# ELL225

## Lecture-15

LTI System

S.S.R

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

It is stable if the natural response goes to 0 as $t \to \infty$

Transfer funt?

$$G(s) = \frac{b(s)}{a(s)}$$

The system is stable if the impulse response goes to 0 as $t \to \infty$

$$G(s) = \frac{c\,\mathrm{adj}(sI-A)\,b}{|sI-A|} = \frac{b(s)}{a(s)}$$

Can we say that the impulse response goes to zero $\Rightarrow$ natural response goes to 0?

• $G(s) = \dfrac{1}{s+1}$ ⟸ ← Impulse response will go to 0, so stable

$$\left\{\begin{aligned} \dot{x} &= \underbrace{\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}_{b} u \\ y &= \underbrace{[0 \quad 1]}_{c}\, x \end{aligned}\right.$$

Unstable

$x(t) = (\ )e^{\lambda_1 t} + (\ )e^{\lambda_2 t}$

e is value of A

$\lambda_1 = -1 \qquad \lambda_2 = 1$

$x(t) = (\ )\underbrace{e^{-1t}}_{\downarrow\, 0} + (\ )\underbrace{e^{t}}_{\downarrow\, \infty}$

$x(t) \longrightarrow \infty$ as $t \to \infty$

$y(t) = c\,x(t) \longrightarrow \infty$ as $t \to \infty$

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & 0 \\ -1 & s-1 \end{bmatrix}^{-1}$$

$$= \frac{1}{(s+1)(s-1)} \begin{bmatrix} s-1 & 0 \\ 1 & s+1 \end{bmatrix}$$

$$G(s) = c(sI - A)^{-1} b = \frac{1}{(s+1)(s-1)} \left\{ [0 \quad 1] \begin{bmatrix} s-1 & 0 \\ 1 & s+1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

$$= \frac{1}{(s+1)(s-1)} \left\{ [0 \quad 1] \begin{bmatrix} -2(s-1) \\ -2 + s + 1 \end{bmatrix} \right\}$$

The unstable pole is cancelled with a zero.

$$= \frac{\cancel{(s-1)}}{(s+1)\cancel{(s-1)}} = \frac{1}{s+1}$$

- Poles : The roots of $a(s) \rightarrow -1$
- Eigenvalues : The roots of $|sI - A| \rightarrow -1, 1$

- $\boxed{\text{The poles of } G(s) \subseteq \text{The eigenvalues of matrix } A}$

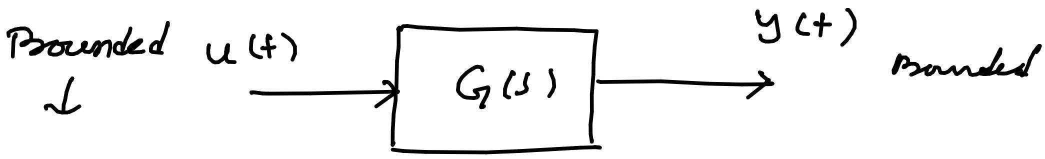
?

Under what condition, the equality will hold ?

- If the transfer function $G(s)$, obtained by performing $G(s) = c(sI - A)^{-1} b$, there is no pole and zero cancellation, then:

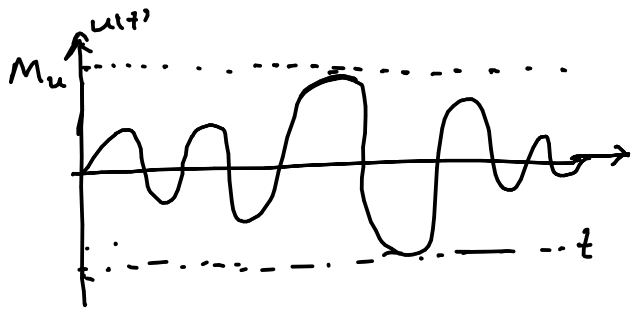
$$\boxed{\text{The poles of } G(s) = \text{The eigenvalues of } A}$$

- Assume that the transfer function of an LTI system is $G(s)$. We say that system is bounded input bounded output (BIBO) stable if for every bounded input $u(t)$ to the system produces bounded output $y(t)$.




$\sup_{t \in [0,\infty)} |u(t)| \leq M_u$

$M_u$ ← a positive scalar




→ Derive a condition such that for every bounded input $u(t)$, the output $y(t)$ of the system is also bounded.

i.e.

- $\sup_t |y(t)| \leq M_y$, for every $u(t)$ satisfying $\sup_t |u(t)| \leq M_u$

$M_u$ & $M_y$ are some +ve scalars.

$$u(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$$y(t) = u(t) * g(t)$$

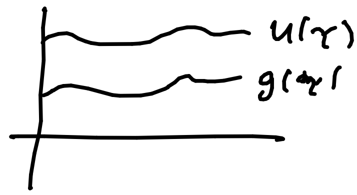
$g(t)$ is impulse response of the system

$$|y(t)| = \left| \int_0^t u(t-\tau)\, g(\tau)\, d\tau \right|$$

$$\leq \int_0^t |u(t-\tau)\, g(\tau)|\, d\tau$$

$|ab| = |a||b|$

$$= \int_0^t |u(t-\tau)|\, |g(\tau)|\, d\tau$$




$$\leq \sup_t |u(t)| \int_0^t |g(\tau)|\, d\tau$$

Since we are using bounded inputs

$\sup_t |u(t)| \leq M_u$ ($M_u$ is a +ve number)

$$\leq M_u \int_0^t |g(\tau)|\, d\tau$$

$$\leq M_u \int_0^t |g(\tau)|\, d\tau + \underbrace{M_u \int_t^\infty |g(\tau)|\, d\tau}$$

$$= M_u \int_0^\infty |g(\tau)|\, d\tau$$

$$|y(t)| \leq M_u \int_0^\infty |g(\tau)|\, d\tau$$

$$|y(t)| \leq M_u \int_0^\infty |g(\tau)| d\tau$$

$$\Downarrow$$

If $\underline{\underline{\int_0^\infty |g(\tau)| d\tau}} \leq M_g$ (a +ve number), then

$$\sup_t |y(t)| \leq M_y \text{ (a +ve number)}$$

$$\underbrace{\int_0^\infty |g(\tau)| d\tau}_{} \leq M_g$$

→ The above integral is finite if and only if the <u>impulse response</u> <u>$g(t)$ goes to 0 as $t \to \infty$</u>.

$$|||$$

- The <u>poles of $G(s)$ are in the open left half of complex plane</u>.