ELL 225

Lecture - 16

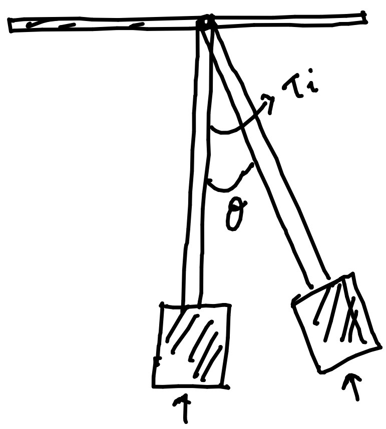
How to study stability of a non-linear system, which is linearized at different equilibrium points?

- For each equillibrium point / steady state operating point we have different state space realizations.

→ When a system has "multiple equillibrium points"

↓

Stability analysis of equillibrium points (instead of a original system)



← Two equillibrium points

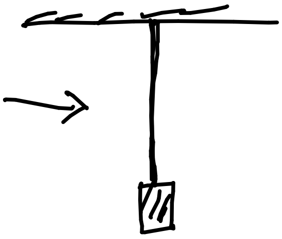
(0, 0)

(π, 0)

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/l \cos x_1 & -D/ml^2 \end{bmatrix}_{\bar{x}} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

→ Linearizing about the point (0, 0)

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/l & -D/ml^2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

→ 

↓ det

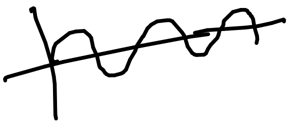
$$s^2 + \frac{D^2}{ml^2} s + g/l = 0$$

$m = 1$ unit
$l = 1$ unit

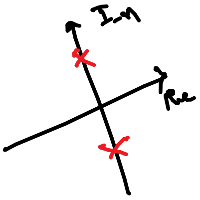
Eigenvalues $\lambda_i = \dfrac{-D \pm \sqrt{D^2 - 4g}}{2}$

D : damping coefficient

when $D = 0$



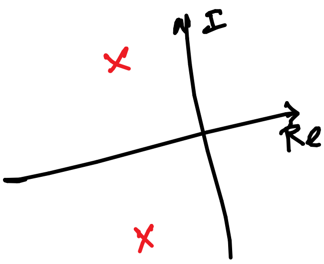
$\lambda_i = \pm j\sqrt{g}$ $\quad \rightarrow \Delta x(t) = K \cos(\omega t - \varphi)$



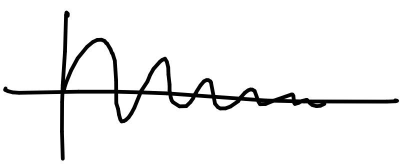
$D = 1 \rightarrow \lambda_i = \dfrac{-1 \pm \sqrt{1 - 4g}}{2}$

$$= -\tfrac{1}{2} \pm j\,\frac{6.18}{2}$$

$\sigma = 1/2$



$$\Delta x = K e^{-\sigma_1 t} \cos(\omega t - \varphi)$$

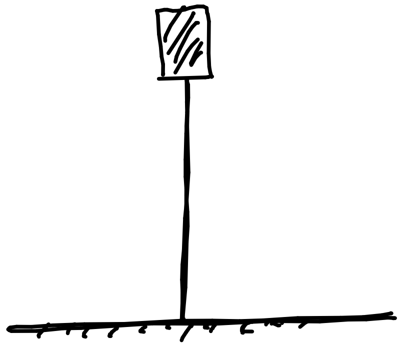


→ The S.S-R is STABLE (Asymptotically stable)

at (0, 0) equilibrium point.

→ The (0,0) equillibrium point is stable.

→ Linearize about $(\pi, 0)$.



$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ g/l & -D/ml^2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$\downarrow$$

$$s^2 + D/ml^2 \div g/l = 0$$

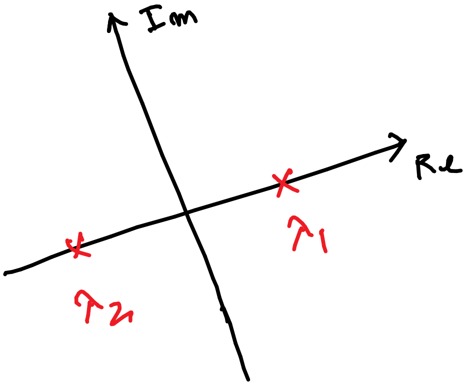
$$\lambda_i = \frac{-D \pm \sqrt{D^2 + 4g}}{2}$$

$D = 1$

$$\lambda_i = \frac{-1 \pm \sqrt{1 + 39.2}}{2}$$

$$= -\tfrac{1}{2} + \frac{6.34}{2}, \quad -\tfrac{1}{2} - \frac{6.34}{2}$$

λ_1 $\qquad\qquad$ λ_2



$$x(t) = (\ )e^{\lambda_1 t} + (\ )e^{\lambda_2 t}$$

$$\downarrow \qquad \downarrow$$

$$\infty \qquad 0$$

$$x(t) \longrightarrow \infty \quad \text{as} \quad t \to \infty$$

→ The S.S.R. obtained at $(\pi, 0)$ is <u>Unstable</u>.

↓

- The <u>equillibrium point $(\pi, 0)$ is unstable</u>.

LTI system

$\dot{x} = Ax + bu$
$y = Cx$

with transfer fun' : $G(s)$

↓ Stable if natural response $y_n(t) \to 0$ as $t \to \infty$

↑ Eigenvalues of A are in open lefhalf of $\mathbb{C}$.

$\Longrightarrow$

$\not\Longleftarrow$

Since poles of $G(s) \subseteq$ eigenvalues of A

↓ Stable/BIBO stable if impulse response $g(t) \to 0$ as $t \to \infty$

↑ Poles of $G(s)$ are in open left half of $\mathbb{C}$.

* We consider a transfer function $G(s)$ to be a valid transfer function if there are NO common factors between numerator & denominator polynomial (No common poles and zeros).