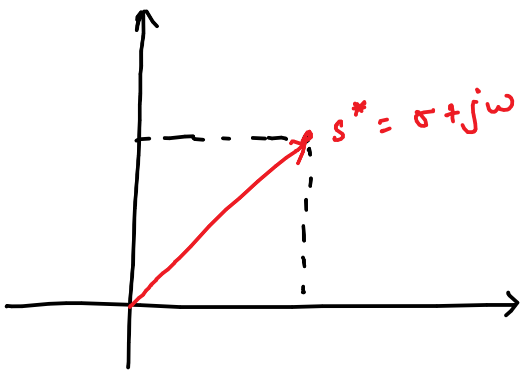
ELL225

Lecture - 21

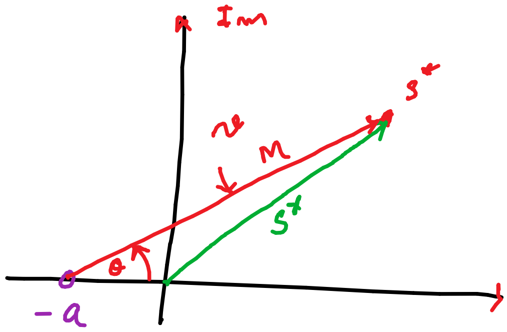
$$F(s) = s + a \qquad s^* = \sigma + j\omega$$



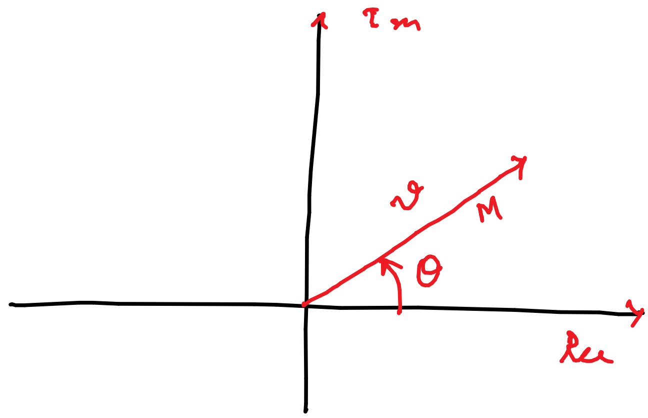
$$v = F(s^*)$$

$$= |F(s^*)| \; \angle F(s^*)$$

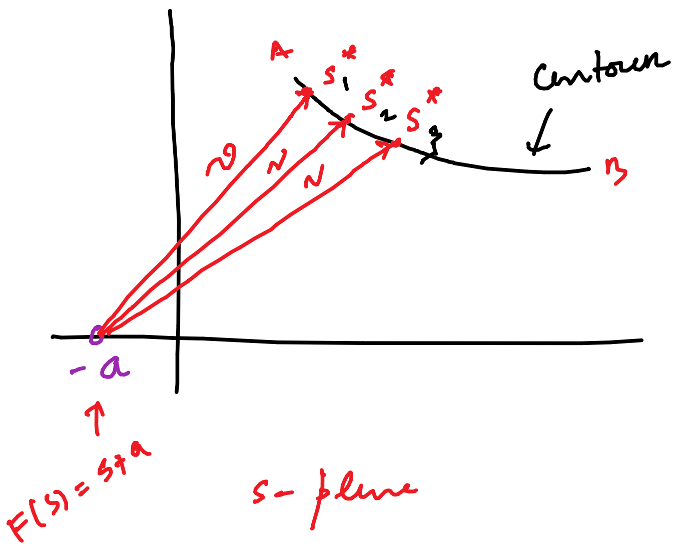
$$\quad\; M \qquad\quad \theta$$



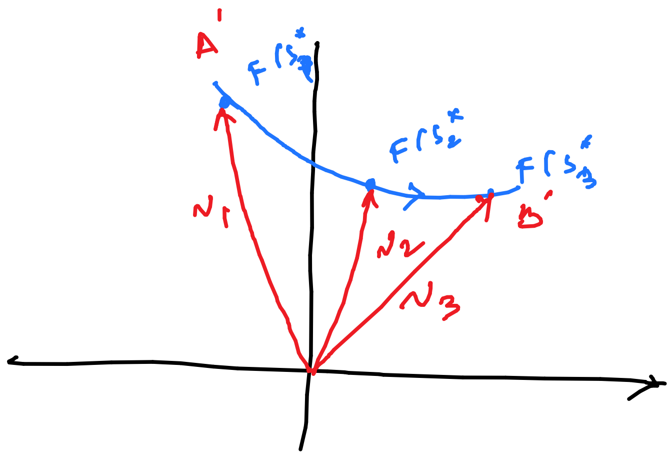
s-plane



F(s)-plane



s-plane



F(s) - plane

$$F(s) = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$r_{z_i} = |s^* + z_i| \qquad \theta_{z_i} = \angle s^* + z_i$$

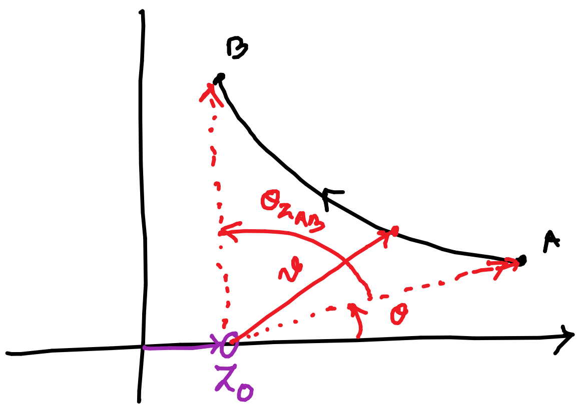
$$r_{p_i} = |s^* + p_i| \qquad \theta_{p_i} = \angle s^* + p_i$$

$$\nu = F(s^*) = M \angle \theta$$

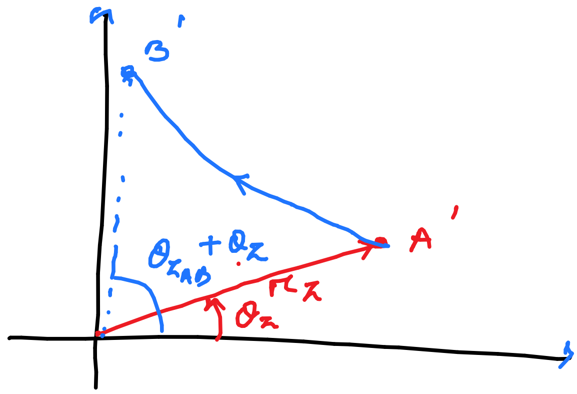
$$M = \frac{r_{z_1} r_{z_2} \cdots r_{z_m}}{r_{p_1} r_{p_2} \cdots r_{p_n}}$$

$$\theta = \sum \theta_{z_i} - \sum \theta_{p_i}$$

$\rightarrow$ $F(s) = s - z_1$



s - plane



F(s) - plane

At point A:

$$M = r_z$$

$$\theta = \theta_z$$

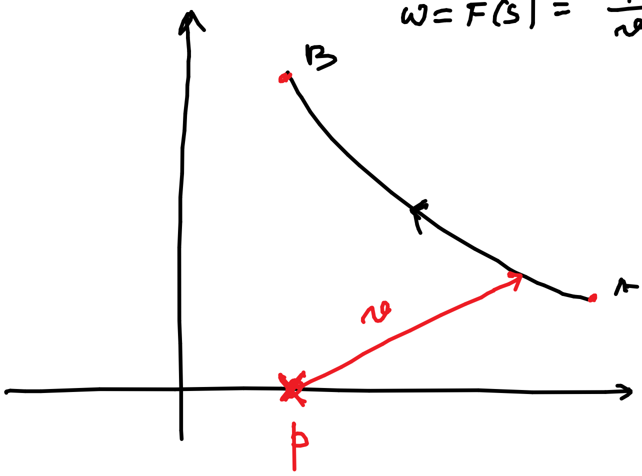
Moving from point A → B

$M = r_z$ (decreasing & then increasing)

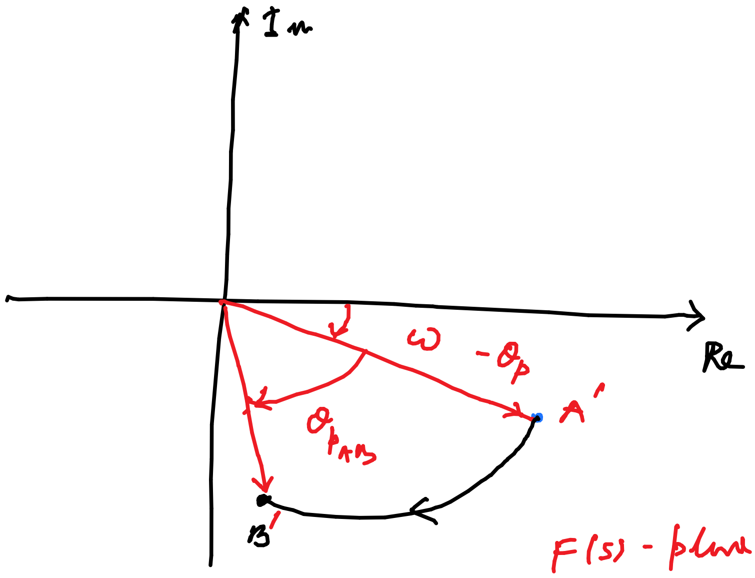
$\theta = \theta_{z_{AB}}$ (in counter clockwise direct)

$$F(s) = \frac{1}{s-p}$$

$$w = F(s) = \frac{1}{v}$$



$v = s - p$ s-plane



At point A

$$M = \frac{1}{r_p}$$

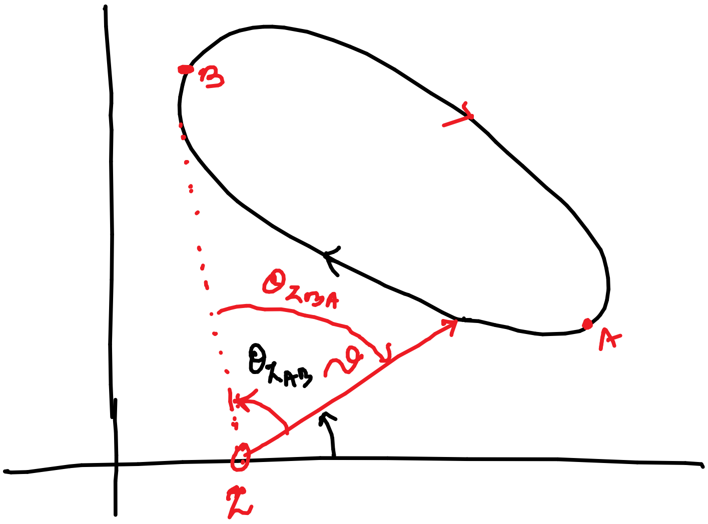
$$\theta = -(\theta_p)$$
$$= -\theta_p$$

Move from A → B

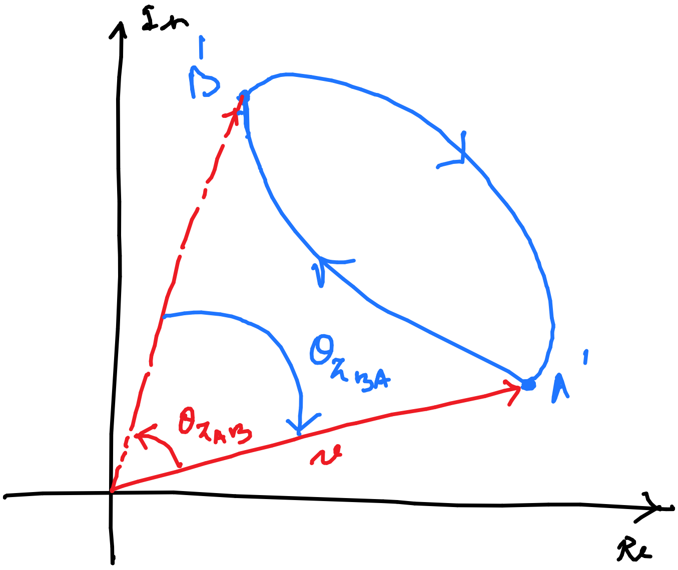
$$M = \frac{1}{r_p}$$

$$\theta = -(\theta_{p_{AB}})$$
$$= -\theta_{p_{AB}}$$

$F(s) = s - z$



s-plane



F(s)-plane

At point A

$M = r_z$

$\theta = \theta_z$

Moving from A → B

$M = r_z$. (decreasing & then increasing)

$\theta = \theta_z$ (in counterclockwise rot')

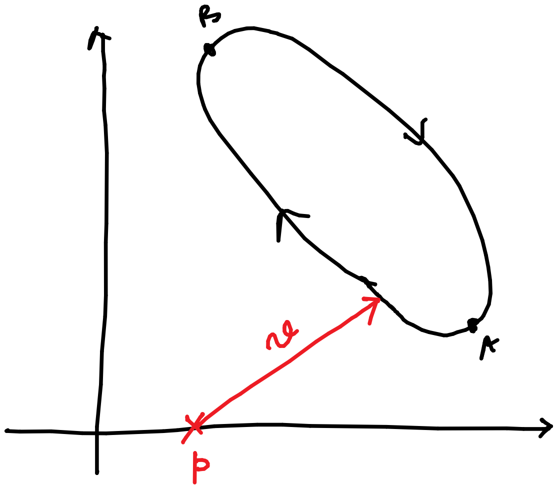
Moving from B → A

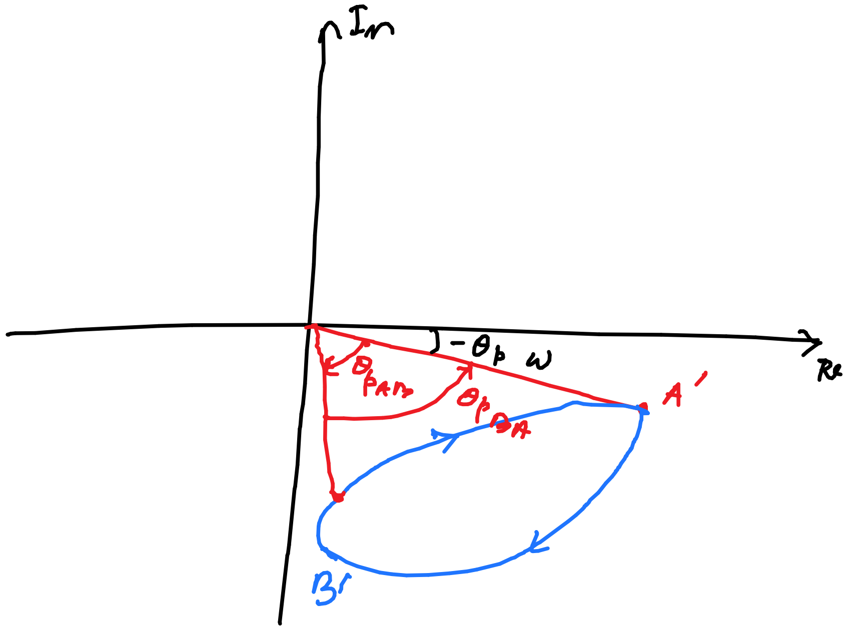
$M = r_z$

$\theta = (-\theta_z)$ since clockwise rotation

$= -\theta_z$

$$F(s) = \frac{1}{s-p} \qquad w = \frac{1}{v}$$





<u>At point A</u>

$$M = \frac{1}{r_p}$$

$$\theta = -(\theta_p)$$

Moving from $A \to B$

$$M = \frac{1}{r_p}$$

$$\theta = -\theta_{p_{AB}}$$

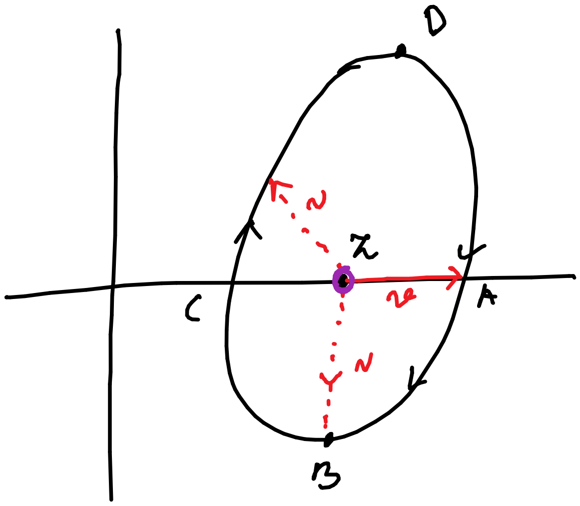
Moving from $B \to A$

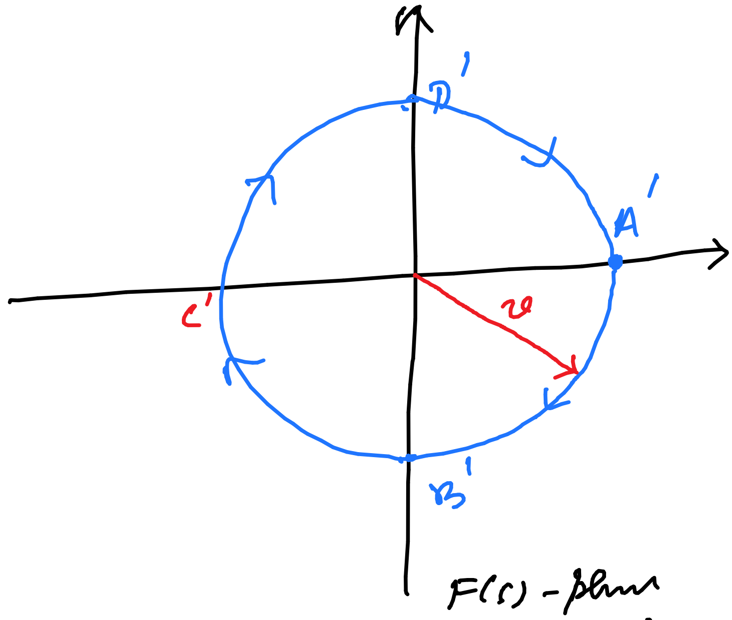
$$M = \frac{1}{r_p}$$

$$\theta = 0 - (-\theta_{p_{BA}})$$

$$= \theta_{p_{BA}}$$

$F(s) = s - z$





F(s) - plane

Point A

$m = r_z$

$\theta = \theta_z = 0$

Moving from A → B

$m = r_z$

$\theta = \theta_z = -90^0$

(90^0 in clockwise dir^n)

Moving from B → D

$m = r_z$

$\theta = \theta_z$

$= -180^0$ (clockwise rot^n of v)

⇓

v will rotate 180^0 in clockwise direct^n in F(s) - plane

Moving from D → A

$m = r_z$

$\theta = \theta_z$

$= -90^0$ (in clockwise rot^n of v)

⇓

v will rotate 90^0 in clockwise in F(s)-plane.

* One has to keep track of the vector $\underline{v}$ in s-plane, in which direction (clock-wise or counter clockwise) it is rotating, while traversing on the given contour.

The direct^n of v or w in F(s) - plane needs to be determined based on that.