ELL225

Lecture - 22

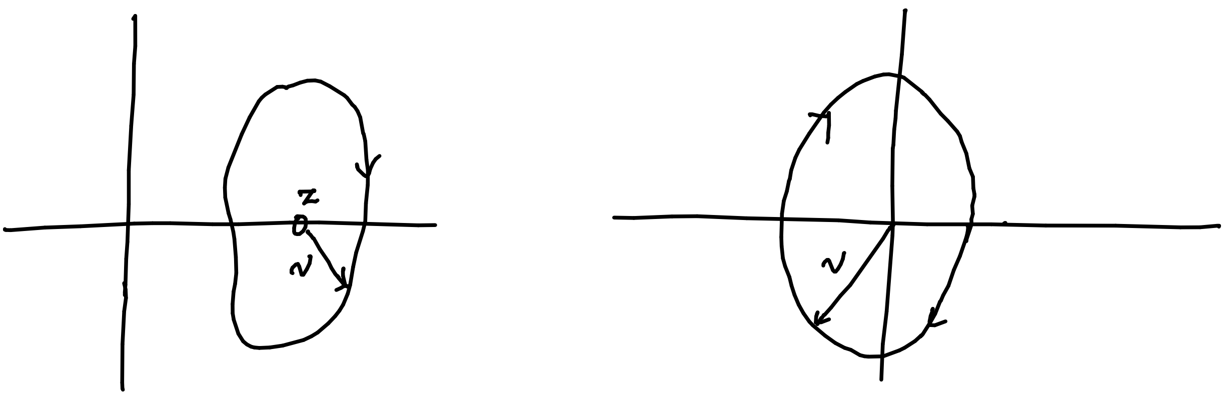
→ Mapping of a contour in s-plane to F(s)-plane through the mapping funct^n F(s).

$$F(s) = \frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

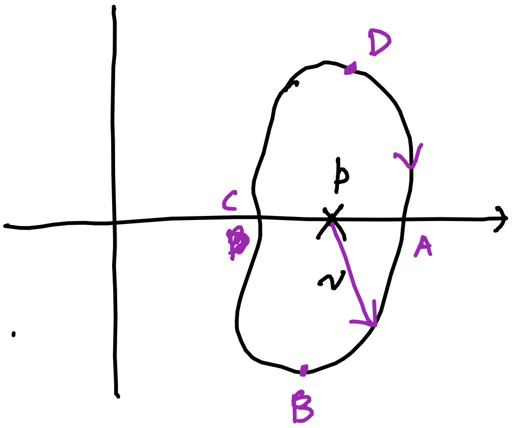
↓

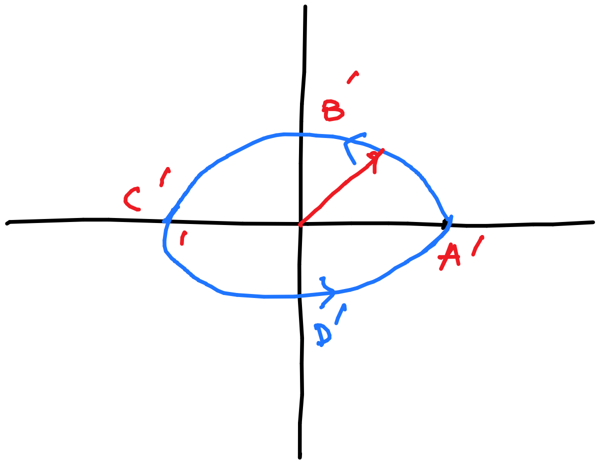
$$v = |F(s)| \angle F(s)$$
$$= M \angle \theta$$

$F(s) = s - z$



$$F(s) = \frac{1}{s-p}$$





F(s) - plane

$v \rightarrow$

At pF A $\rightarrow$ $M = \frac{1}{r_p}$ $\quad \theta = -\theta_p = 0$

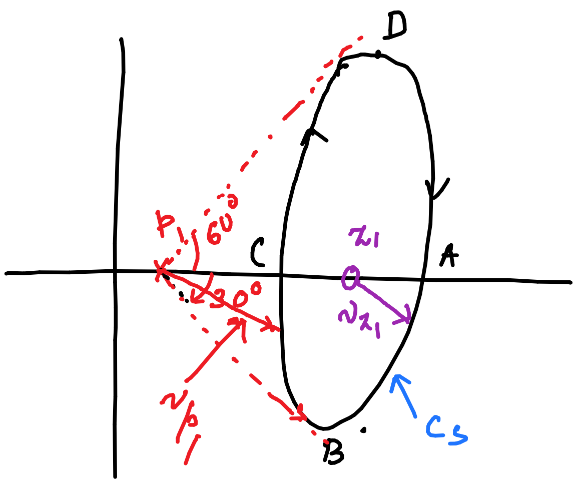
$A \xrightarrow{B} C \rightarrow M = \frac{1}{r_p}$ $\quad \theta = -(\theta_p) = -(-180^\circ)$

$= 180^\circ$

$C \xrightarrow{D} A \rightarrow M = \frac{1}{r_p}$ $\quad \theta = -(\theta_p)$

$= -(-180^\circ) = 180^\circ$

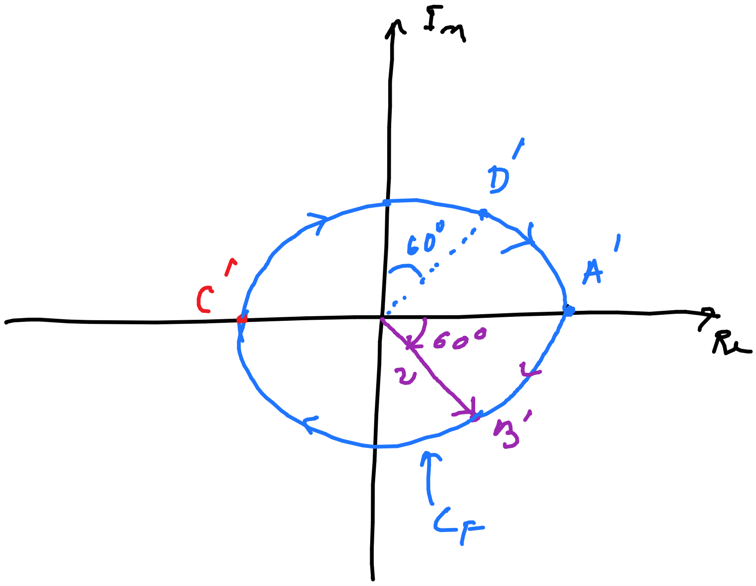
$\rightarrow \quad F(s) = \dfrac{s - z_1}{s - p_1}$



s - plane

$v_{z_1} = r_{z_1} \angle \theta_{z_1}$

$v_{p_1} = r_{p_1} \angle \theta_{p_1}$



F(s) - plane

At pt. A

$v = M \angle \theta$

$M = \dfrac{r_{z_1}}{r_{p_1}}$ ✓

$\theta = \theta_{z_1} - \theta_{p_1}$
$= 0^\circ - 0^\circ$
$= 0$

For B → C

$M = \dfrac{r_{z_1}}{r_{p_1}}$

$\theta = \theta_{z_1} - \theta_{p_1}$
$= (-90^\circ) - (30^\circ)$
$= -120^\circ$
$\Downarrow$
v will rotate 120° in clockwise direct' in $F(s)$-plo

Movmg for D → A

$M = \dfrac{r_{z_1}}{r_{p_1}}$

$\theta = \theta_{z_1} - \theta_{p_1}$
$= (-90^\circ) - (-60^\circ)$
$= -30^\circ$

Movmg for pt A → B

$M = \dfrac{r_{z_1}}{r_{p_1}}$ (Dominated by the vector v_{p_1})

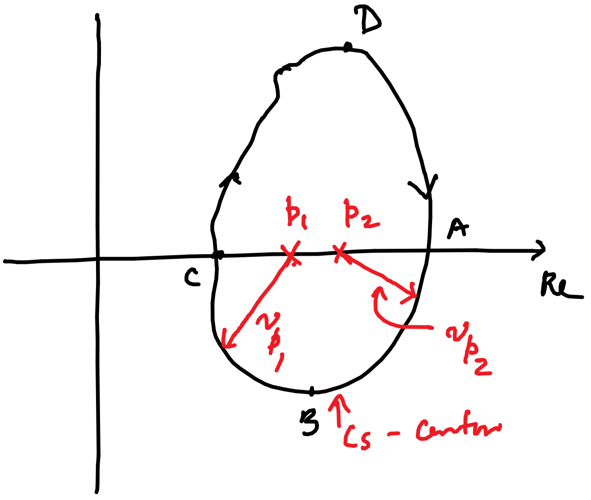
$\theta = \theta_{z_1} - \theta_{p_1}$
$= (-90^\circ) - (-30^\circ)$
$= -90^\circ + 30^\circ$
$= -60^\circ$
$\Downarrow$
the vector v will rotate 60° in clockwise in $F(s)$-pla

Move for C → D

$M = \dfrac{r_{z_1}}{r_{p_1}}$

$\theta = \theta_{z_1} - \theta_{p_1}$
$= -90^\circ - (60^\circ)$
$= -150^\circ$
$\Downarrow$
150° rot' in clockwi in $F(s)$ pla

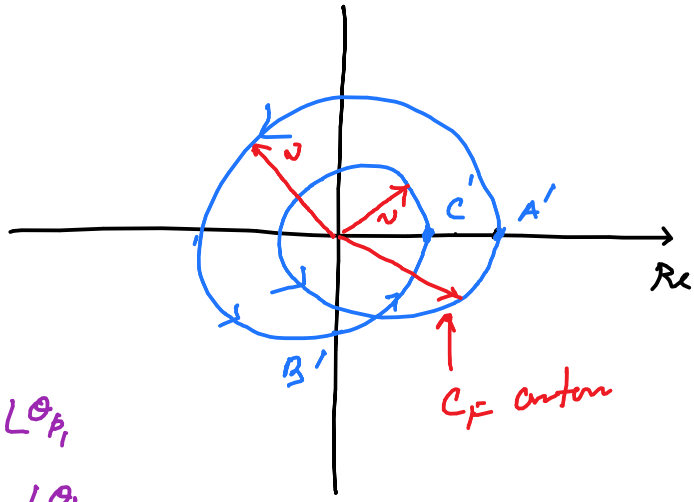
$$F(s) = \frac{1}{(s-p_1)(s-p_2)}$$



S-plane

$v_{p_1} = r_{p_1} \angle \theta_{p_1}$

$v_{p_2} = r_{p_2} \angle \theta_{p_2}$



F(s)-plane

At point A

$$M = \frac{1}{r_{p_1} r_{p_2}}$$

$$\theta = -(\theta_{p_1} + \theta_{p_2})$$
$$= -(0^\circ + 0^\circ)$$
$$= 0^\circ$$

Moving from A → C thru B

$$M = \frac{1}{r_{p_1} r_{p_2}}$$

$$\theta = -(\theta_{p_1} + \theta_{p_2})$$
$$= -(-180^\circ - 180^\circ)$$
$$= 360^\circ$$

$$\Downarrow$$

The vector v will rotate 360° in counter clockwise direction in F(s)-plane.

Moving from C → A thru D

See how v_{p_1} & v_{p_2} are moving along the contour

$$M = \frac{1}{r_{p_1} r_{p_2}}$$

$$\theta = -(\theta_{p_1} + \theta_{p_2})$$
$$= -(-180^\circ - 180^\circ) = 360^\circ$$

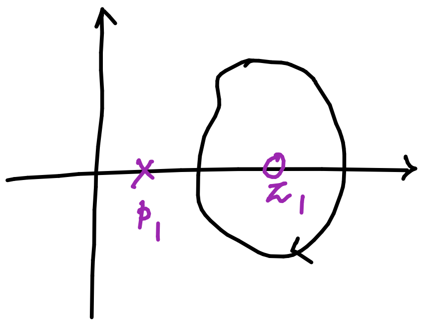
360° rotation of v in counter clockwise direction in F(s)-plane

→ **Cauchy's Result** $F(s) = \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$

Principle of the Arguement :

If a contour C_s in the s-plane encircles Z - zeros and P - poles of the mapping funct? $F(s)$, and does not pass through any poles and zeros of $F(s)$, and the traverse is in the clockwise direction along the contour, the corresponding contour C_F (mapped contour) in $F(s)$-plane, encircle the origin of $F(s)$-plane : $\boxed{N = Z - P}$ times in the clockwise direction.

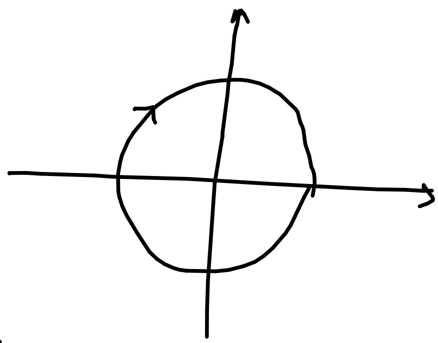
← +ve N is clockwise
-ve N is counter clockwise



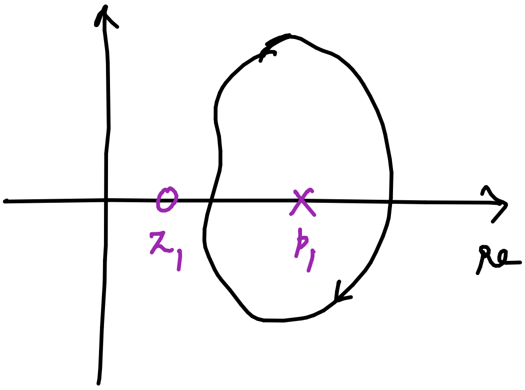
s - plane

$$N = Z - P$$
$$= 1 - 0$$
$$= 1$$

↑
One rotation of origin in clockwise direct?

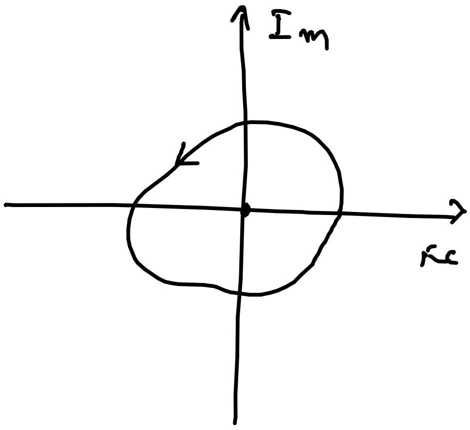


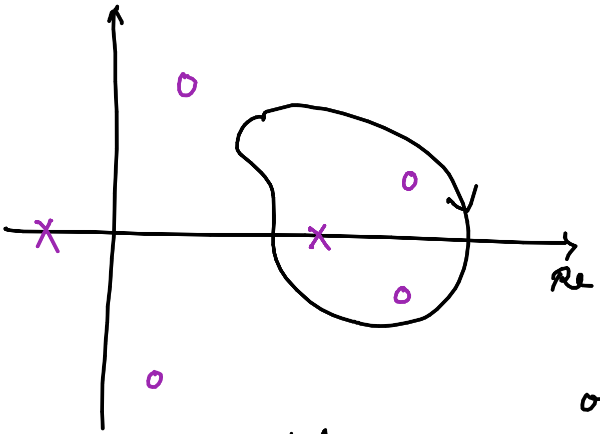
F(s) - plane



$$N = Z - P$$
$$= 0 - 1$$
$$= -1$$

One rotation in counter clockwise direction.

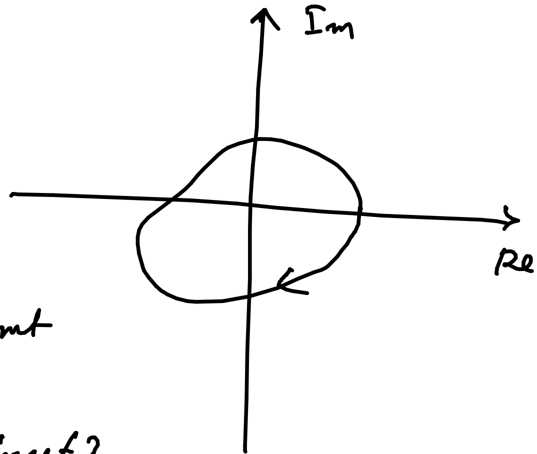




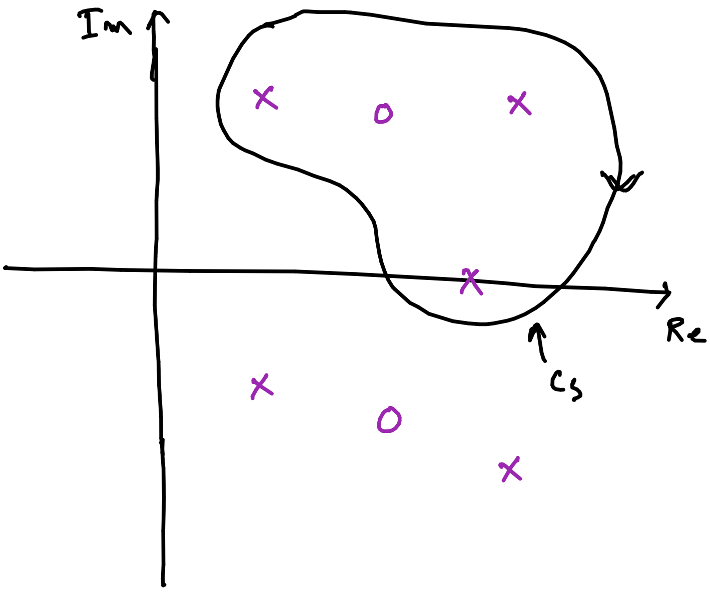
s-plane

$$N = Z - P$$
$$= 2 - 1$$
$$= 1$$

One encirclement of origin in clockwise direct^n.

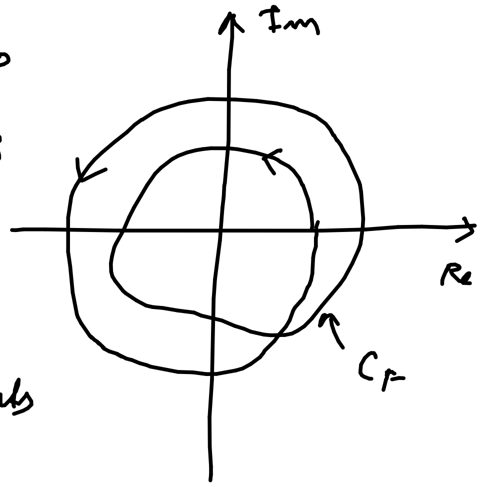


F(s)-plane



$$N = Z - P$$
$$= 1 - 3$$
$$= -2$$

2 encirclements of origin in counter clockwise direct^n



F(s)-plane.