Lecture - 19

→ Routh Table (Special Cases):

- zero in the 1^{st} column

↓ proceed

→ Using ϵ method

→ Reversing the coefficients.

→ Another Special Case:

$$\alpha(s) = s^4 + 2s^3 + 3s^2 + 4s + 2$$
$$= \underbrace{(s^2+2)}(s^2+2s+1)$$

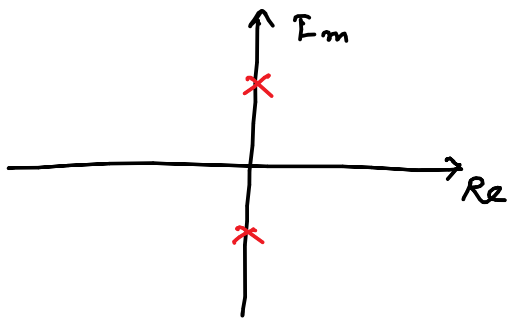
An even polynomial is factor of the original polynomial $\alpha(s)$.

s^4	1	3	2
s^3	2	4	0
s^2	1	2	0
s^1	0	0	0

← Zero row

The roots of even polynomial s^2+2

On the imaginary axis.



We write the original polynomial $\alpha(s)$ as follows:

$$\alpha(s) = \underset{\uparrow}{\alpha_e(s)} \cdot \tilde{\alpha}(s)$$

Even polynomial, which is a factor of original polynomial.

→ Another Example:

$$G(s) = \frac{1}{\underbrace{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}_{\alpha(s)}}$$

Routh Table:

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	$\cancel{-10}$ -1	$\cancel{-20}$ -2	$\cancel{10}$ 1	$\cancel{20}$ 2	0
s^5	$\cancel{20}$ 1	$\cancel{60}$ 3	$\cancel{40}$ 2	0	0
s^4	1	3	2	0	0
s^3	0 $\cancel{4}$ 2	0 $\cancel{6}$ 3	0	0	0
s^2	3	4	0	0	0
s^1	$1/3$	0	0	0	0
s^0	4				

} Two sign changes

← Entire row is zero

→ is only for even polynomial $\alpha_e(s)$, which is factor of $\alpha(s)$.

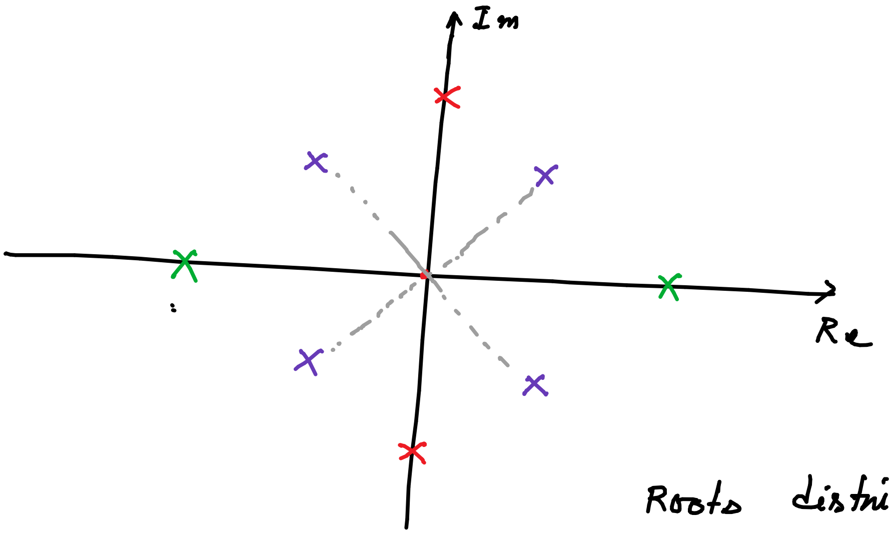
$\alpha_e(s) = s^4 + 3s^2 + 2 \leftarrow$ Even Polynomial

Even Polynomial $\rightarrow$ $s^8 + 3s^4 + 5s^2 + 33$

Odd Polynomial $\rightarrow$ $s^7 + 2s^5 + 3s^3 + s$

Property of an Even Polynomial

$\rightarrow$ All the roots of an even polynomial are symmetrical about the origin in the complex plane.



Roots distribution of an even polynomial

$$\alpha(s) = \underline{\alpha_e(s)}\,\underline{\tilde{\alpha}(s)}$$

$\alpha(s) \downarrow$ the roots set S_α

$\alpha_e(s) \downarrow$ root set S_{α_e}

$\tilde{\alpha}(s) \downarrow$ root set $S_{\tilde{\alpha}}$

$\underline{\alpha(s)} = (s^2+1)(s+1)$

$(s^2+1) \downarrow \pm j1$

$(s+1) \downarrow -1$

The relation : $S_\alpha = S_{\alpha_e} \cup S_{\tilde{\alpha}}$

- The roots of $\alpha(s)$ is equal to the union of roots of $\alpha_e(s)$ & $\tilde{\alpha}(s)$.

x

To complete the Routh table, we construct another polynomial from $P(s)$ as follows:

$$\alpha_e(s) = s^4 + 3s^2 + 2 \rightarrow \text{roots: } \begin{cases} \pm 1i \\ \pm 1.4142i \end{cases}$$

$$\frac{d\alpha_e(s)}{ds} = \underline{4}s^3 + \underline{6}s \leftarrow \text{Odd polynomial}$$

Use these coefficients to fill the zero row.

$\rightarrow$ $\alpha(s) = \tilde{\alpha}(s)\,\alpha_e(s)$

- There are two sign changes in 1^{st} column

$\Rightarrow$ there are two roots of $\alpha(s)$ in open right half of $\mathbb{C}$.

- There is no sign change in $\alpha_e(s)$

$\Downarrow$

No roots of $\alpha_e(s)$ are in open left half of Complex plane.

$\Downarrow$ Due to symmetricity about origin.

All 4 roots are on imaginary axis.

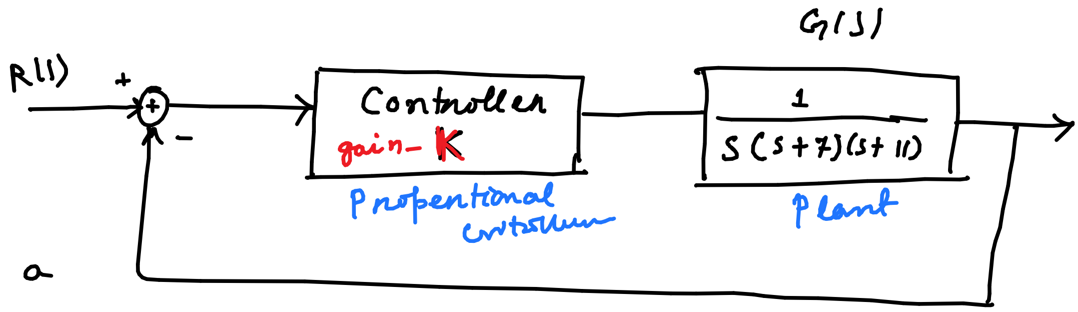
- System is <u>unstable</u>.

→ The row previous to the row of zeros contains the coefficients of even polynomial $\alpha_e(s)$, which is a factor of original polynomial $\alpha(s)$.

$\Downarrow$

Starting from the row, containing the coefficient of even polynomial $\alpha_e(s)$ (above zero row), to the end of Routh table, is a test only for even polynomial $\alpha_e(s)$.

→ Controller Design using Routh Criterion:



Gain K is a real number.

Is it possible to design K s.t. the unity negative feedback closed loop system is stable.

To study stability

The closed loop T.f. $T(s) = \dfrac{k}{s^3 + 18s^2 + 77s + k}$

To ensure stability: all the poles should belong to the open left half of $\mathbb{C}$.

$$
\begin{array}{c|cc}
s^3 & 1 & 77 \\
s^2 & 18 & K \\
s^1 & \dfrac{1386-K}{18} & 0 \\
s^0 & K &
\end{array}
$$

← For $K = 1386$ zero row.

To ensure stability → all elements must be +ve

Stable → $0 < K < 1386$

$K > 1386$ → Unstable stable

$K = 1386$ →

For $K = 1386$, the even polynomial

$$\alpha_e(s) = 18s^2 + K = 18s^2 + 1386$$

$$\frac{d\alpha_e(s)}{ds} = 36s$$

For $K = 1386$

$$
\begin{array}{c|cc}
s^3 & 1 & 77 \\
s^2 & 18 & 1386 \\
s^1 & \cancel{0}\ 36 & \cancel{0} \\
s^0 & 1386 &
\end{array}
$$

$\alpha_e(s)$

$\alpha_e(s) \rightarrow$ No sign change

$\Rightarrow$ No roots on right half of $\mathbb{C}$.

$\Rightarrow$ two roots are on imaginary axis.

$\Downarrow$

The system is "marginally stable".

Not BIBO stable.