Tutorial Sheet: Maxima-Minima

1. Using Taylor's formula, find quadratic and cubic approximations $e^x \sin y$ at origin. Estimate the error in approximations if $|x| \leq 0.1\,, |y| \leq 0.2$.

2. Find all the critical points of $f(x, y) = \sin x \sin y$ in the domain $-2 \leq x \leq 2,\ -2 \leq y \leq 2$.

3. Find local minima and local maxima points of the function $f(x, y) = xye^{-(x^2+y^2)}$.

4. Let $f(x, y) = (y - 4x^2)(y - x^2)$. Verify that $(0, 0)$ is a saddle point of f.

5. Let $f(x, y) = (x - y)^2$. Find all critical points of f and categorize them according as they are either saddle points or the location of local extreme values. Is the second derivative test useful in this case?

6. In each of the cases below, discriminant is zero. Find the critical points and their nature by imagining the surface $z = f(x, y)$ (a) $f(x, y) = x^2y^2 \qquad (b)1 - x^2y^2 \quad (c)f(x, y) = xy^2$.

7. Find the maximum and minimum values of $f(x, y) = 3x + 4y$ subject to the constraint $x^2 + 4xy + 5y^2 = 10$.

8. Find the points on the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ which are nearest and farthest from the straight line $3x + y - 9 = 0$.

9. Find the maximum volume of a rectangular solid in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with one vertex at the origin and the other vertex on the plane $x + y + \dfrac{z}{2} = 1$.

10. Show that the closed cylinder (with lids) with the greatest surface area that can be inscribed in a sphere of radius a has the altitude $h = a\sqrt{2 - \frac{2}{\sqrt{5}}}$ and the radius of the base $r = \frac{a}{2}\sqrt{2 + \frac{2}{\sqrt{5}}}$.

11. A farmer wishes to build a rectangular storage bin, without a top, with a volume of 500 cubic meters. Find the dimensions of the bin that will minimize the amount of material needed in its construction.

12. A company produces steel boxes at three different plants in amounts x, y and z, respectively, producing an annual revenue of $f(x, y, z) = 8xyz^2 - 200(x + y + z)$. The company is to produce 100 units annually. How should the production be distributed to maximize revenue?