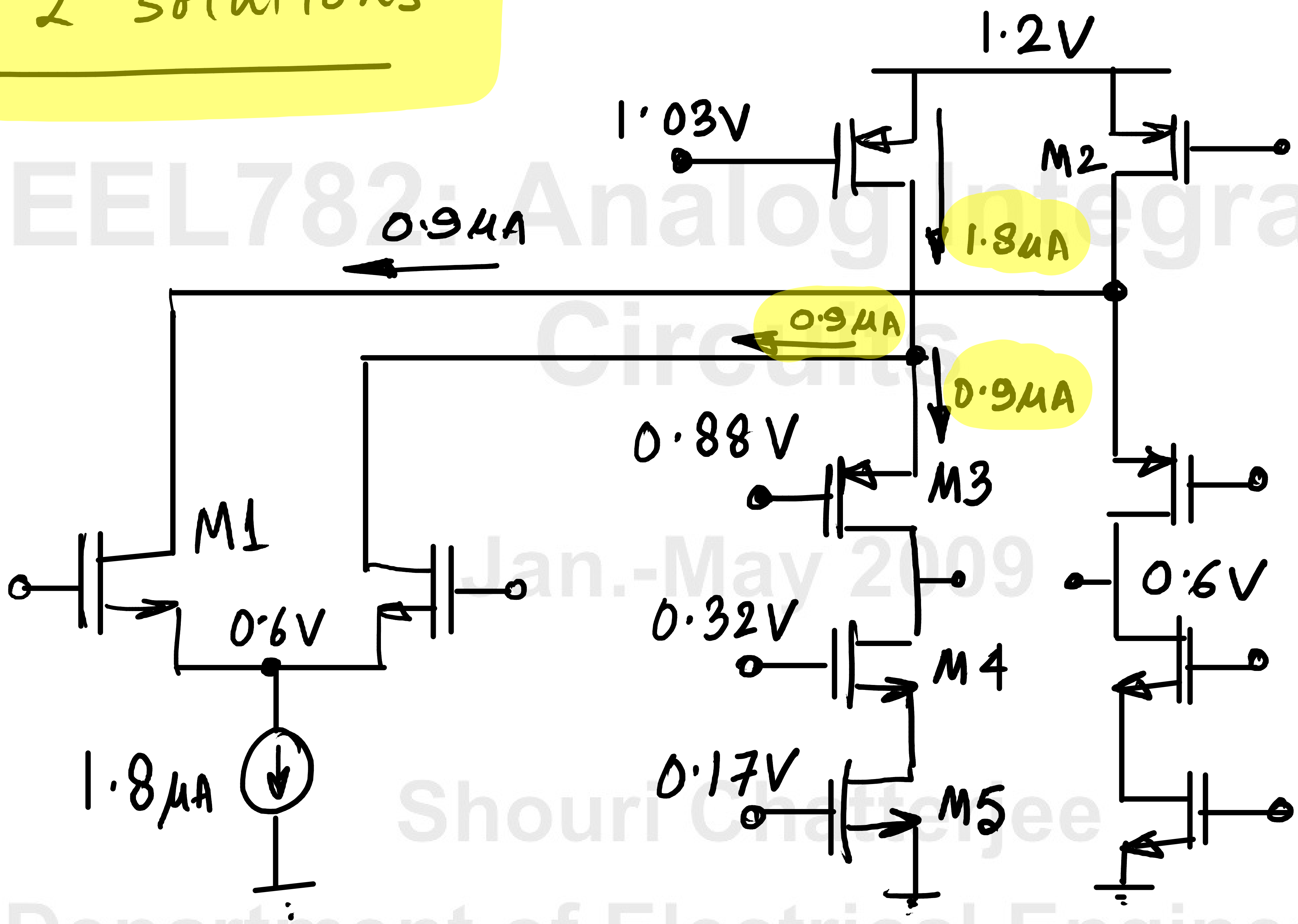
Minor 2 solutions



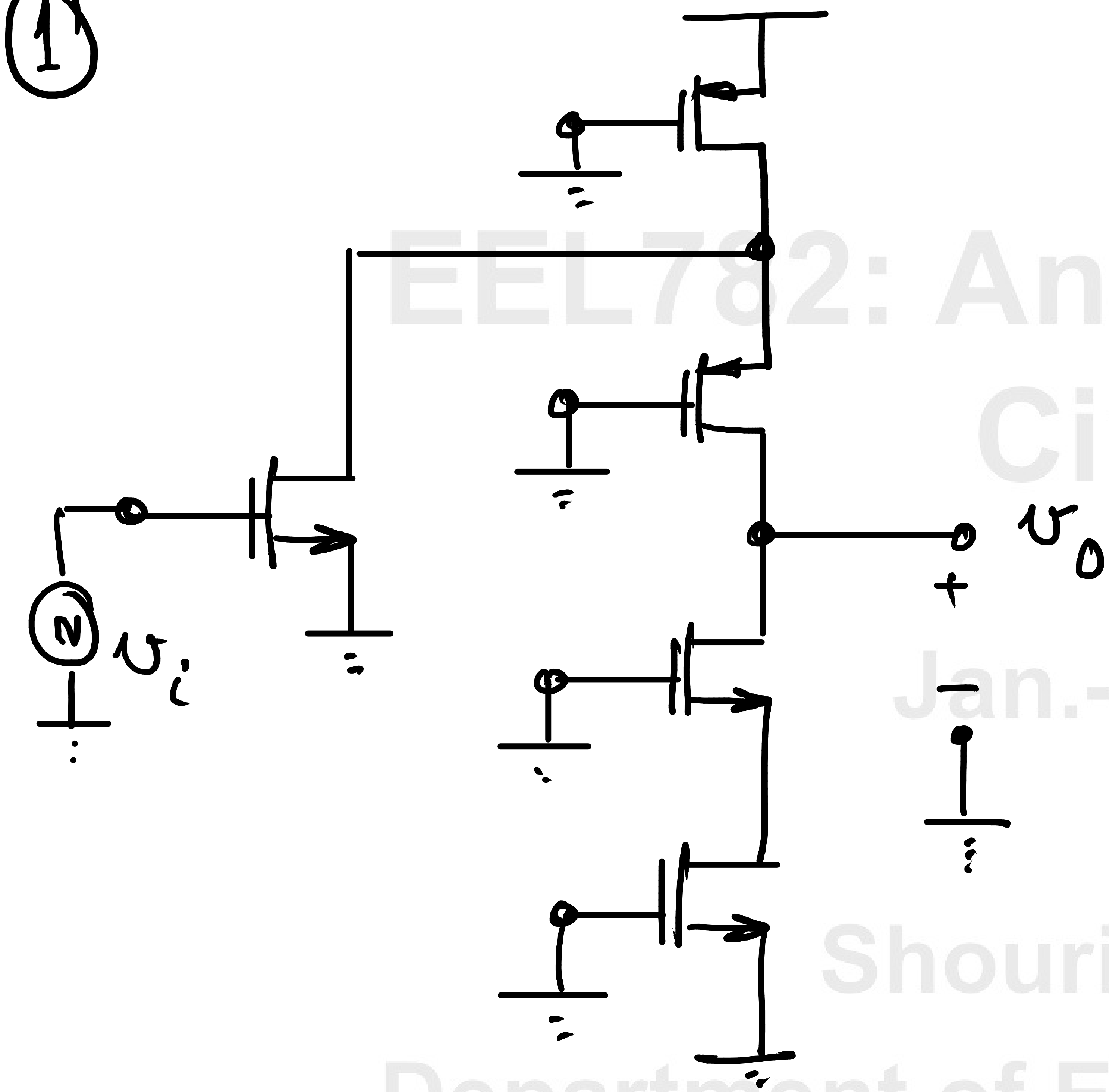
Current through M2 $\Rightarrow$ $I_2 = 4 \times 0.5\,\text{nA} \times \exp\left(\frac{0.17}{0.025}\right)$

$$\approx 1.8\,\mu A$$

So, by KCL, current through M3, M4, M5 = $0.9\,\mu A$

(1)

Small signal differential mode half circuit



②

$$C_{GS} = C_{GD} = \frac{1}{2} W \cdot L \cdot 8fF/sq.\mu m$$

	W	L	C_{GS}	C_{GD}
M1	1 µm	1 µm	4fF	4fF
M2	4µm	1µm	16fF	16fF
M3	2µm	1µm	8fF	8fF
M4	1µm	1µm	4fF	4fF
M5	1µm	1µm	4fF	4fF

Capacitance at X

$$C_{GD_1} + C_{GD_2} + C_{GS_3} = 28fF$$

Capacitance at Z

$$C_{GD_5} + C_{GS_4} = 8fF$$

Capacitance at output

$$C_{GD_3} + C_{GD_4} + C_L = 1.012\ pF$$

Resistance at output

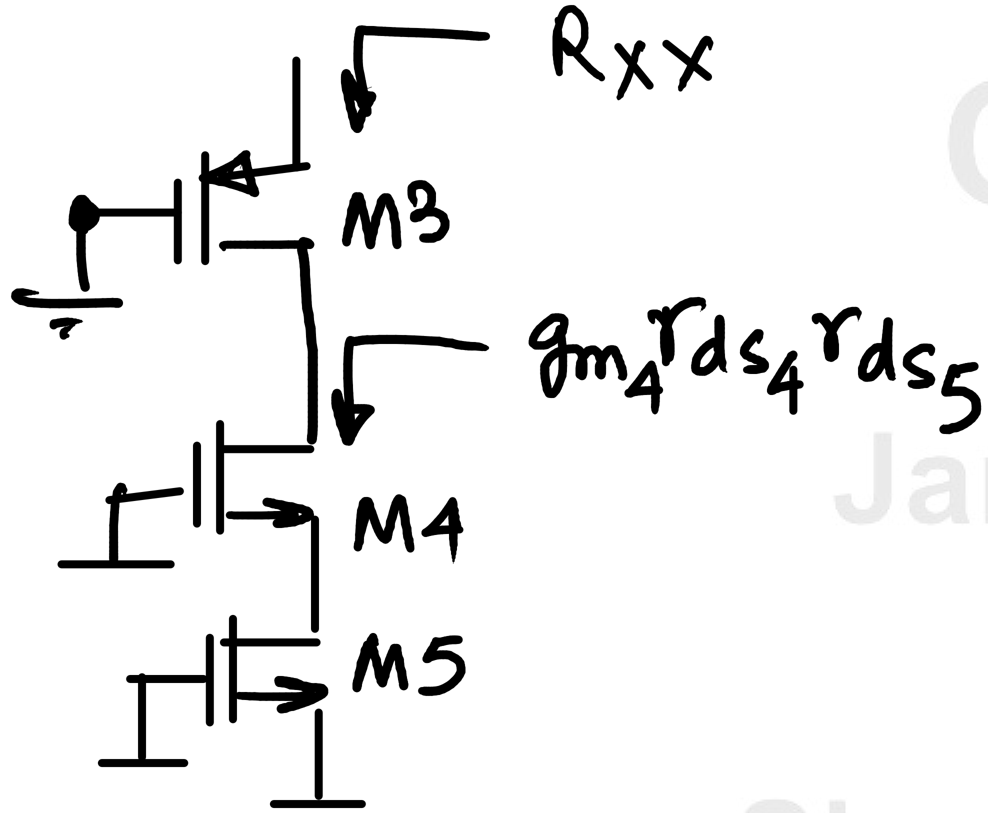
$$\left(g_{m_4} r_{ds_4} r_{ds_5}\right) \| \left(g_{m_3} r_{ds_3} \left(r_{ds_1} \| r_{ds_2}\right)\right)$$

$$= (36\mu \cdot 10M \cdot 1.2M) \| (36\mu \cdot 10M \cdot (10M \| 0.6M))$$

$$= 430M\Omega \| 200M\Omega = 136\ M\Omega$$

Resistance at X :

$R_X = r_{ds_1} \| r_{ds_2} \|$ Resistance looking into source of M3



$$R_{XX} = \frac{r_{ds_3} + g_{m_4} r_{ds_4} r_{ds_5}}{1 + g_{m_3} r_{ds_3}}$$

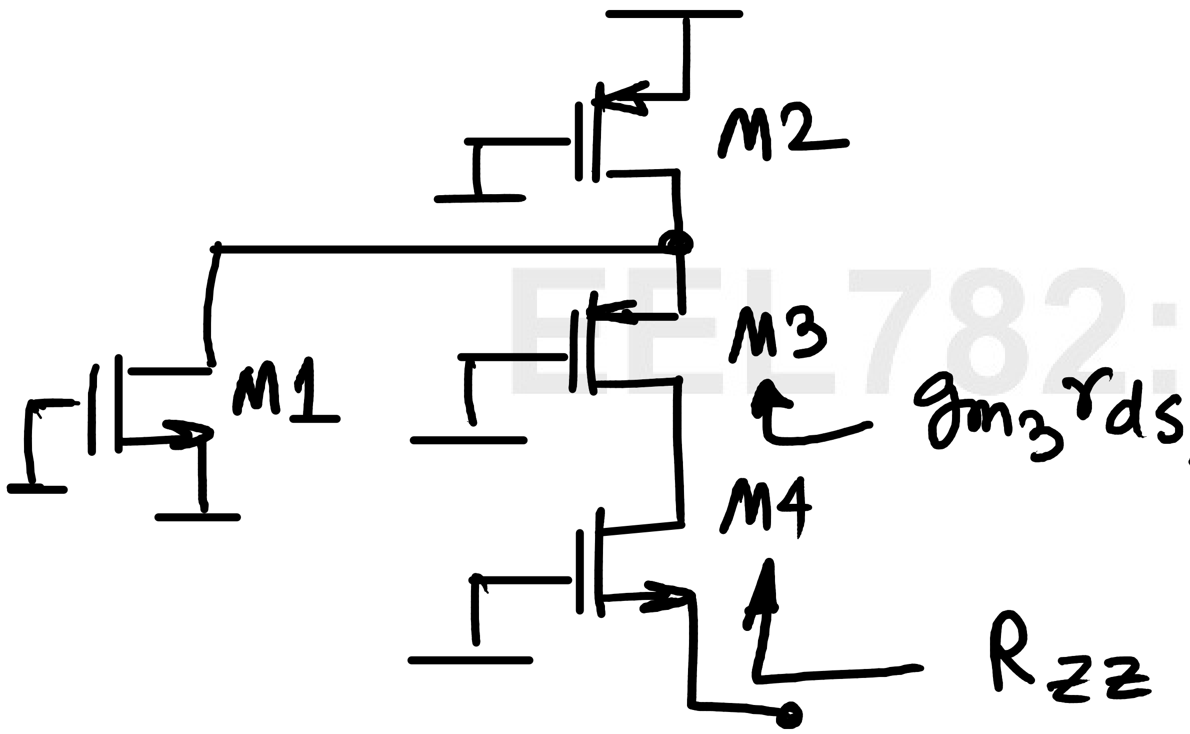
$$= \frac{10M\Omega + 430M\Omega}{1 + 36\mu \cdot 10M.}$$

$$= 1{\cdot}2\,M\Omega$$

$$R_X = 10M\Omega \| 0{\cdot}6M\Omega \| 1{\cdot}2M\Omega = 380\,k\Omega$$

Resistance at Z (Similar to X)

$R_Z = r_{ds_5} \|$ Resistance looking into source of M4



$g_{m_3} r_{ds_3} (r_{ds_2} \| r_{ds_1})$

$$R_{zz} = \frac{r_{ds_4} + g_{m_3} r_{ds_3} (r_{ds_2} \| r_{ds_1})}{1 + g_{m_4} r_{ds_4}}$$

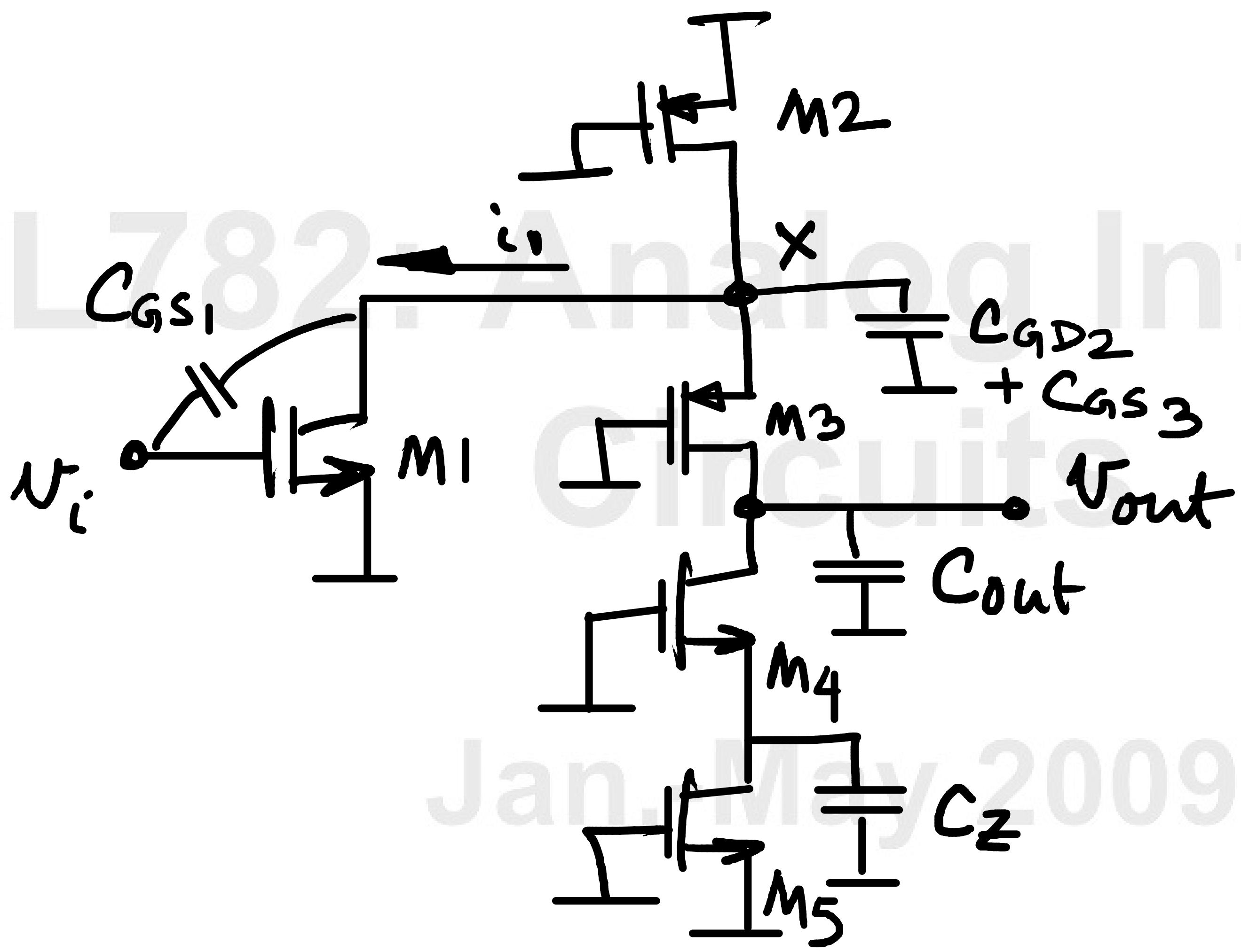
$$= \frac{10\,M\Omega + 200\,M\Omega}{1 + 36\mu \cdot 10M} = 580\,k\Omega$$

$$R_Z = 1.2\,M\Omega \,\|\, 580\,k\Omega = 390\,k\Omega$$

Pole frequencies

	Cap	Res	Pole	
X	28 fF	380 kΩ	94 Mrad/sec	15 MHz
Z	8 fF	390 kΩ	320 Mrad/sec	51 MHz
Output	1.012 pF	136 MΩ	7.2 krad/sec	1.15 kHz

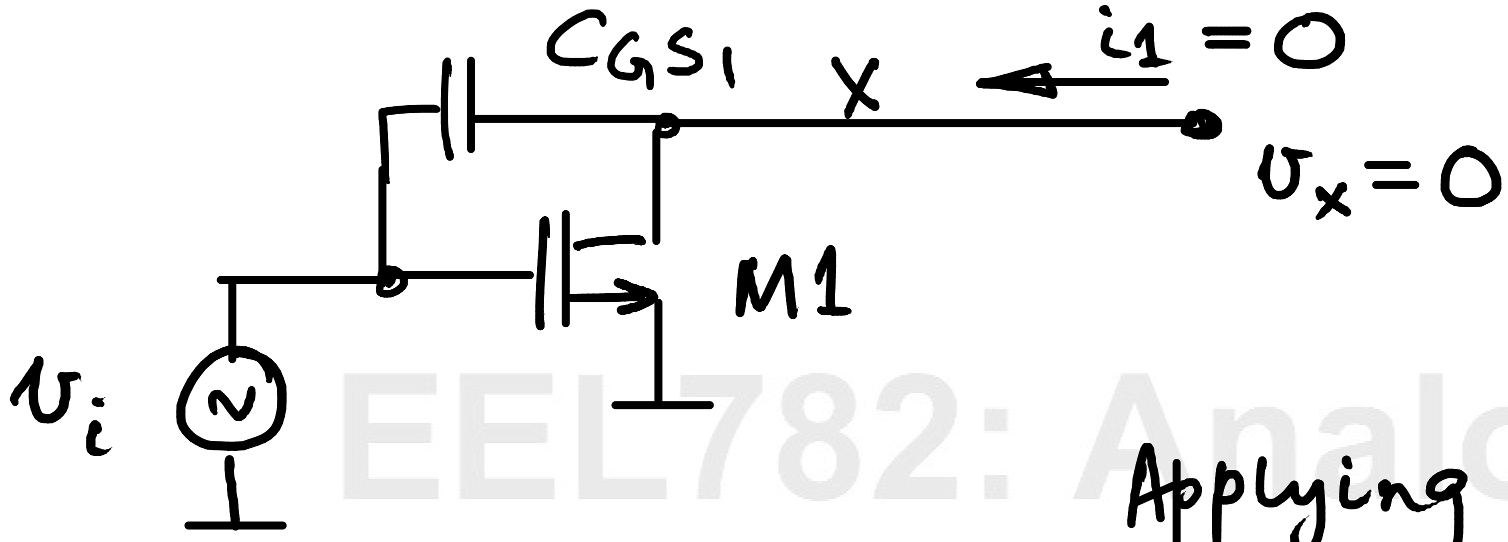
(3)



Zero in the transfer function means that at some complex frequency, say 's_z', $v_{out} = 0$

Consider applying v_i at the input. If $i_1 = 0$ at 's_z', then $v_x = 0$ ($v_x = i_1 \cdot$ some impedance), which means $v_{out} = 0$.

So is there a frequency 's_z' at which $i_1 = 0$?



Applying KCL at X,

$$g_m v_i = v_i \cdot s_z C_{GS1}$$

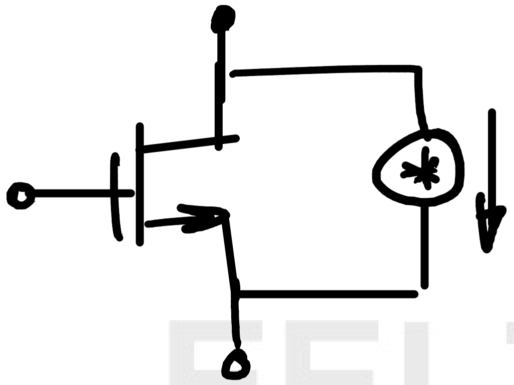
$$\therefore \; s_z = g_{m1} / C_{GS1}$$

There exists an RHP zero at g_{m1}/C_{GS1} or at $\frac{36\,\mu S}{4\,fF}$

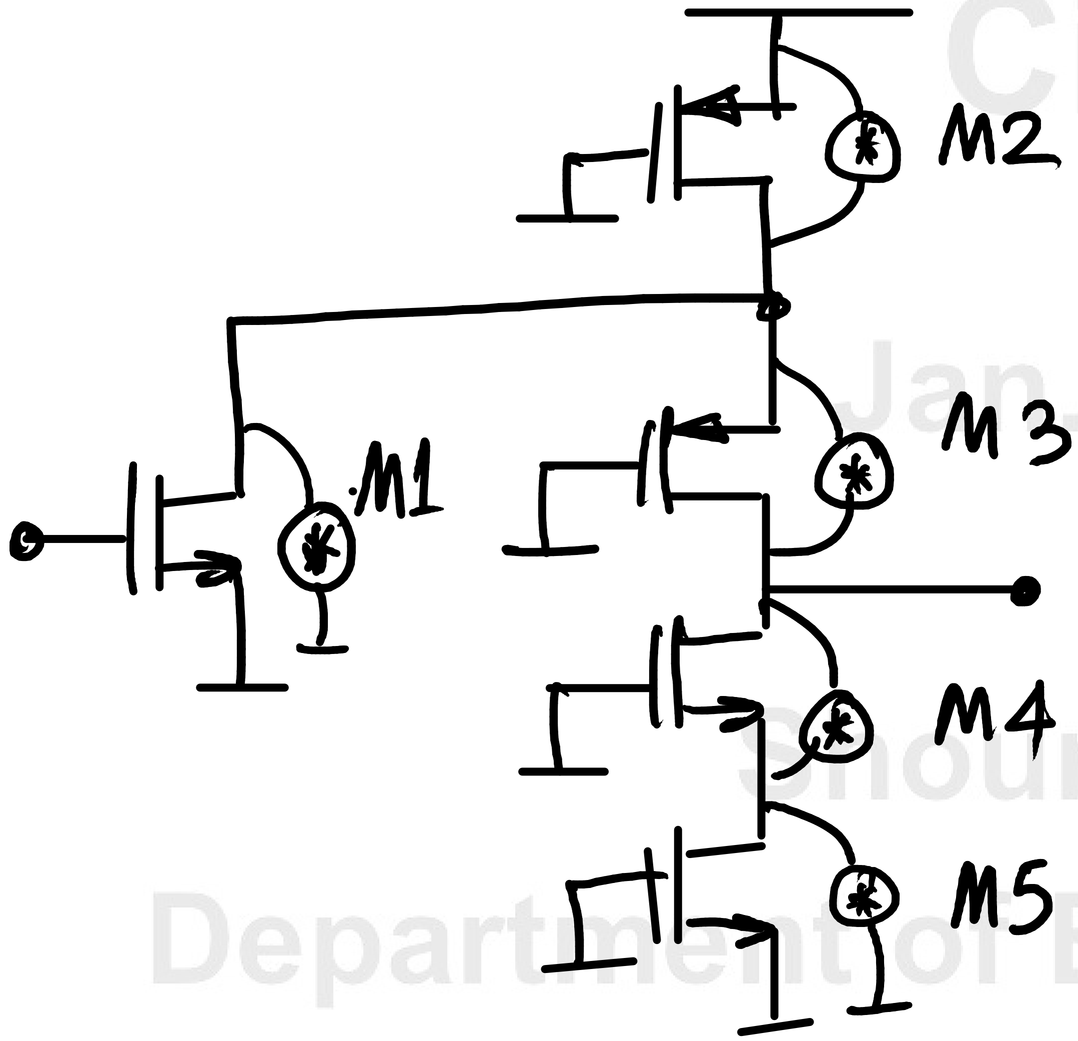
$$= 9 \text{ GRad/sec}$$

$$= 1.4 \text{ GHz}.$$

④



$\overline{i_n^2} = 2qI$



<u>M1</u>

Noise current sq. $= \overline{i_n^2}$

Input refd. noise voltage sq. $= \dfrac{\overline{i_n^2}}{g_{m_1}^2}$

$$= \frac{2q \cdot 0.9\mu A}{36\mu S \cdot 36\mu S}$$

$$= 222 \times 10^{-18}\ V^2/Hz$$

<u>M2</u> Any noise current will go to the output (source of M3 is low impedance).

$$v_{out} \approx i_{n_2} \cdot R_{out}$$

Input referred noise voltage sq. $= \dfrac{v_{out}^2}{G_m^2 R_{out}^2} = \dfrac{i_{n_2}^2 \cdot R_{out}^2}{G_m^2 \cdot R_{out}^2}$

$$= \frac{i_{n_2}^2}{G_m^2} \approx i_{n_2}^2 / g_{m_1}^2$$

$$\overline{i_{n_2}^2}/g_{m_1}^2 = \frac{2q \cdot 1.8\,\mu A}{36\,\mu S \cdot 36\,\mu S} = 222 \times 10^{-18}\ V^2/Hz$$

<u>For M5</u> All the noise current will go to the output node (source of M4 is low impedance compared to r_{ds_5})

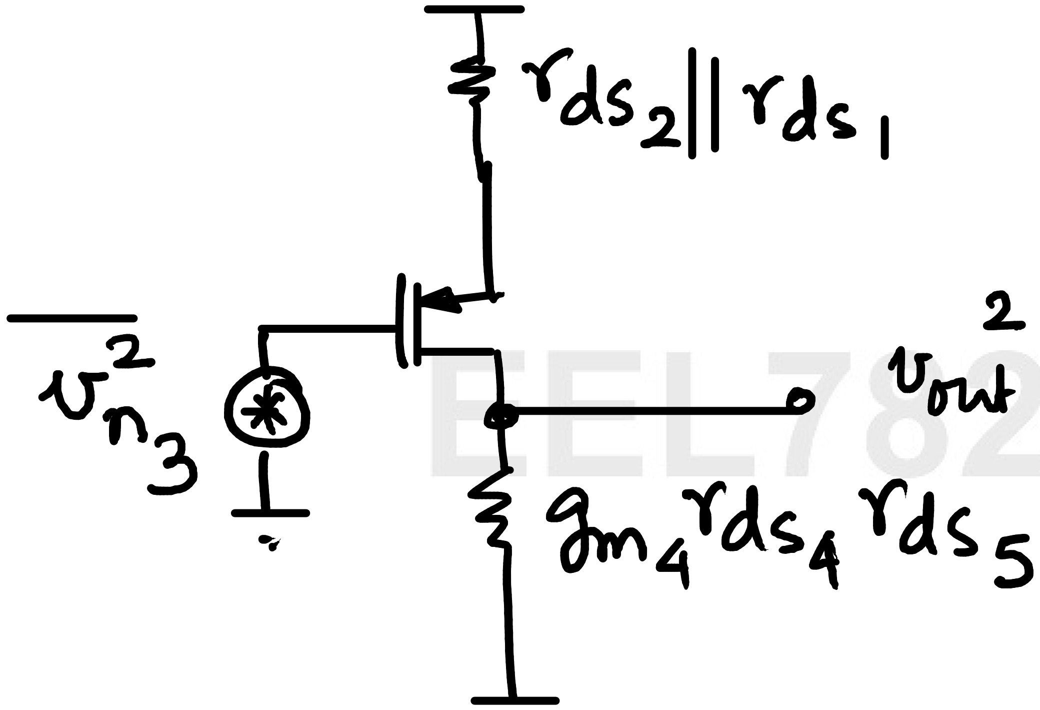
$$\therefore\ \overline{v_{out}^2} \approx \overline{i_{n_5}^2} \cdot R_{out}^2$$

Input referred noise voltage $= \dfrac{\overline{v_{out}^2}}{G_m^2 R_{out}^2} = \dfrac{\overline{i_{n_5}^2}}{G_m^2} \approx \dfrac{\overline{i_{n_5}^2}}{g_{m_1}^2}$

$$= \frac{2q \cdot 0.9\,\mu A}{36\,\mu S \cdot 36\,\mu S} = 222 \times 10^{-18}\ V^2/Hz$$

<u>For M3</u> Let us consider the equivalent noise voltage of the device M3 (as opposed to noise current)

$$\overline{v_n^2} = \overline{i_n^2}/g_m^2 = 2qI/g_m^2$$



$$v_{out}^2 \approx \frac{v_{n_3}^2}{(r_{ds_2} \| r_{ds_1})^2} (g_{m_4} r_{ds_4} r_{ds_5})^2$$

Input referred noise :

$$\frac{v_{out}^2}{G_m^2 R_{out}^2}$$

$$= 5.4 \times 10^{-18} \, V^2/Hz$$

$$G_m \approx g_{m_1} = 36 \mu S$$

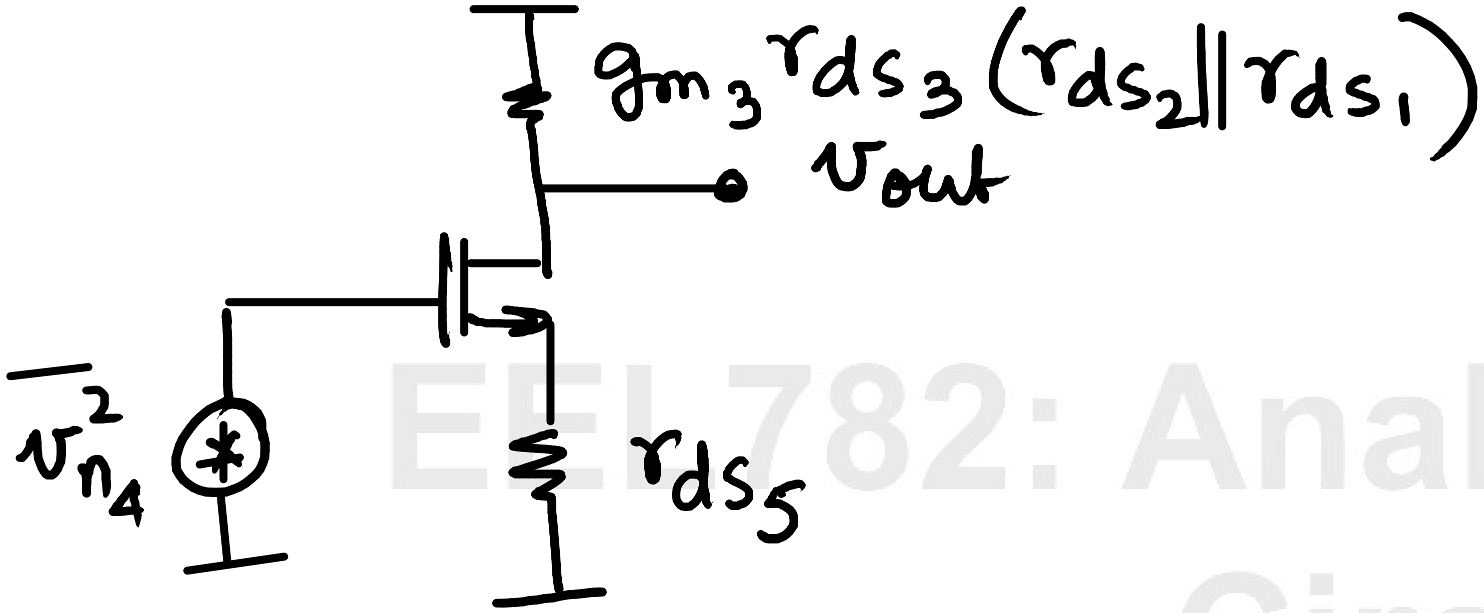
$$R_{out} \approx 136 \, M\Omega$$

$$g_{m_4} r_{ds_4} r_{ds_5} = 432 \, M\Omega$$

$$r_{ds_2} \| r_{ds_1} = 566 \, k\Omega$$

$$v_{n_3}^2 = \frac{2q \cdot 0.9 \mu A}{36 \mu S \cdot 36 \mu S} = 222 \times 10^{-18} \, \frac{V^2}{Hz}$$

M4 Consider the equivalent noise voltage



$$\overline{v_{out}^2} = \frac{\overline{v_{n_4}^2}}{r_{ds_5}^2} \left[g_{m_3} r_{ds_3} (r_{ds_2} \| r_{ds_1}) \right]^2$$

$G_m \approx 36\,\mu S$

$R_{out} = 136\,M\Omega$

$g_{m_3} r_{ds_3} (r_{ds_2} \| r_{ds_1}) = 200\,M\Omega$

$r_{ds_5} = 1.2\,M\Omega$

$$\overline{v_{n_4}^2} = \frac{2q \cdot 0.9\,\mu A}{36\,\mu S \cdot 36\,\mu S} = 222 \times 10^{-18}\,V^2/Hz$$

$$\text{Input refd. noise} = \frac{\overline{v_{out}^2}}{G_m^2 R_{out}^2}$$

$$= 0.25 \times 10^{-18}\,V^2/Hz$$

<u>Total input referred noise</u>

$$\underset{(M1)}{222 \times 10^{-18}} + \underset{(M2)}{444 \times 10^{-18}} + \underset{(M5)}{222 \times 10^{-18}} + \underset{(M3)}{5.4 \times 10^{-18}} + \underset{(M4)}{0.25 \times 10^{-18}}\ V^2/Hz$$

$$= 894 \times 10^{-18}\,V^2/Hz$$

(5) Input signal : 0.1 V amplitude

Gain-Bandwidth of design : 1.2 kHz × 36 μS × 136 MΩ = 5.9 MHz

Noise voltage within GBW : $\sqrt{v_n^2 \times 5.9\,\text{MHz}}$

= 73 μV

Out of a 0.1 V signal, input referred 73 μV noise → SNR of about 63 dB

not bad