ELL225

Lecture - 10

$$G(s) = \frac{b(s)}{a(s)}$$

The "order" of $G(s)$ or the system is equal to the degree of $a(s)$ (after cancellation of common factors in numerator & denominator)

$$\frac{A_1}{s+\alpha} + \frac{A_2}{(s+\alpha)^2} + \cdots + \frac{As+B}{s^2+as+b}$$

first order — second order — second order

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\xi \rightarrow$ scalar damping coefficient

roots of $\boxed{s = -\sigma_d \pm j\omega_d}$

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

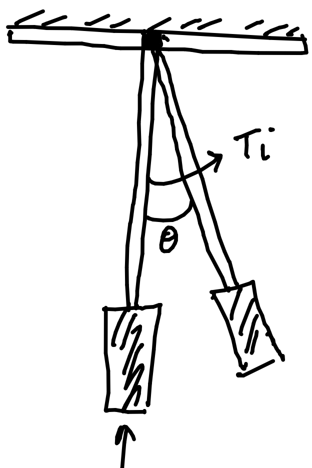
$\omega_n \leftarrow$ natural frequency

$\sigma_d = \xi\omega_n$ (Real part of the roots)

$\omega_d = \omega_n\sqrt{\xi^2-1}$ (Imaginary part)

Poles of 2nd order system	Damping ratio ξ - value	Impulse Response $y(t)$
• Two distinct real roots $-\sigma_1, -\sigma_2$	$\xi > 1$	Over damped system $y(t) = k_1 e^{-\sigma_1 t} + k_2 e^{-\sigma_2 t}$
• Two real roots at $-\omega_n$	$\xi = 1$	Critically damped $y(t) = k_1 e^{-\sigma_1 t} + k_2 t e^{-\sigma_1 t}$
• Two complex conjugate pair $-\sigma_d \pm j\omega_d$	$0 < \xi < 1$	Under damped $y(t) = A e^{-\sigma_d t} \cos(\omega_d t - \varphi)$
• Purely imaginary roots	$\xi = 0$	Undamped system $y(t) = A\cos(\omega_n t - \varphi)$

→



equillibrium point

Linearized model

(Linearizing about equillibrium point $(0,0)$)

$$\Delta \dot{x}_1 = \Delta x_2$$

$$\Delta \dot{x}_2 = -\underbrace{\tfrac{g}{l}}_{\left(-\frac{g}{l}\cos x_1\right)} \Delta x_1 - \frac{D}{ml^2}\Delta x_2 + \frac{1}{ml^2}\Delta T_i$$

$$s\,\Delta X_1(s) = \Delta X_2(s)$$

$$s\,\Delta X_2(s) = -\frac{g}{l}\Delta X_1(s) - \frac{D}{ml^2}\Delta X_2(s) + \frac{1}{ml^2}\Delta T_i(s)$$

$$\Delta X_2(s) = -\left(\frac{ml^2}{s\,ml^2 + D}\right) g/_l \,\Delta X_1(s) + \frac{1}{ml^2}\Delta T_i(s)$$

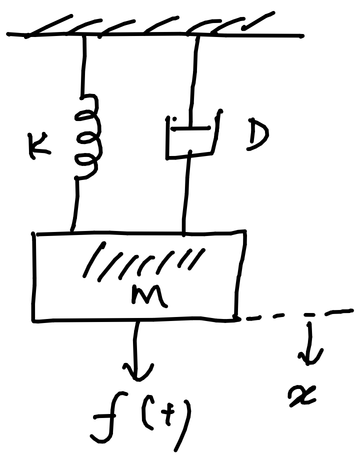
$\Delta x_1 = \Delta\theta$

$m = 1 \qquad l = 1$

$$\boxed{\Delta X_1(s) = \frac{s + D}{s^2 + Ds + 9.8}\,\Delta T_i(s)}$$

Impulse response $\rightarrow \qquad \mathcal{L}^{-1}\left(\dfrac{s + D}{s^2 + Ds + 9.8}\right)$

step response $\rightarrow \qquad \mathcal{L}^{-1}\left(\dfrac{s + D}{s(s^2 + Ds + 9.8)}\right)$

- 

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K_D s + K_s}$$

Let $M = 1$

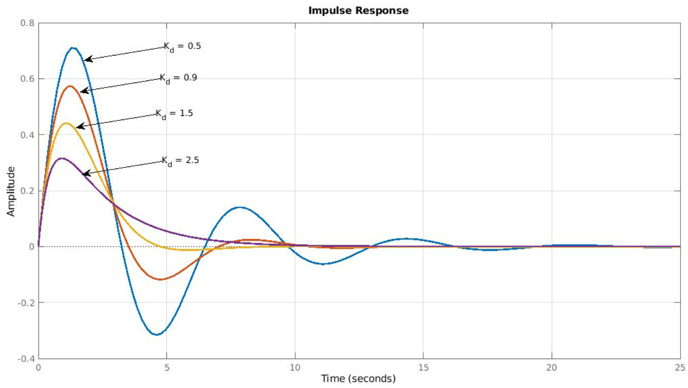
& $K_s = 1$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + K_D s + 1}$$

Impulse respons $\qquad \mathcal{L}^{-1}\left(\dfrac{1}{s^2 + K_D s + 1}\right)$

Step Response $\qquad \mathcal{L}^{-1}\left(\dfrac{1}{s(s^2 + K_D s + 1}\right)$

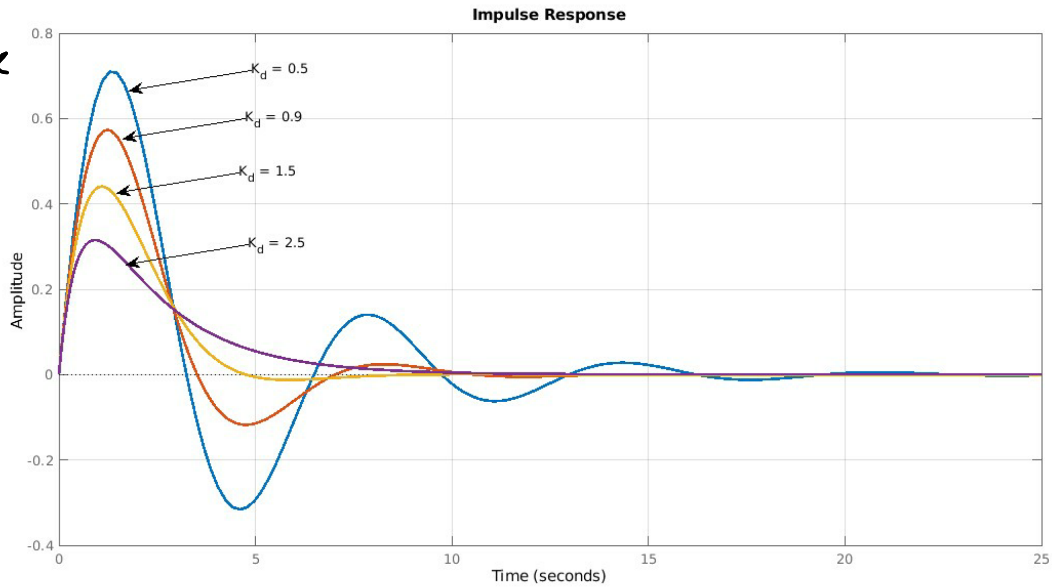
Impulse Response of Spring-mass-damper System



↓ magnified

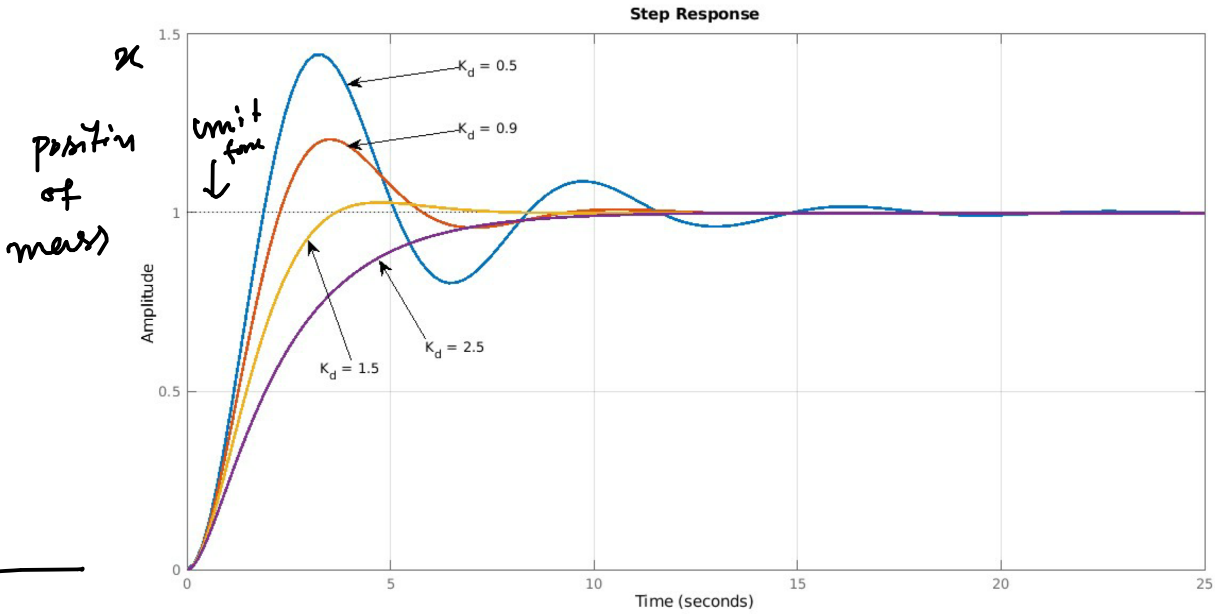
x

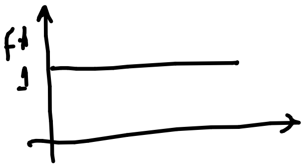
Position of mass



Time →

Step Response for Spring-mass-damper system.





A unit amplitude force is given to the mass

→ For second order System :

- <u>Rise Time</u> : The time required for the <u>response</u> $y(t)$ to go from 10% of final value to 90% of the final value of the response.

- Peak Time : The time required to reach the maximum peak of the response.

- % Overshoot : $$\frac{y_{mp} - y_{ss}}{y_{ss}} \times 100$$

  y_{mp} : Maximum peak value
  y_{ss} : Steady state or final value

- settling time : The time required for the response to reach & stay within $\pm 2\%$ of the final value of $y(t)$.

- Step respons of spring-mass-damper System.: