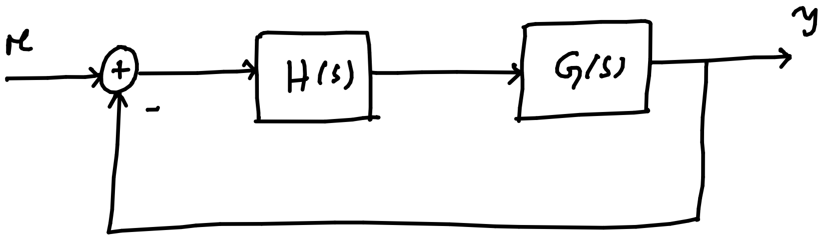
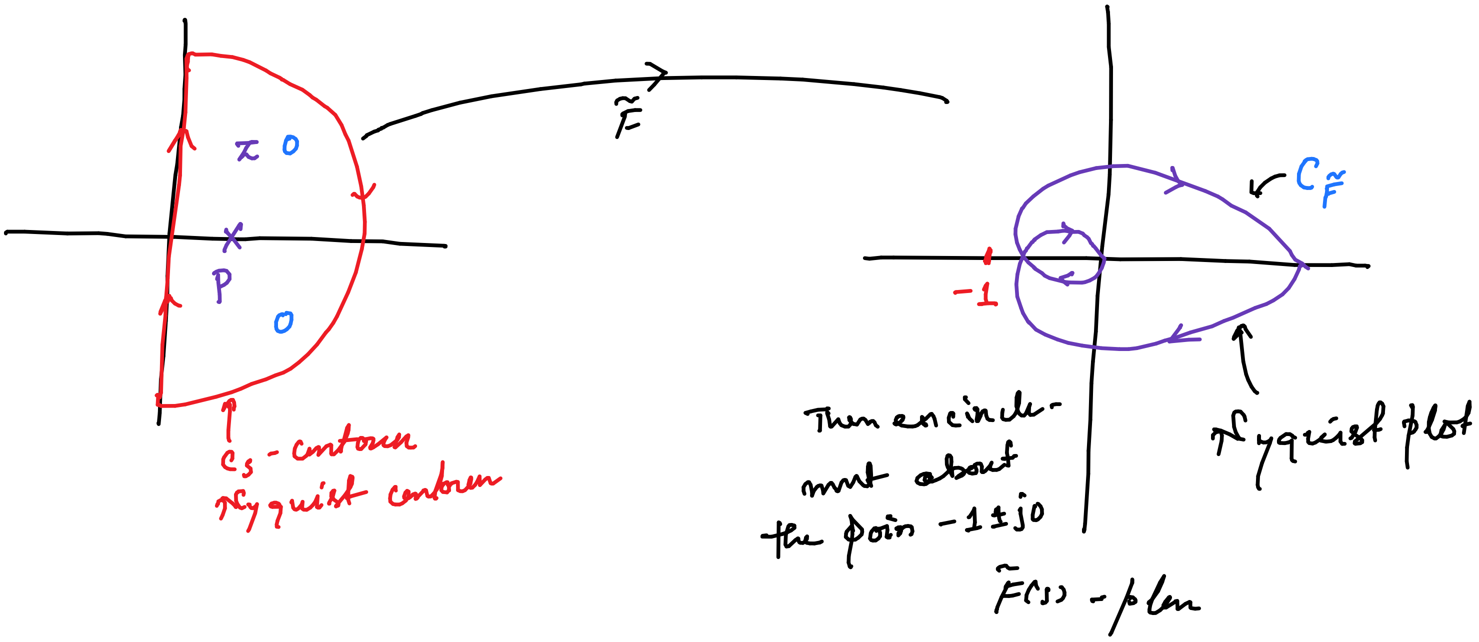
ELL 225

Lecture - 24



To study the stability using Nyquist Criterion :

- We defined a mapping fn^n $\tilde{F}(s) = G(s)H(s)$
- We observe the encirclement of $C_{\tilde{F}}$ contour in $\tilde{F}(s)$ - plane about the point $-1 \pm j0$, instead of origin.



?

needs to be identified by plotting $C_{\tilde{F}}$ contour

$$Z = N + P \leftarrow \text{given}$$

closed loop poles that belongs to $\mathbb{C}^+$

↑

closed right half of complex plan

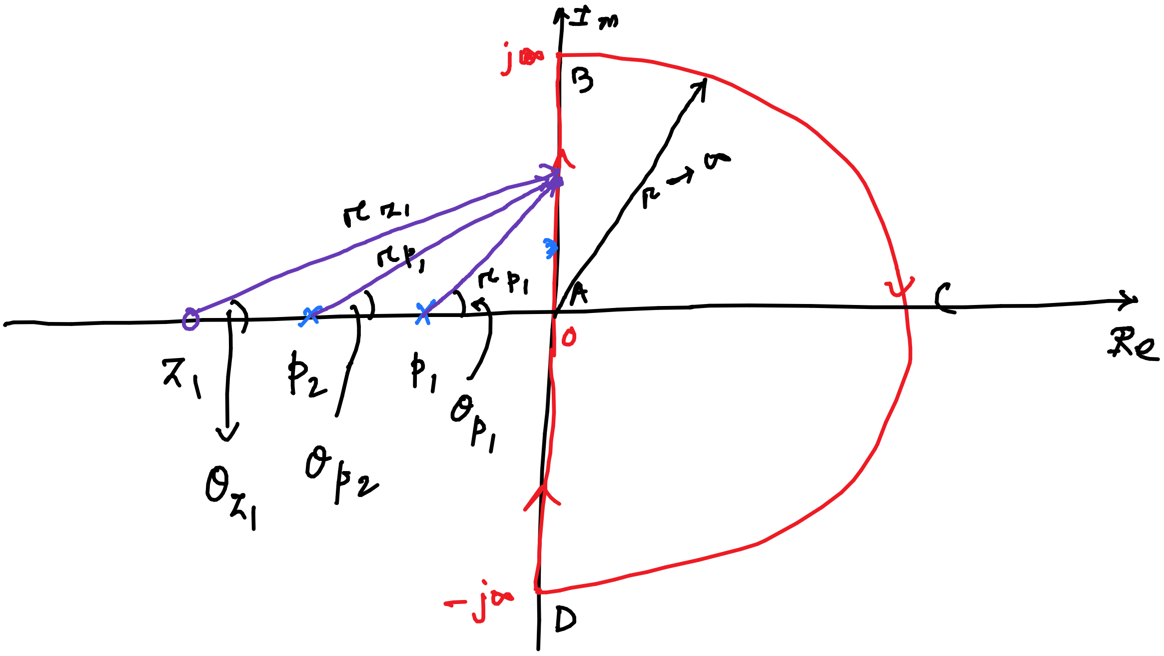
≡ inside the Nyquist contour

No of encirclement about $-1 \pm j0$ by $C_{\tilde{F}}$ contour

No. of poles of $\tilde{F}(s) = G(s)H(s)$ that are inside the Nyquist contour.

→ Nyquist Plot

Let $H(s) = 1$ $\quad G(s) = \dfrac{s+z_1}{(s+p_1)(s+p_2)}$

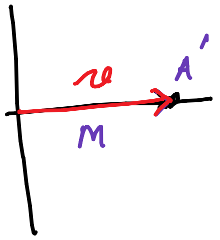


At point 'A'

$$\tilde{F}(s) = v = m e^{i\theta}$$

$$M = \frac{r_{z_1}}{r_{p_1} r_{p_2}} \qquad \theta = \theta_{z_1} - \theta_{p_1} - \theta_{p_2}$$

$$\theta = 0 - 0 - 0 = 0$$



Moving from A to B

$$\theta = \theta_{z_1} - \theta_{p_1} - \theta_{p_2}$$

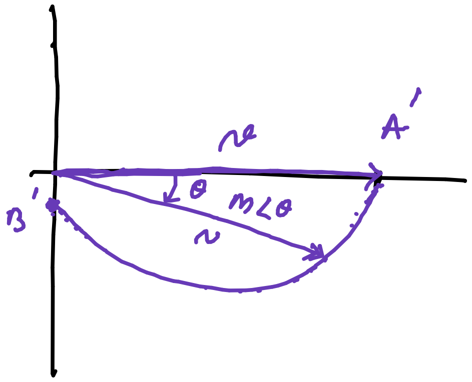
$$= 90^\circ - (90^\circ) - (90^\circ)$$

all pole & zero vectors rotated almost 90° in counter-clockwise direct'

$$= -90^\circ$$

↑ vector v has to rotate 90° in clock wise direct'

$$M = \frac{r_{z_1}}{r_{p_1} r_{p_2}} \rightarrow \epsilon, \text{ is a small number}$$



Moving from B to D through C

$$\theta = \theta_{z_1} - \theta_{p_1} - \theta_{p_2}$$

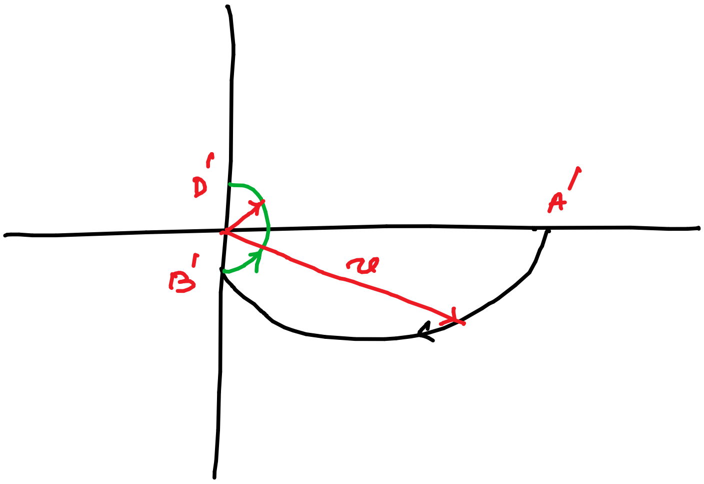
$$= (-180^\circ) - (-180^\circ) - (-180^\circ)$$

$$= -180 + 180 + 180 = 180^\circ$$

↑ in counter clock wise

all the pole zero vectors will rotate around 180° in clockwise direct'

$$M = \frac{r_{z_1}}{r_{p_1} r_{p_2}} \longrightarrow \epsilon \text{ a small number}$$



Moving from D to A

$$\theta = \theta_{z_1} - \theta_{p_1} - \theta_{p_2}$$

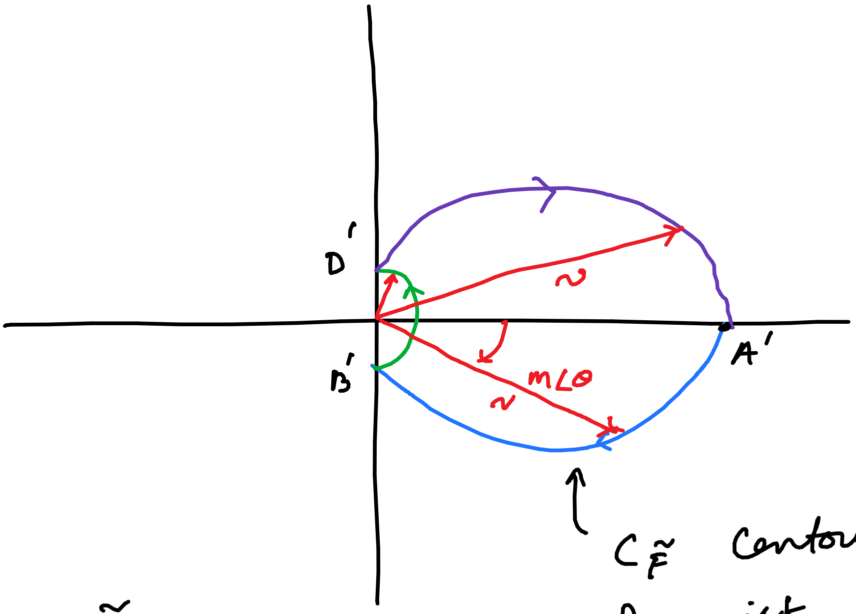
$$= 90^\circ - (90^\circ) - (90^\circ)$$

$$= -90^\circ$$

↑ 90° in clock wise direct'

all pole zero vectors will rotate 90° in counter-clockwise direct.

$$M = \frac{r_{z_1}}{r_{p_1} r_{p_2}}$$



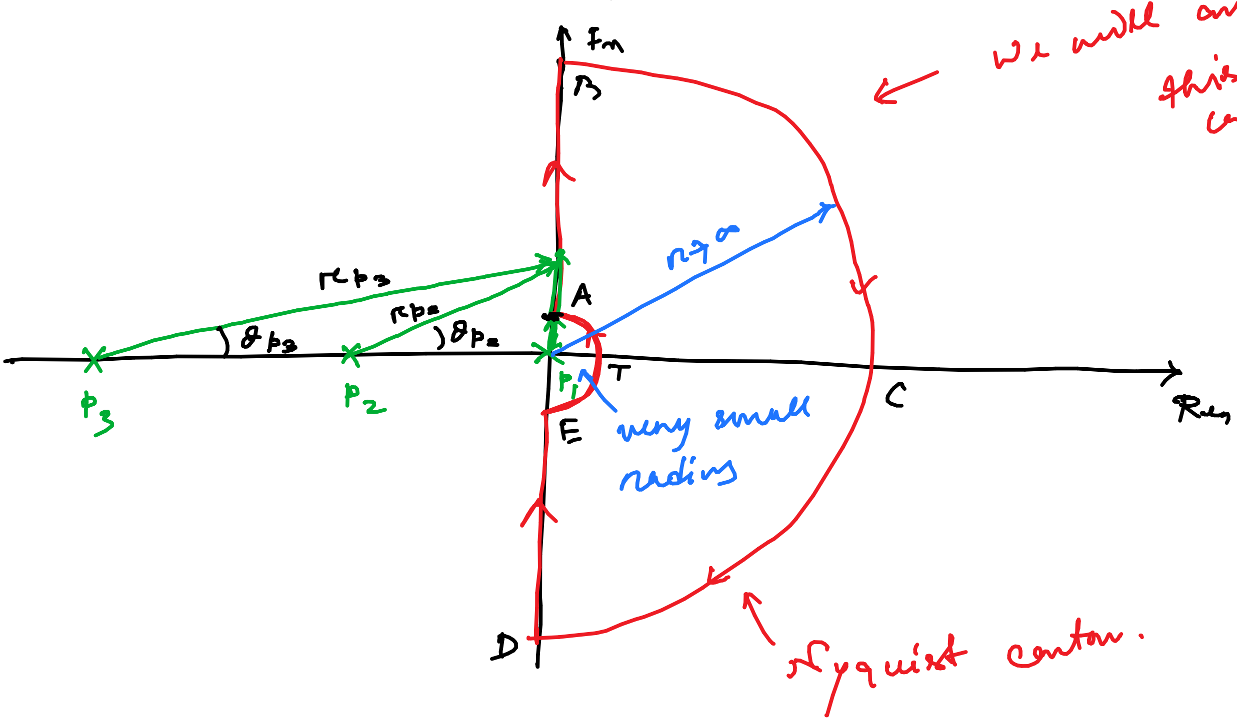
$C_{\tilde{F}}$ Contour

Nyquist Plot

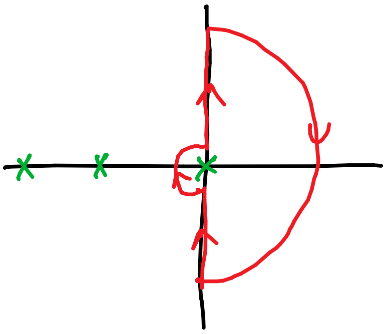
$\tilde{F}(s)$ - plane

→ Let $H(s) = 1$ $\quad G(s) = \dfrac{1}{s(s+p_1)(s+p_2)}$

$$\tilde{F}(s) = \frac{1}{s(s+p_1)(s+p_2)}$$



We will consider this type contour..



Both are fine

At point A

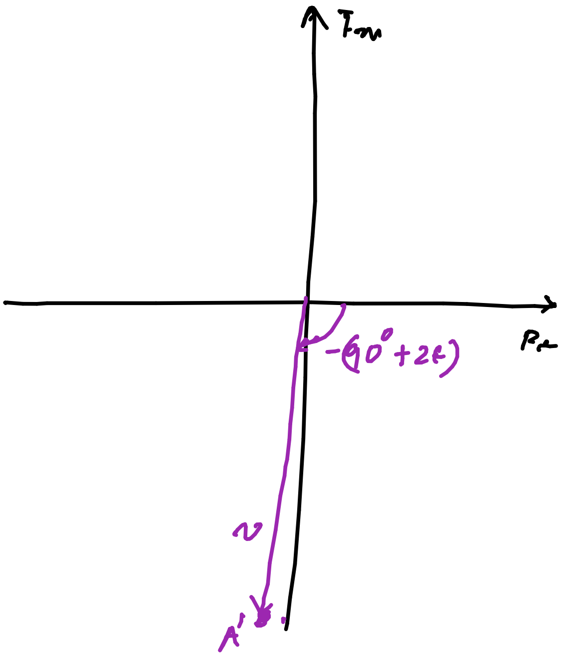
$$\theta = -\theta_{p_1} - \theta_{p_2} - \theta_{p_3}$$

$$= -(90^\circ) - (\underset{\epsilon}{0^\circ}) - (\underset{\epsilon}{0^\circ})$$

$$= -90^\circ \qquad -(90 + 2\epsilon)$$

$$M = \frac{1}{r_{p_1} r_{p_2} r_{p_3}}$$ → very large magnitude

since r_{p_1} is very small (close to 0)



Moving from A to B

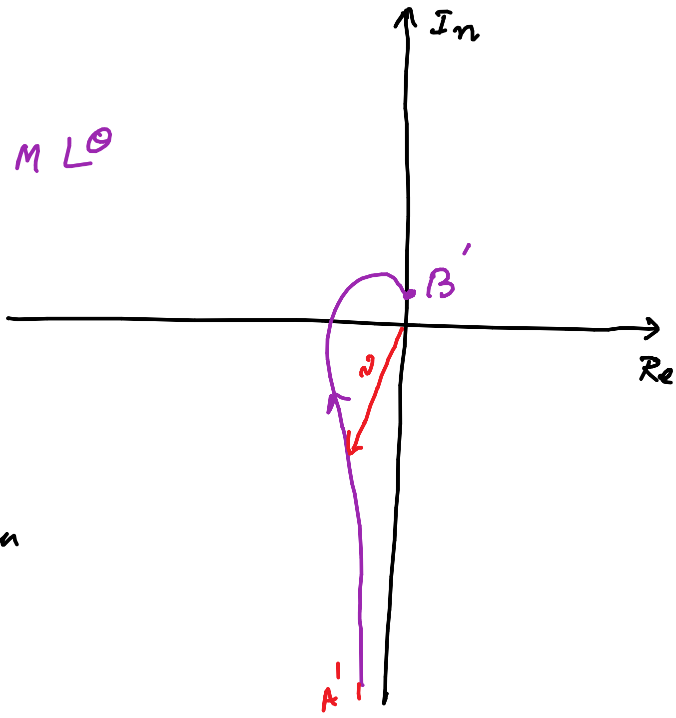
$$\theta = -\theta_{p_1} - \theta_{p_2} - \theta_{p_3} \qquad \nu = M \angle \theta$$

$$= -(0^\circ) - (90^\circ) - (90^\circ)$$

$$= -180^\circ$$

$$M = \frac{1}{\underset{\uparrow}{r_{p_1}} r_{p_2} r_{p_3}}$$ → ϵ very small

they are large



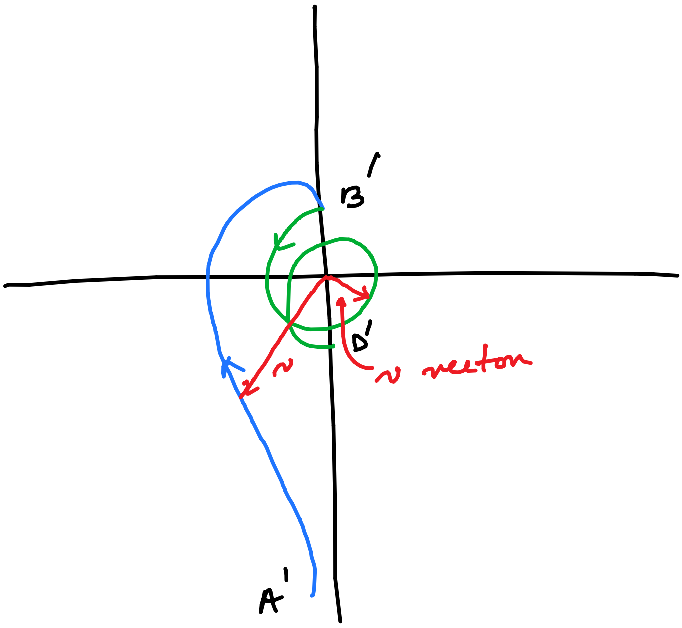
Moving from B to D through C

$$\theta = -\theta_{p_1} - \theta_{p_2} - \theta_{p_3}$$

$$= -(-180^\circ) - (-180^\circ) - (-180^\circ)$$

$$= 540^\circ$$

$$M = \frac{1}{r_{p_1} r_{p_2} r_{p_3}} \rightarrow \epsilon \text{ small quantity}$$



Moving from D to E

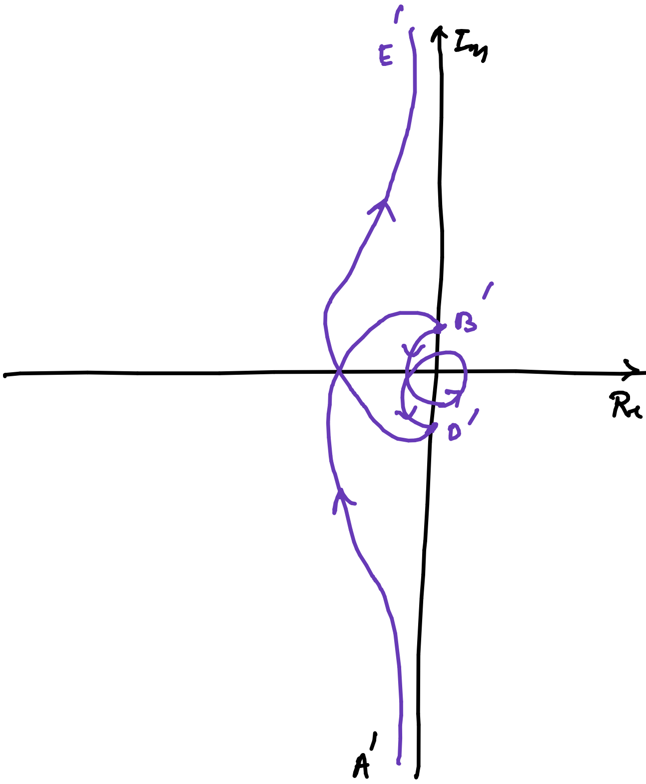
$$\theta = -\theta_{p_1} - \theta_{p_2} - \theta_{p_3}$$

$$= -(0^\circ) - (90^\circ) - (90^\circ)$$

$$= -180^\circ$$

$$M = \frac{1}{r_{p_1} \underbrace{r_{p_2} r_{p_3}}_{\text{will converge to some finite num}}} \rightarrow \text{very large value}$$

will converge to a very small num



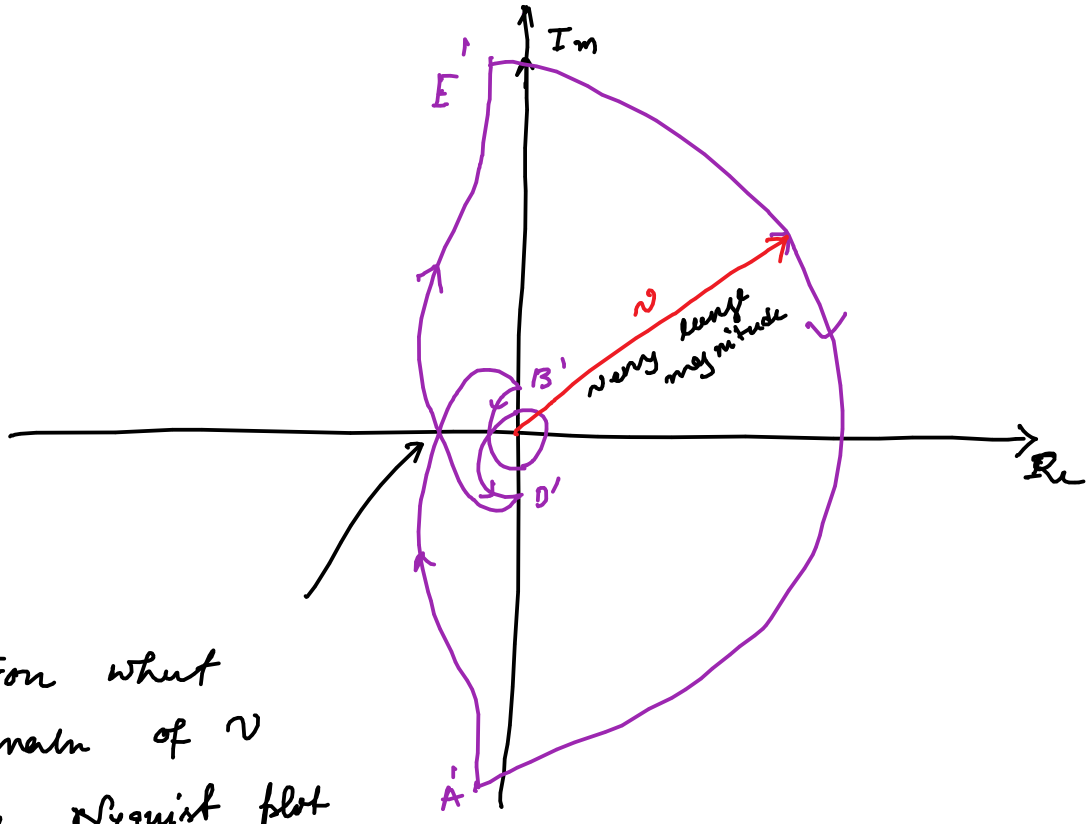
Moving from E to A through T

$$\theta = -\theta_{p_1} - \theta_{p_2} - \theta_{p_3}$$

$$= -(180^\circ) - (\underset{\epsilon}{0^\circ}) - (\underset{\epsilon}{0^\circ})$$

$$= -180^\circ \qquad -(180 + 2\epsilon)$$

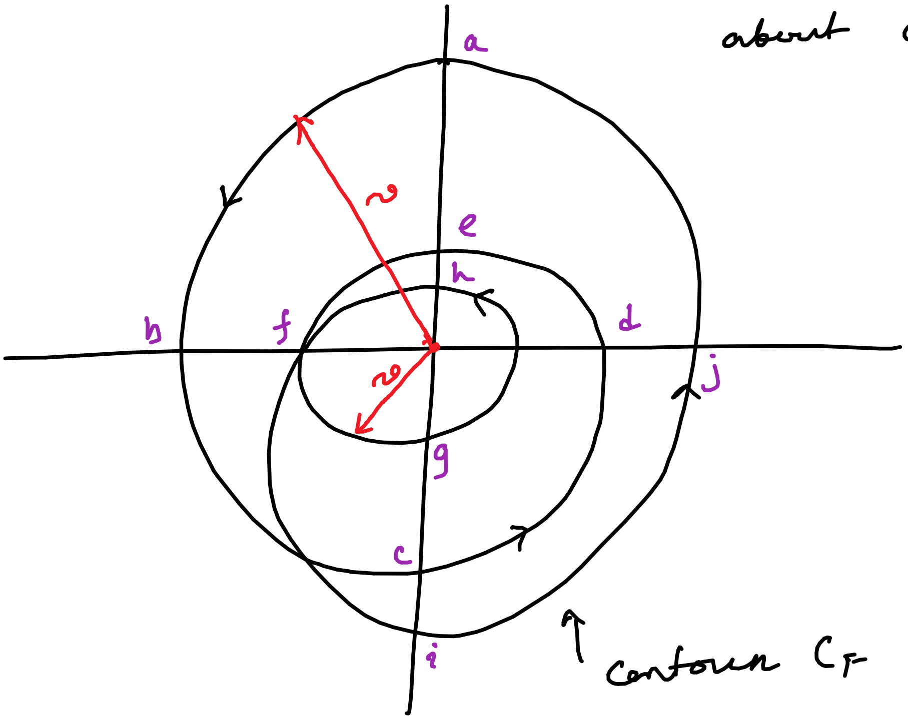
$$M = \frac{1}{\underset{\text{very small}}{r_{p_1}}\ \underbrace{r_{p_2} r_{p_3}}_{\text{some finite number}}} \rightarrow \text{very large}$$



For what value of v the Nyquist plot cross the real axis?

Encirclement Counting

Number of encirclement about origin.



- Keep track of the angle made by vector v while traversing on the contour C_F

$a \xrightarrow{b} c = 180^0$

$c \xrightarrow{d} e = 180^0$

$e \xrightarrow{f} g = 180^0$

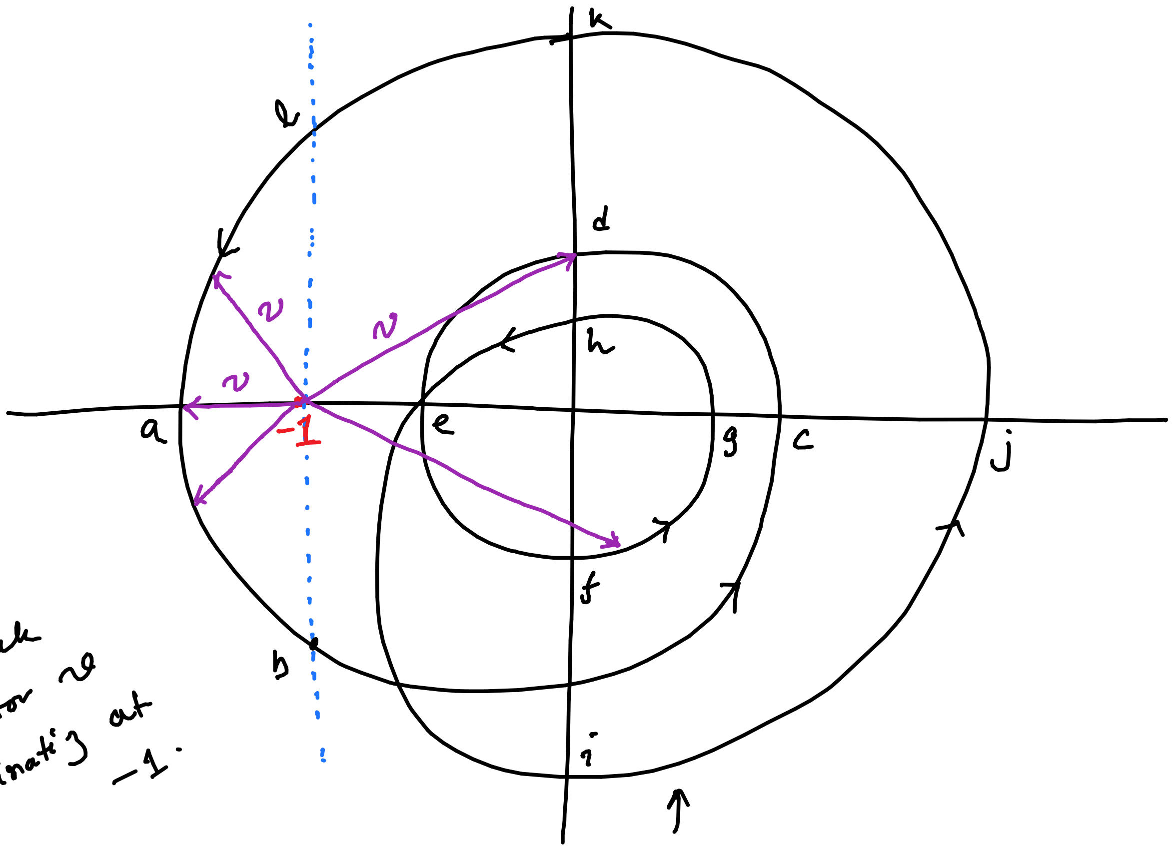
$g \rightarrow h = 180^0$

$h \rightarrow i = 180^0$

$i \rightarrow a = 180^0$

3×360^0

So 3 rotations about the point origin in counter-clockwise directⁿ.



- keep track of vector v originating at -1.

Countour $C\tilde{F}$

- Encirclement about the point $-1+j0$.

$a \to b = 90^\circ$

$b \to c = 90^\circ$

$c \xrightarrow{d} e = 0^\circ$

$e \xrightarrow{f} g = 0^\circ$

$g \xrightarrow{h} e = 0^\circ$

$e \xrightarrow{i} j = 0^\circ$

$j \xrightarrow{k} l = 90^\circ$

$l \to a = 90^\circ$

360°

↑

One rotation about the point -1 in counter-clockwise direction.