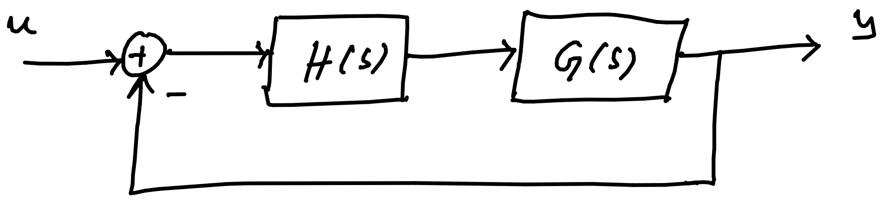
ELL225

Lecture - 20

→ Study of stability using Routh-Hurwitz criterion

↑ Algebraic method

→ Graphical Approach to study stability :



→ Contour Mapping in Complex Plane

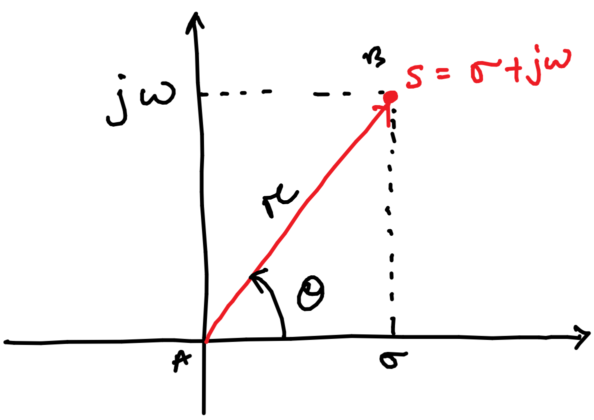
A complex no. $s \rightarrow \sigma + j\omega$ (Rectangular form)

$\equiv r\angle\theta$ (Polar form)

$\equiv re^{i\theta}$ ↑ (Euler form)

$r = |s| = \sqrt{\sigma^2+\omega^2}$

$\theta = \tan^{-1}\left(\frac{\omega}{\sigma}\right)$



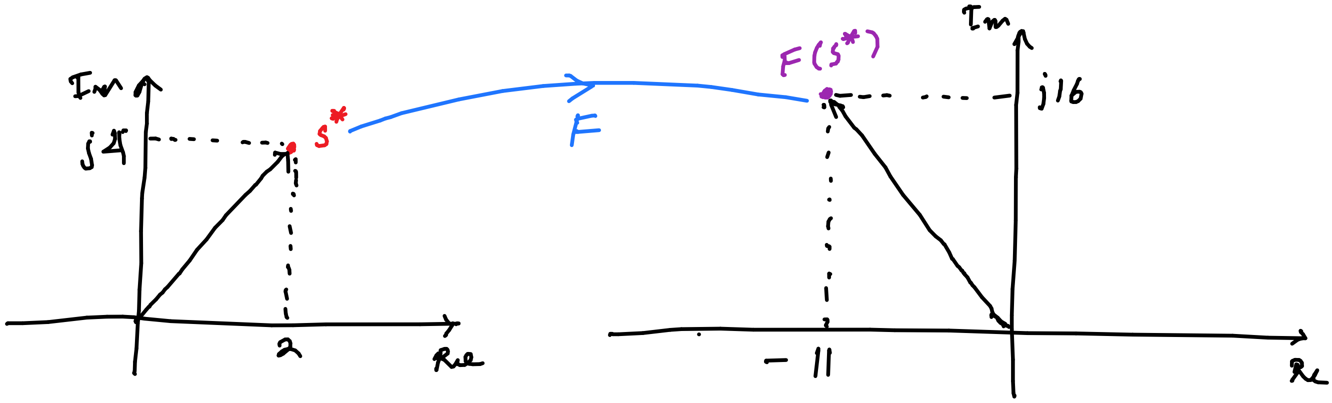
$r = |AB|$

Consider a mapping function : $F : s \longrightarrow v$

$$s = \sigma + j\omega \qquad v = x + jy$$

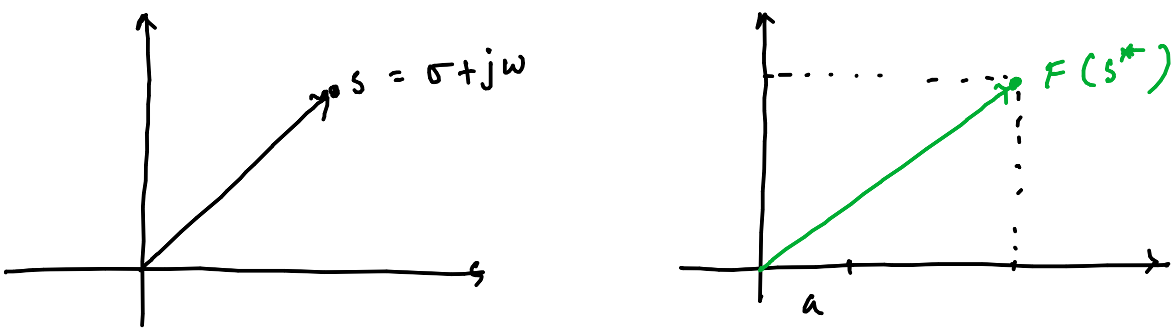
- Evaluate or map the point $s^* = 2 + j4$ to the point v through mapping funct^n: $F(s) = s^2 + 1$

$$F(s^*) = (2 + j4)^2 + 1$$
$$= \underset{x}{-11} + j\,\underset{y}{16}$$

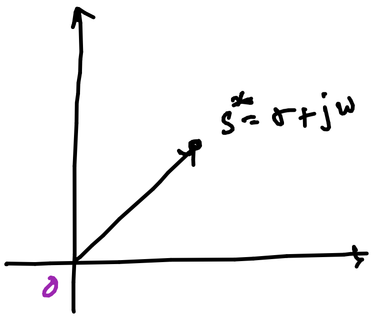


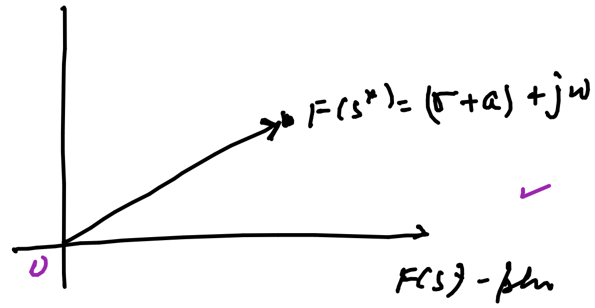
s-plane (complex plane) F(s)-plane (complex plane)

$$F(s) = s + a \qquad F(s^*) = (\sigma + a) + j\omega$$

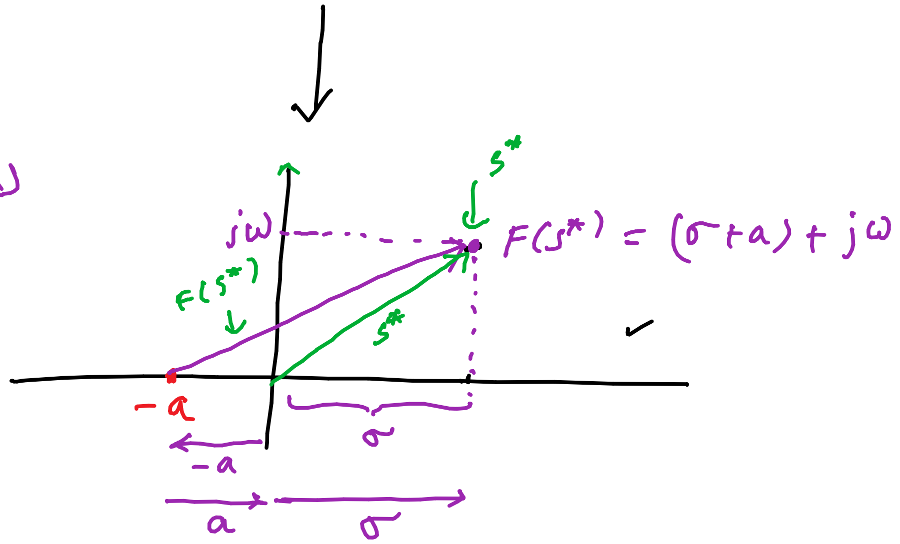


s-plane F(s)-plane





$F(s) = s + a$



$$F(s) = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)} \qquad n \geq m$$

m zeros
n poles

What is $F(s^*)$ for given s^*

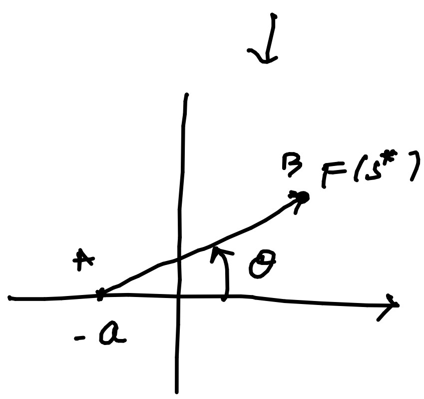
Let $r_{z_i} = |s^* - z_i|$

$r_{p_i} = |s^* - p_i|$

$\theta_{z_i} = \angle(s^* - z_i)$

$\theta_{p_i} = \angle(s^* - p_i)$

$F(s) = s + a$



$|F(s^*)| = |AB|$
$\angle F(s^*) = \theta$

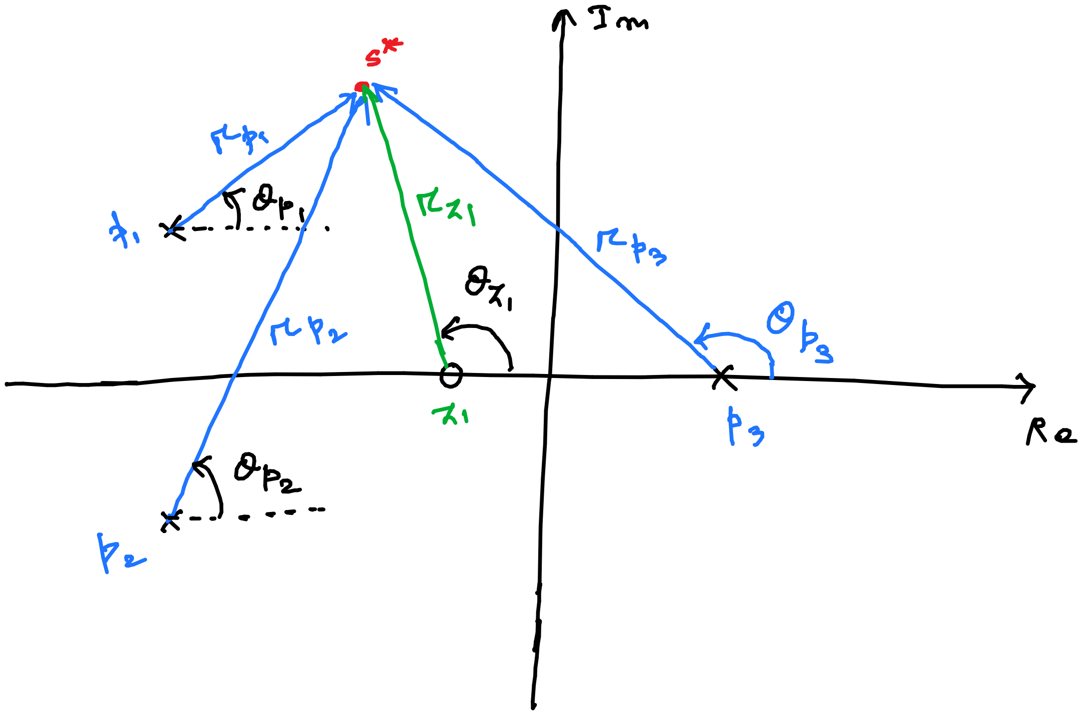
$$M = |F(s^*)| = \frac{r_{z_1}\, r_{z_2} \cdots r_{z_m}}{r_{p_1}\, r_{p_2} \cdots r_{p_n}} \quad \left(\frac{\text{(multiplicat}^{\text{n}}\text{ of zero lengths)}}{\text{(multiplication of poles lengths)}} \right)$$

$$\theta = \angle F(s^*) = (\theta_{z_1} + \theta_{z_2} + \cdots + \theta_{z_m}) - (\theta_{p_1} + \theta_{p_2} + \cdots + \theta_{p_n})$$

$$= \sum \theta_{z_i} - \sum \theta_{p_i}$$

$$F(s) = \frac{s + z_1}{(s+p_1)(s+p_2)(s+p_3)}$$

Map the point s^* to $v = F(s^*)$



$$M = \frac{r_{z_1}}{r_{p_1}\, r_{p_2}\, r_{p_3}} \qquad \theta = \theta_{z_1} - (\theta_{p_1} + \theta_{p_2} + \theta_{p_3})$$

$$v = F(s^*) = M \angle \theta$$

→ In both s - plane and F(s) - plane :

- If a vector rotates in counter clockwise direction, the angle $(\theta_{z_i}, \theta_{p_i}) \rightarrow +ve$
- If a vector rotates in clock wise direct^n then the angle $(\theta_{z_i}, \theta_{p_i}) \rightarrow -ve$

→ Assume that a complex valued function F(s) is defined in an open set D.

$D \subseteq \mathbb{C}$

$[a, b] \rightarrow$ closed set

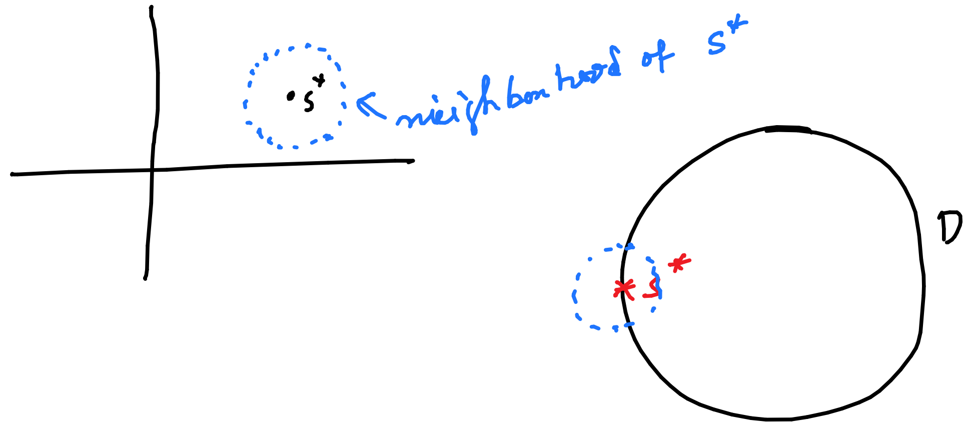
$(a, b) \rightarrow$ open set

If the following limit exists

$$F'(s) = \lim_{s \to s^*} \frac{F(s) - F(s^*)}{s - s^*}$$

then we say that F(s) is differentiable at point $s^* \in D$.

→ The funct^n F(s) is said to be "Analytic" at point s^* if $F'(s)$ exists at point s^* and also at each point in the neighborhood of s^*.



→ The funct^n $F(s)$ is <u>analytic in D</u> if it is analytic at each point in D.

→ Properties of $F(s)$:

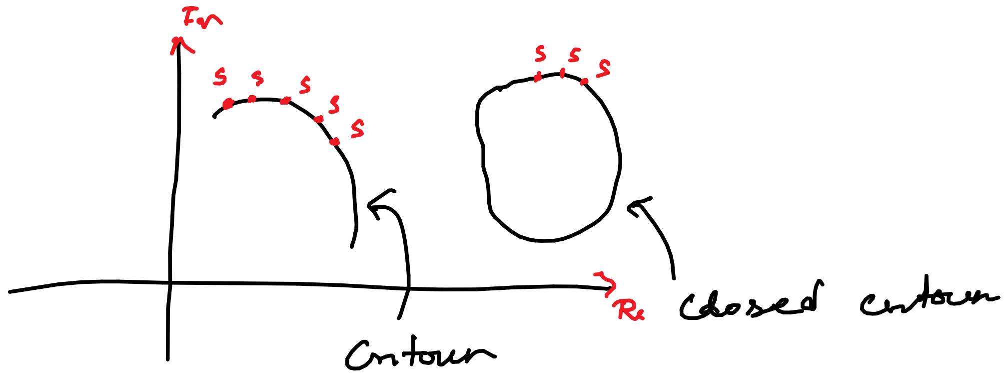
- Every point s^* in s-plane is mapped into only one point $F(s^*)$ in $F(s)$-plane.
  ($F(s)$ is one-to-one)

- $F(s)$ is analytic at every points in s-plane, except at poles of $F(s)$.

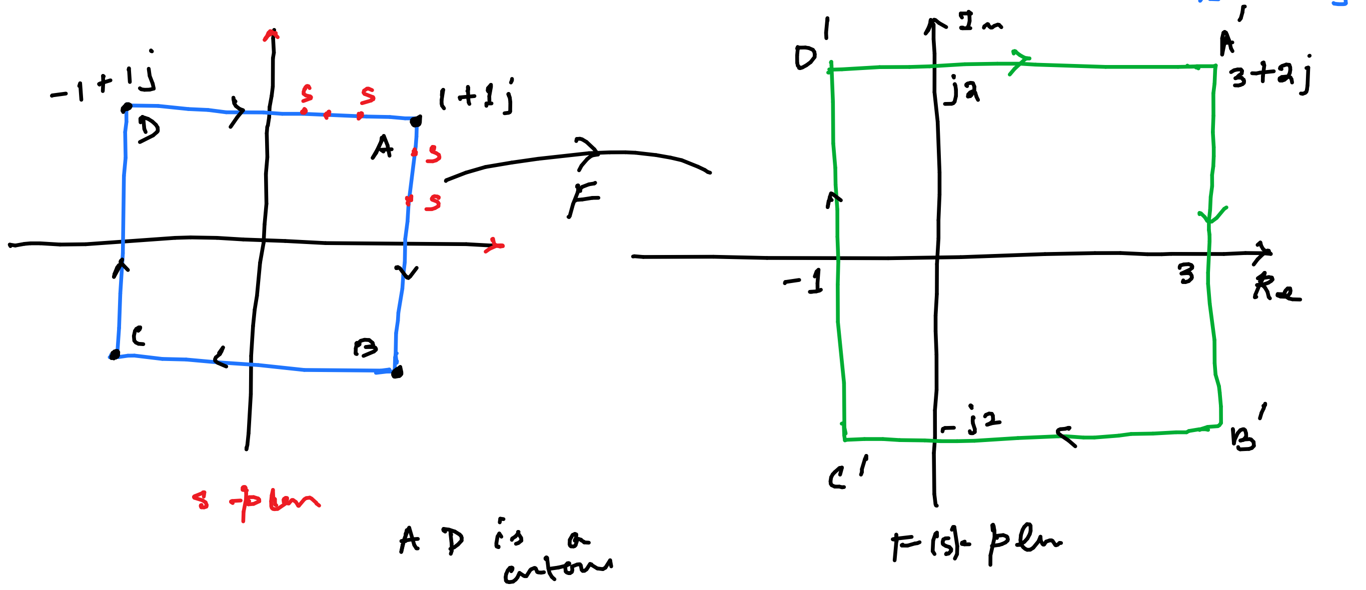
  A point $p \in \mathbb{C}$, at which $F(s)$ is not analytic, is called <u>singular point</u> of $F(s)$.

→ A <u>contour</u> in the complex plane, $\mathbb{C}$ is a continuous path in $\mathbb{C}$.
(collection of set of points)

→ A <u>closed-contour</u> in $\mathbb{C}$ is a contour that starts and ends at same point in $\mathbb{C}$.



Let a mapping funct $F(s) = 2s + 1 = 2(\sigma + j\omega) + 1$
$= (2\sigma + 1) + j2\omega$
$x \quad y$

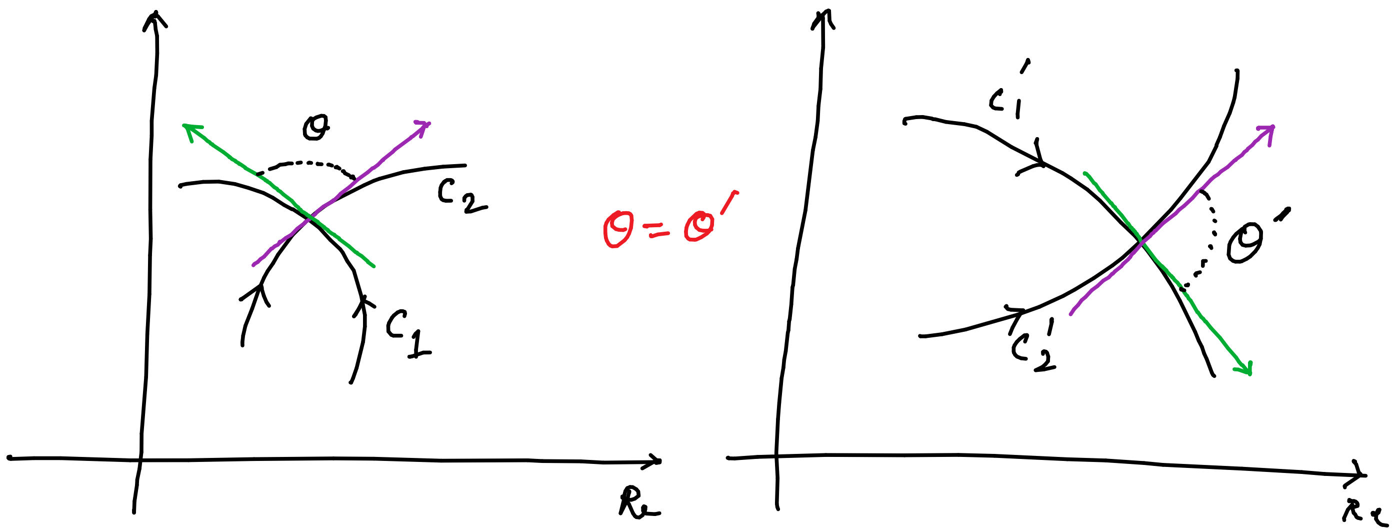


→ Every closed contour in s-plane maps to a closed contour in F(s)-plane through the mapping funct^n F(s).

→ $F(S)$ is <u>Conformal mapping</u>:

↓

The angle and the direction of any two intersecting curves in s-plane are <u>preserved</u> by the mapping of these curves to $F(s)$-plane.



s-plane

F(s) - plane