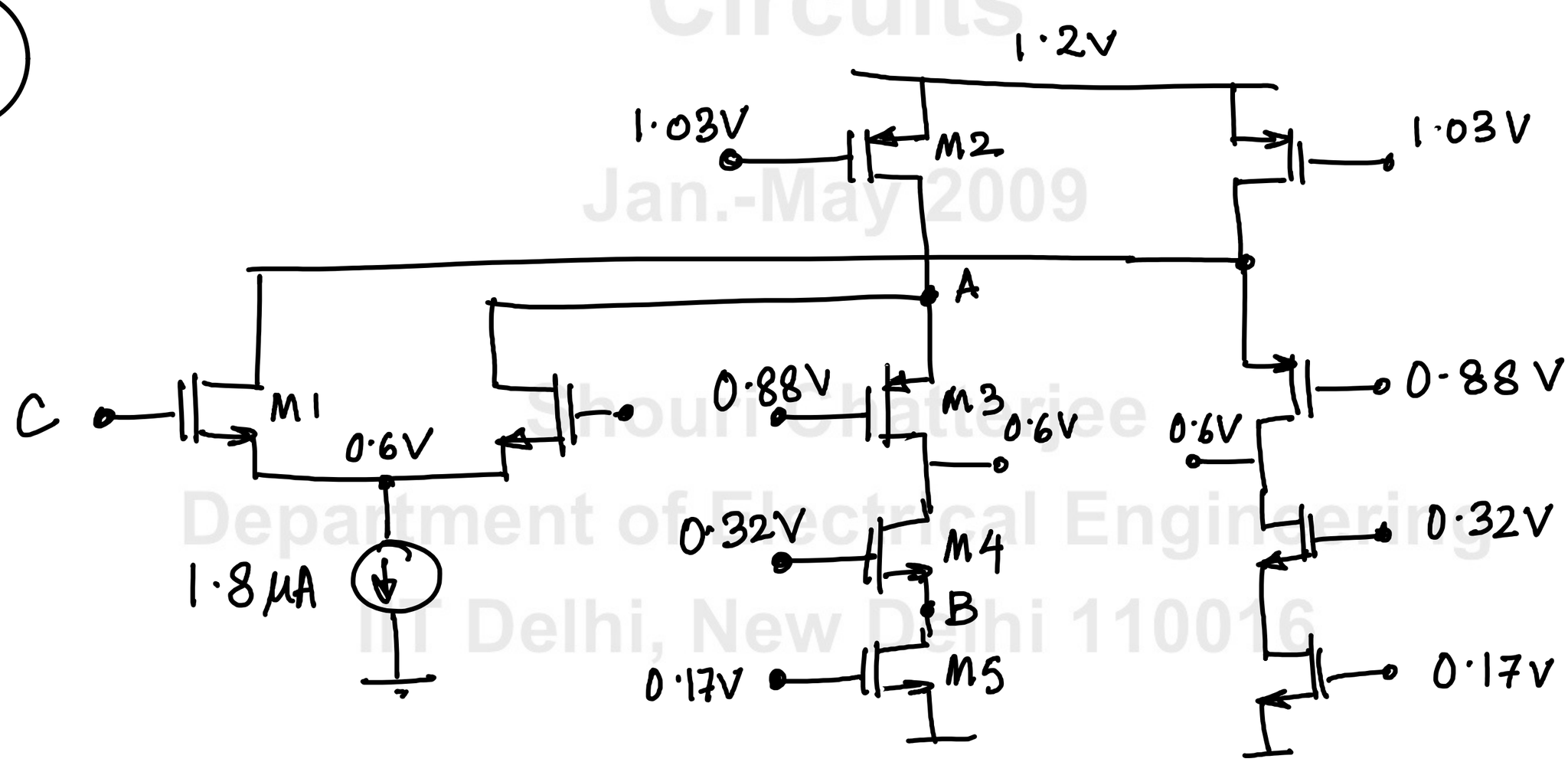
Likewise, for pMOS devices, we will arrive at the same result, with sign adjustments.

$v_T = 25\,mV$ @ 300 K ; $\zeta = 1$; $\eta = 2$

nMOS $\Rightarrow$ $I_0 = \frac{W}{L} \cdot 1\,nA$ pMOS $\Rightarrow$ $I_0 = \frac{W}{L} \cdot 0.5\,nA$

So, $g_m = \frac{I_{DS}}{25\,mV}$ $g_{ds} = \frac{I_{DS}}{50mV} \exp\left(-\frac{|V_{DS}|}{50mV}\right)$

(2)



Current through M2 assuming saturation : $\approx I_0 \exp\left(\frac{0.17\,V}{25\,mV}\right) = 4 \times 0.5\,nA \times 900$

$= 1.8\,\mu A$

Current through M5

assuming saturation : $\simeq I_0 \exp\left(\frac{0.17\,V}{25\,mV}\right) = 1 \times 1nA \times 900$

$= 0.9\,\mu A$

Check : Current through M1 : $\frac{1.8\,\mu A}{2} = 0.9\,\mu A \longrightarrow$ verified

assuming saturation

	Current	V_{GS}
M1	$0.9\,\mu A$	$0.17\,V$
M2	$1.8\,\mu A$	$0.17\,V$
M3	$0.9\,\mu A$	$0.17\,V$
M4	$0.9\,\mu A$	$0.17\,V$
M5	$0.9\,\mu A$	$0.17\,V$

This means :

node A is at

$0.88\,V + 0.17\,V$

$= 1.05\,V$

node B is at

$0.32\,V - 0.17\,V$

$= 0.15\,V$

node C is at

$0.6\,V + 0.17\,V$

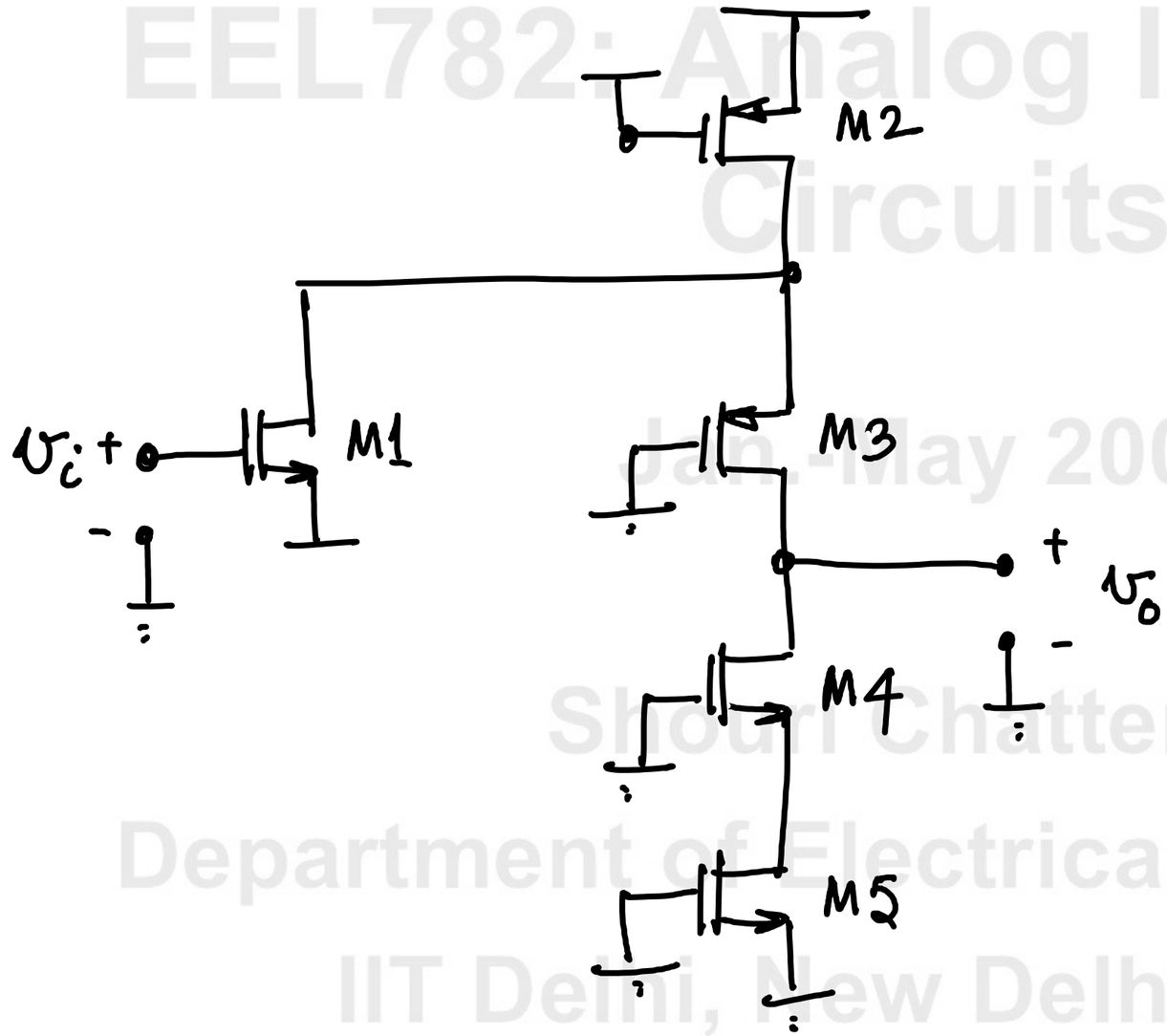
$= 0.77\,V$

Device	I_D	$\lvert V_{DS}\rvert$	g_m	g_{ds}	r_{ds}
M1	0·9μA	0·45 V	36 μS	2·22 nS	450 MΩ
M2	1·8μA	0·15 V	72 μS	1·8 μS	550 kΩ
M3	0·9μA	0·45V	36μS	2·22nS	450 MΩ
M4	0·9μA	0·45V	36μS	2·22nS	450 MΩ
M5	0·9μA	0·15V	36μS	0·9μS	1·1 MΩ

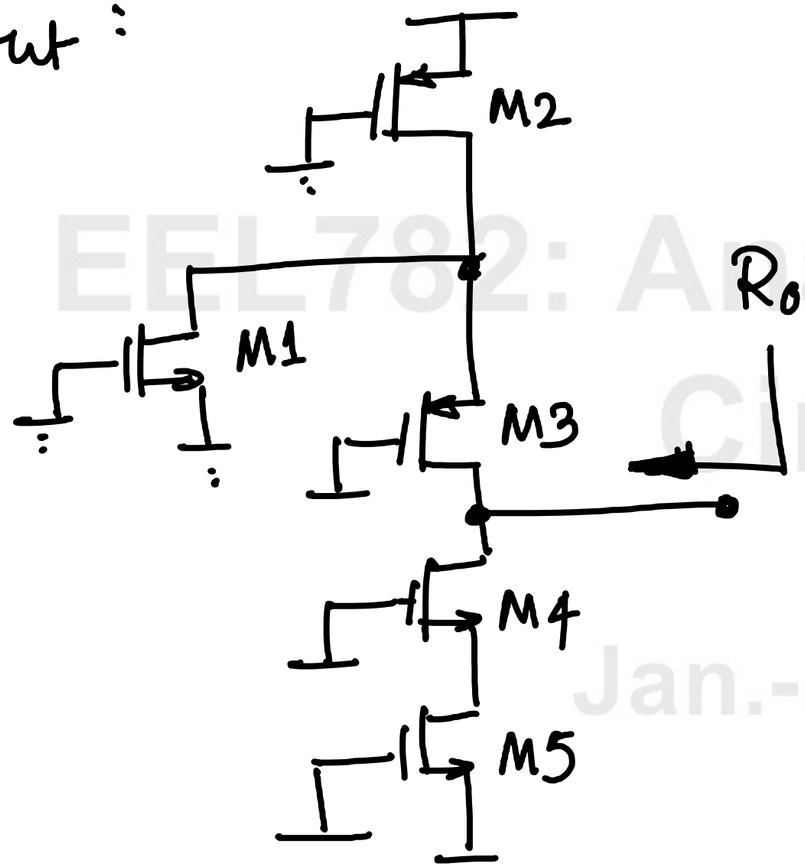
$$g_m = I_{DS}/25mV$$

$$g_{ds} = \frac{I_{DS}}{50mV} \cdot \exp\left(-\frac{V_{DS}}{50mV}\right)$$

(3) Differential mode half circuit :



For R_{out}:



Looking into drain of M4:

$\approx g_{m_4} r_{ds_4} r_{ds_5}$

Looking into drain of M3:

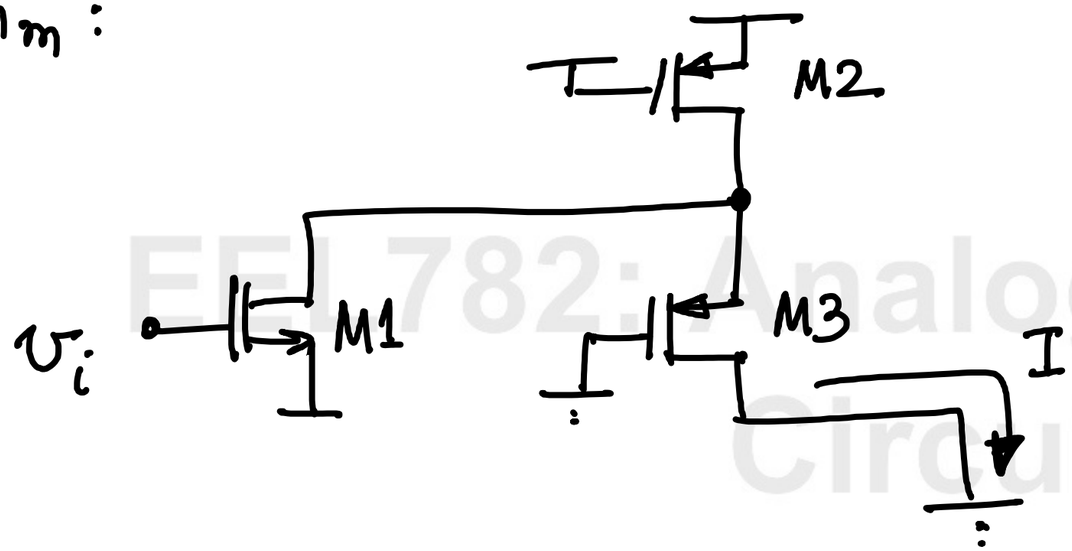
$\approx g_{m_3} r_{ds_3} (r_{ds_2} \| r_{ds_1})$

$R_{out} \approx (g_{m_4} r_{ds_4} r_{ds_5}) \| (g_{m_3} r_{ds_3} (r_{ds_2} \| r_{ds_1}))$

$= (36\mu S \cdot 450 M\Omega \cdot 1.1 M\Omega) \| (36\mu S \cdot 450 M\Omega \cdot (1.1 M\Omega \| 550 k\Omega))$

$= 17.8 G\Omega \| 5.94 G\Omega = \boxed{4.45 G\Omega}$

For G_m:



Current $g_{m_1} v_i$ sees r_{ds_2}, $1/g_{m_3}$ and r_{ds_3}.

$r_{ds_2} = 550\,k\Omega$; $r_{ds_3} = 450\,M\Omega$;

$1/g_{m_3} = 27.78\,k\Omega$

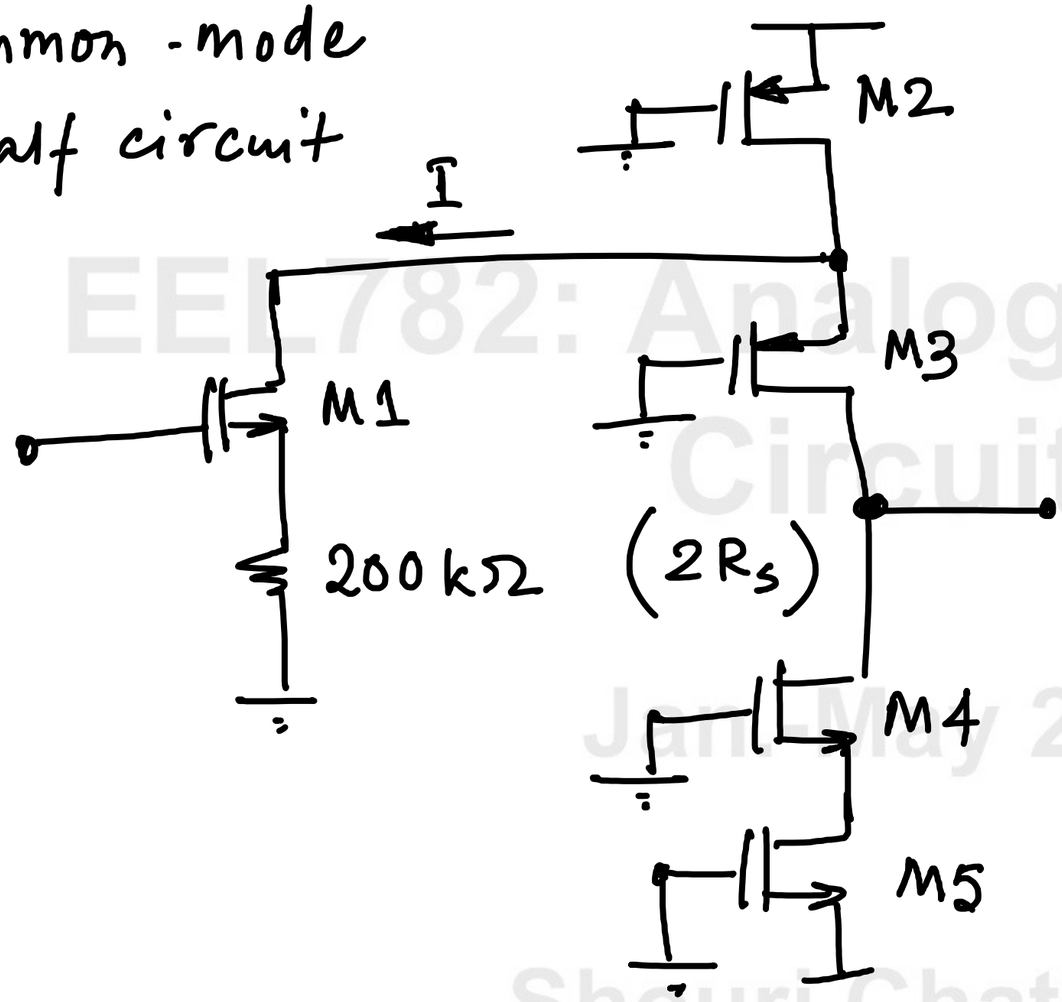
→ almost all the current goes through M3.

$\therefore I \approx -g_{m_1} \cdot v_i$

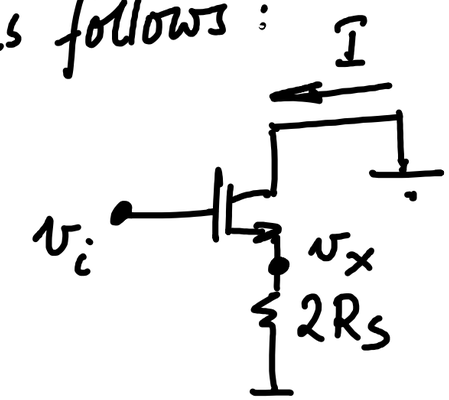
$\therefore \boxed{G_m \approx -g_{m_1} = -36\,mS.}$

Gain = $\boxed{G_m R_{out} = 160,000 = A_v}$

(4) Common-mode half circuit



A model for the first stage transconductance is as follows:



$$I = g_{m_1}(v_i - v_x) = \frac{v_x}{2R_S}$$

$$\therefore v_x = \frac{g_{m_1}}{g_{m_1} + \frac{1}{2R_S}} \cdot v_i$$

$$\text{and } I = \frac{g_{m_1}}{1 + 2R_S \cdot g_{m_1}} \cdot v_i$$

So, common-mode $G_m \approx \dfrac{\text{diff-mode } G_m}{1 + 2R_S g_{m_1}} \Rightarrow A_c \approx \dfrac{A_v}{8.2}$

(5) $V_{DS_4} = |V_{DS_3}| = 0.45\,V$

$|V_{DS_2}| = V_{DS_5} = 0.15\,V$

So output node can increase by upto 0.3V, decrease by upto 0.3V] pk-pk swing of 0.6V

differential pk-pk swing of 1.2 V

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