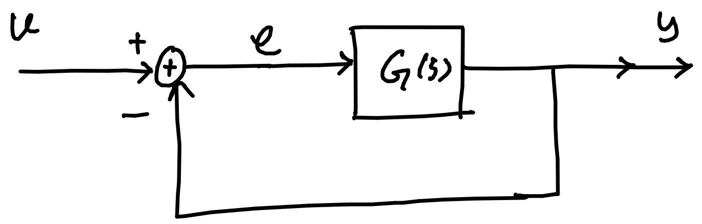
Lecture - 16 Continued ...

→ Steady State Error



$e(t) = u(t) - y(t)$ error signal

$$E(s) = U(s) - Y(s)$$

$$= U(s) - G(s)E(s)$$

$$\boxed{\Rightarrow \quad E(s) = \frac{1}{1+G(s)} U(s)}$$

steady state error $\quad e(\infty) := \lim_{t \to \infty} e(t)$

To compute $e(\infty)$, one can proceed as follows:

(i) Obtain Laplace inverse of $E(s) \rightarrow e(t)$

(ii) Compute $\lim_{t \to \infty} e(t) = e(\infty)$

→ We are interested in finding a condition, under which the steady state error (SSE) can be made 0 as $t \to \infty$.

↓

We consider the following approach:

→ $$\boxed{\lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,E(s)}$$

Final Value Theorem

This relation holds for $E(s)$, which has poles in the open left half of cmplx plane or at most one pole at origin.

→ $$\boxed{e(\infty) = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \frac{s}{1 + G(s)} U(s)}$$

Test signals:

- Step → $U(s) = \frac{1}{s}$
- Ramp → $U(s) = \frac{1}{s^2}$
- Parabolic → $U(s) = \frac{1}{s^3}$

→ For Step-input

$$G(s) = \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$ with $n \geq m$.

$$e(\infty) = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \frac{1}{1 + G(s)}$$

$$e(\infty) = \lim_{s \to 0} \frac{1}{1 + G(s)}$$

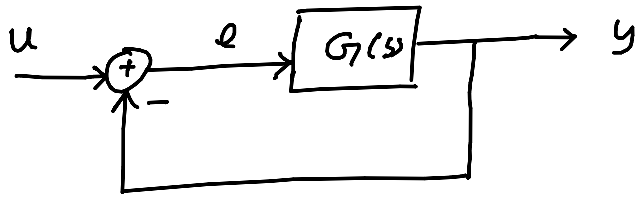
$$= \frac{1}{1 + \lim_{s \to 0} G(s)}$$

Under what condition $e(\infty)$ can be made 0?

ELL225

Lecture - 17

→ steady state error



$$e(\infty) = \lim_{t \to \infty} e(t)$$

We need $e(\infty) = 0$

We write $G(s) = \dfrac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$

with $\underline{\underline{n \geq m}}$

z_i : zeros of $G(s)$

p_i : poles of $G(s)$

→ step input $\mathcal{L}(u(t)) \to \frac{1}{s}$

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{1}{1 + G(s)}$$

$$= \frac{1}{1 + \underbrace{\lim_{s \to 0} G(s)}_{K_p}}$$

$$K_p = \lim_{s \to 0} G(s)$$ ← Position error constant.

$$e(\infty) = \frac{1}{1 + K_p}$$

To make $e(\infty) = 0$ we need $K_p = \infty$

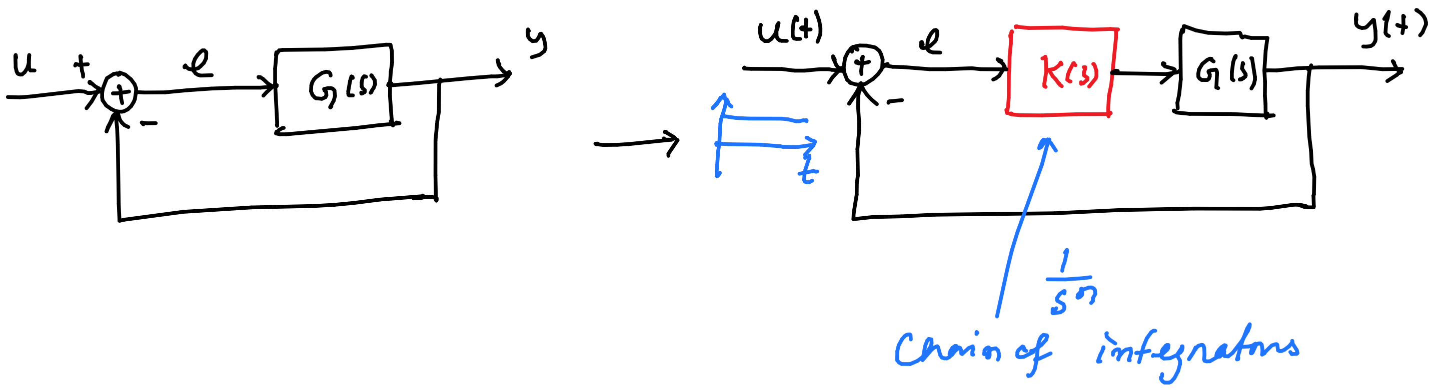
$$\lim_{s \to 0} G(s) = \infty$$

$$\tilde{G}(s) = K(s)\, G(s) \qquad G(s) = \frac{s+1}{(s+3)(s+2)}$$

$$= \frac{1}{s^n} \cdot G(s)$$

For $\boxed{n \geq 1}$ $\quad \lim_{s \to 0} \tilde{G}(s) = \infty$

- We need at least one integrator in the forward path to make SSE to 0 for step input.



→ For ramp input $u(t) = t$

$$\mathcal{L}(u(t)) = \frac{1}{s^2}$$

$$e(\infty) = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} s \cdot \frac{1}{1 + G(s)} \cdot \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{s + s\,G(s)}$$

$$= \frac{1}{\lim_{s \to 0} s\,G(s)}$$

We need $\lim_{s \to 0} s\,G(s) = \infty$

$$\tilde{G}(s) = \frac{1}{s^n} \cdot G(s) \quad \text{when} \quad \boxed{n \geq 2}$$

- We need at least 2 integrators to make SSE $= 0$ for ramp input.

$K_v := \lim_{s \to 0} s\,G(s)$ ← Velocity error constant.

→ Parabolic input $u(t) = t^2$

$$\mathcal{L}(u(t)) = \frac{1}{s^3}$$

$$e(\infty) = \lim_{s \to 0} s \cdot \frac{1}{1 + G(s)} \cdot \frac{1}{s^3}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

To make $e(\infty) = 0$, we need at least 3 - more integrators in the forward path.

$$\tilde{G}(s) = \frac{1}{s^n} \cdot G(s) \qquad \boxed{n \geq 3}$$

$$K_a := \lim_{s \to 0} s^2 G(s) \quad \leftarrow \text{Accelaration const.}$$

→ <u>Order of a transfer fnt?</u>:

$$G(s) = \frac{b(s)}{a(s)} \quad \text{with no poles at } 0.$$

Degree of denominator polynomial is order of $G(s)$ or system.

→ <u>Type of a system</u>

It is equal to the number of integrators in $G(s)$

$$\tilde{G}(s) = \frac{1}{s^n} \cdot \frac{b(s)}{\alpha(s)}$$

n : Type of $\tilde{G}(s)$

η := the degree of denominator polynomial ($s^n \alpha(s)$) is equal to the order of the plant or $G(s)$.

Type 0 : $n = 0$ (no integrators or no poles at origin)

Type 1 : $n = 1$ (One pole at origin or one integrator in the forward path)