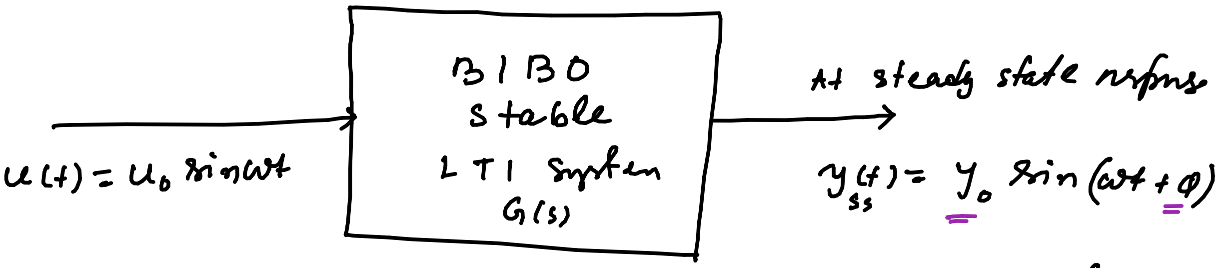
ELL225

Lecture - 27



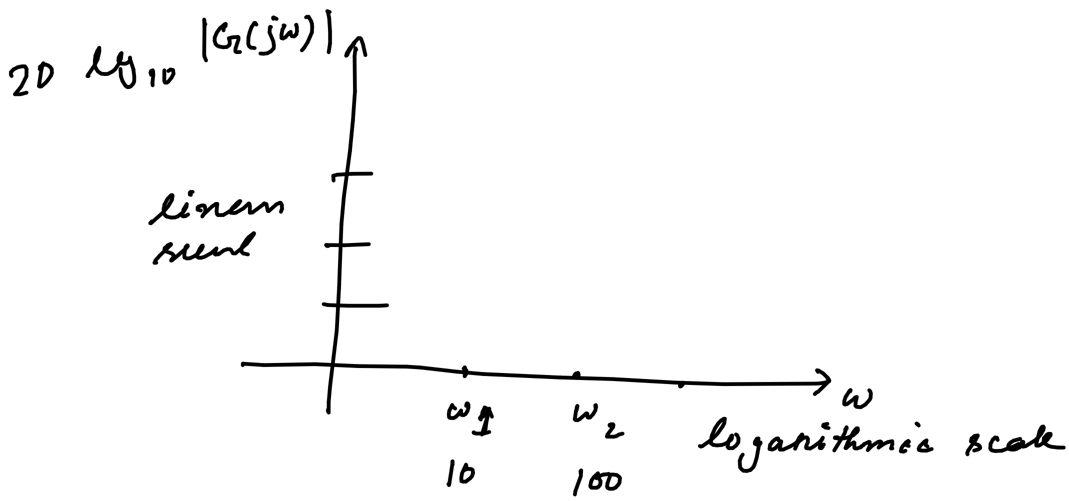
Frequency response

$$y_0 = u_0 |G(j\omega)|$$

$$\varphi = \angle G(j\omega)$$

→ Bode Plot

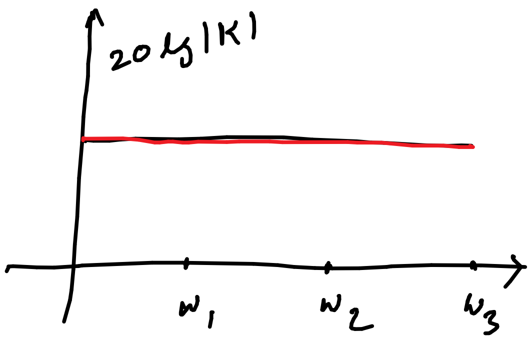
- Plotting $|G(j\omega)|$ & phase φ
- Use semilog paper

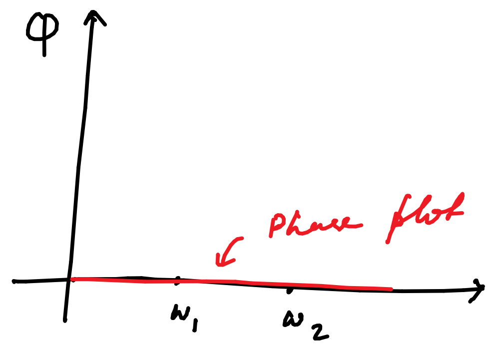


- We will express the magnitude in dB
  ||
  $20 \log_{10} |G(j\omega)|$

$$20 \log |G(j\omega)| = 20 \log |K| + 20 \log |j\omega + z_1| + \dots + 20 \log |j\omega + z_m|$$
$$- 20 \log |(j\omega)^N| - 20 \log |j\omega + p| - \dots - 20 \log |j\omega + p_n|$$

- Gain K
- Derivative & integral term s or $\frac{1}{s}$





Magnitude plot

→ $s \longrightarrow j\omega$

$$\boxed{M := |G(j\omega)|}$$

$$|G(j\omega)| = \omega$$

$$\varphi = 90^\circ$$

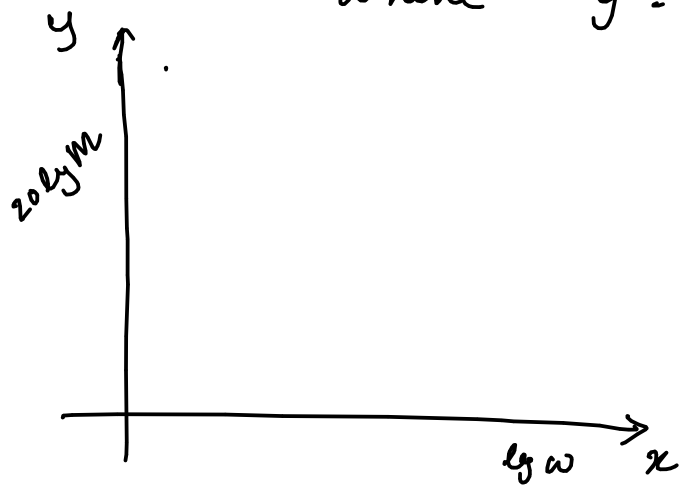
$$\boxed{20 \log M = 20 \log \omega}$$

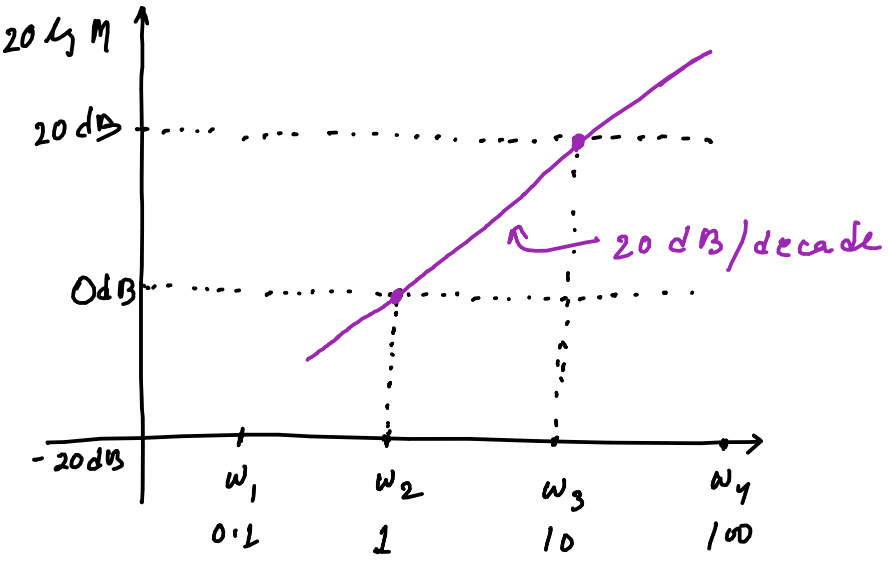
at $\omega = 1 \rightarrow 20 \log M = 0$ dB

$\omega = 10 \rightarrow 20 \log M = 20$ dB

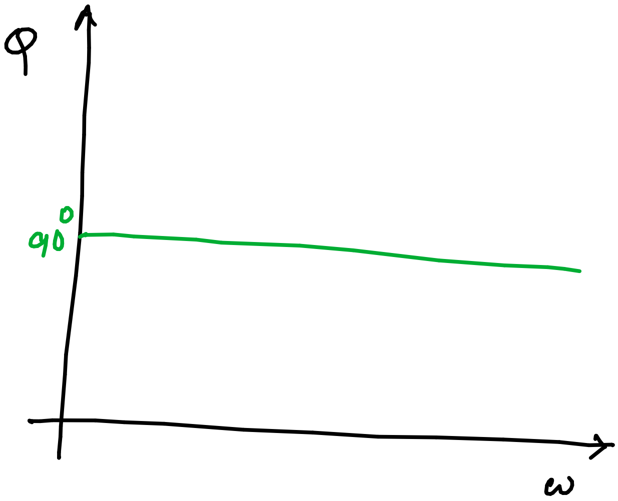
$$\boxed{y = 20x}$$ ← Equation of a straight line with slope 20

where $y = 20 \log M$ & $x = \log \omega$





Magnitude Plot



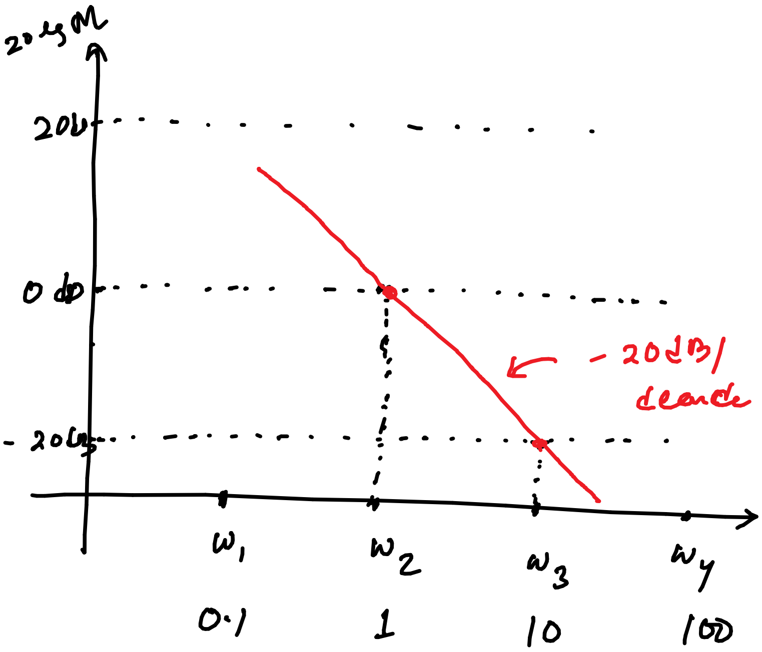
Phase plot.

→ For $\frac{1}{s} \rightarrow \frac{1}{j\omega}$

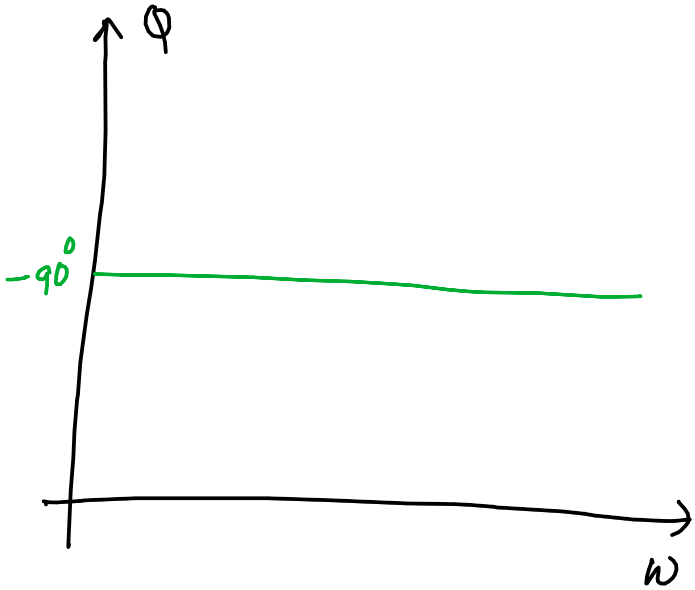
$$M = \frac{1}{\omega} \qquad \varphi = -90^\circ$$

$$20 \log M = -20 \log \omega$$

$$(y = -20x)$$



Magnitude plot



Phase plot.

• Term $(s+z)$: $s \to j\omega \to z + j\omega$

$z \to$ Position of a zero

$$M = \sqrt{\omega^2 + z^2}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{z}\right)$$

$\phi \approx 0^0$ (for $\omega << z$)

$\phi \approx 90^0$ (for $\omega >> z$)

$\phi = 45^0$ (for $\omega = z$)

• For $\omega << z$

$M \approx z$

$20 \log M \approx 20 \log z$

Since z is constant at lower frequency range $20 \log M$ is approximately const. with value $20 \log z$

• For $\omega >> z$

$M \approx \omega$

$20 \log M \approx 20 \log \omega$

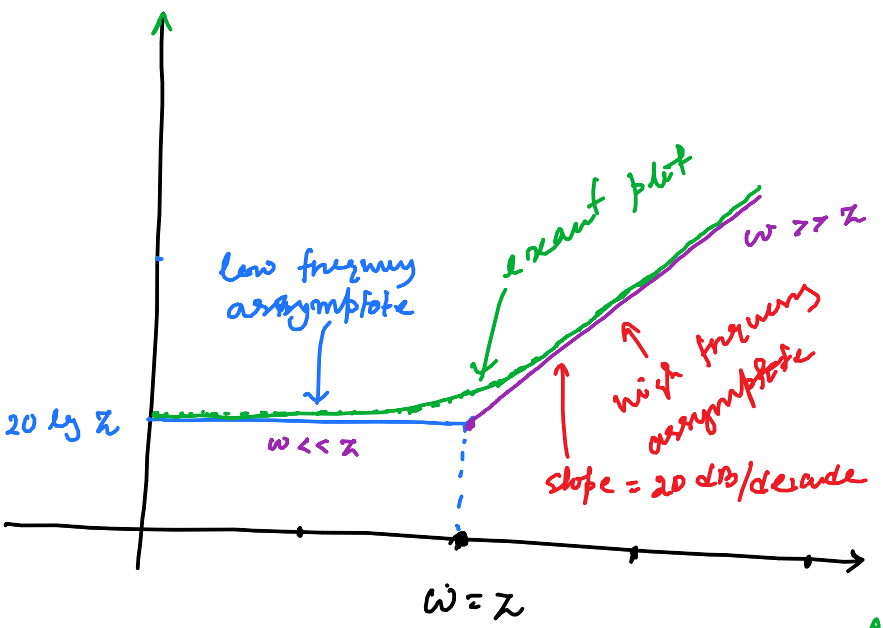
↳ is a straight line with slope 20 dB/decade.

• At $\omega = z$

$M = \sqrt{2} z$

$\approx z$

$20 \log M \approx 20 \log z$

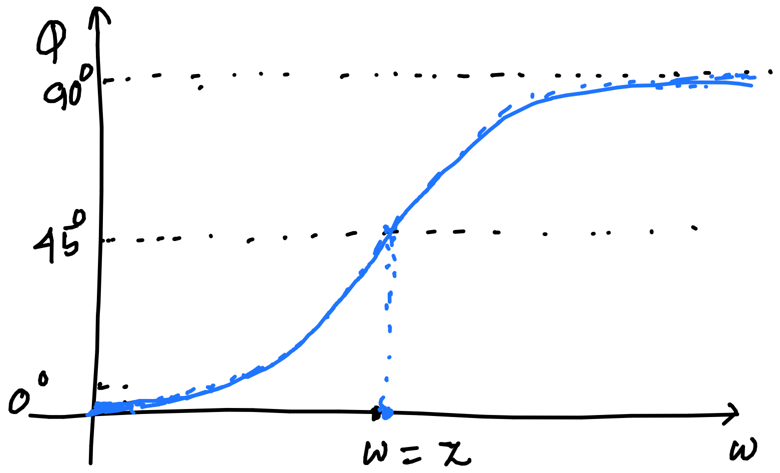


• For $\omega << z$ we have one straight line

• $\omega >> z$ we have another straight line

• $\omega = z$ both straight lines are intersecting

• Asymptotes are approximate magnitude plot.



w = z is reffered to as "corner frequency" or break frequency.

- Term $\frac{1}{s+p} \xrightarrow{s \to j\omega} \frac{1}{p+j\omega}$

$$M = \sqrt{\left(\frac{p}{p^2+\omega^2}\right)^2 + \left(\frac{\omega}{p^2+\omega^2}\right)^2}$$

→ p is a pole, whose value is fixed.

For $\omega << p$

$$M \approx \sqrt{\frac{1}{p^2}} = \frac{1}{p}$$

$$20 \lg M = -20 \lg p$$

↑ straight line, const., since p is fixed

- For $\omega >> p$

$$M \approx \sqrt{\frac{1}{\omega^2}} = \frac{1}{\omega}$$

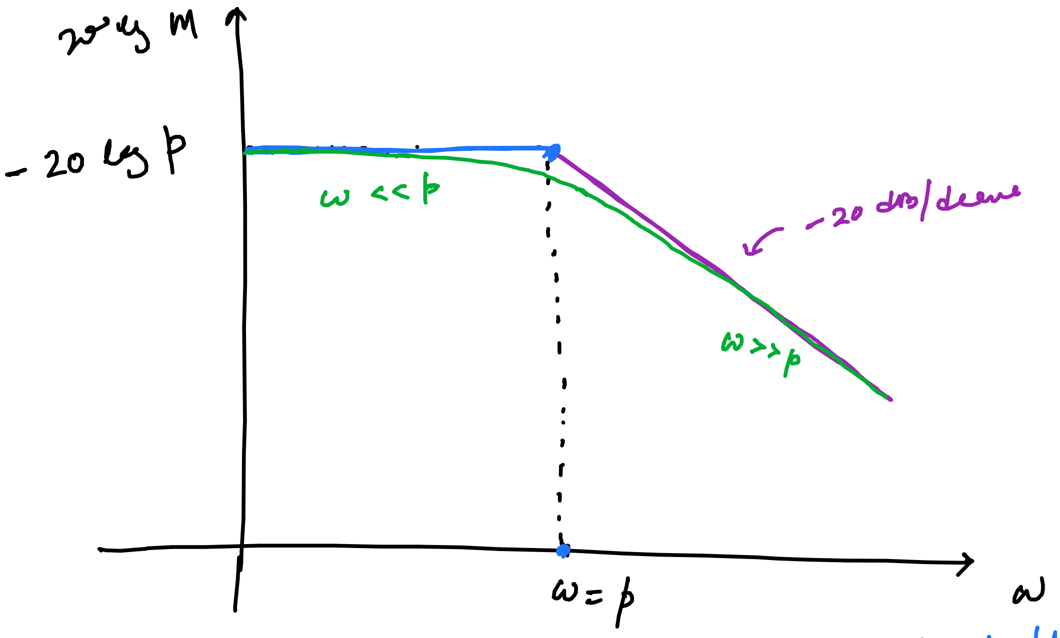
$$20 \lg M \approx -20 \lg \omega$$

↑ is a straight line with slope = −20 dB/d.

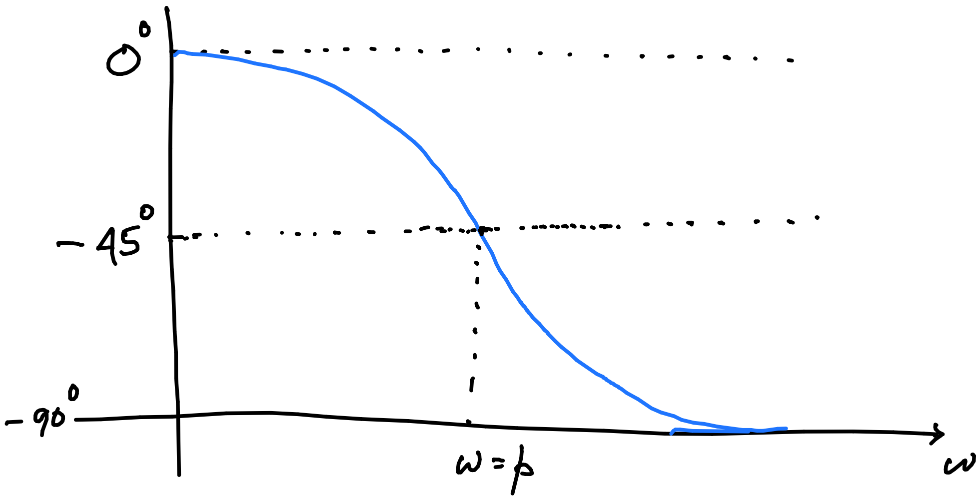
For $\omega = p$

$$M = \sqrt{\frac{2}{4p^2}} = \frac{1}{\sqrt{2}p}$$

$$M \approx \frac{1}{p} \rightarrow 20 \lg M \approx -20 \lg p$$



Magnitude plot



Phase plot

→ Quadratic term:

$$s^2 + 2\xi\omega_n s + \omega_n^2 \xrightarrow{s \to j\omega} -\omega^2 + j2\xi\omega_n\omega + \omega_n^2$$

$$= (\omega_n^2 - \omega^2) + j\,2\xi\omega_n\omega$$

- $\xi < 1$

$$\boxed{M = \sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2}}$$

- For $\omega << \omega_n$

$$M \approx \sqrt{(\omega_n^2)^2} = \omega_n^2$$

$$20 \log M = 2 \times 20 \log \omega_n$$

$$= 40 \log \omega_n$$ ← Does not depend on ξ

ω_n is fixed, so we have a straight line

- For $\omega = \omega_n$

$$\boxed{M = 2\xi\omega_n^2}$$

↑ depends on damping ratio ξ.

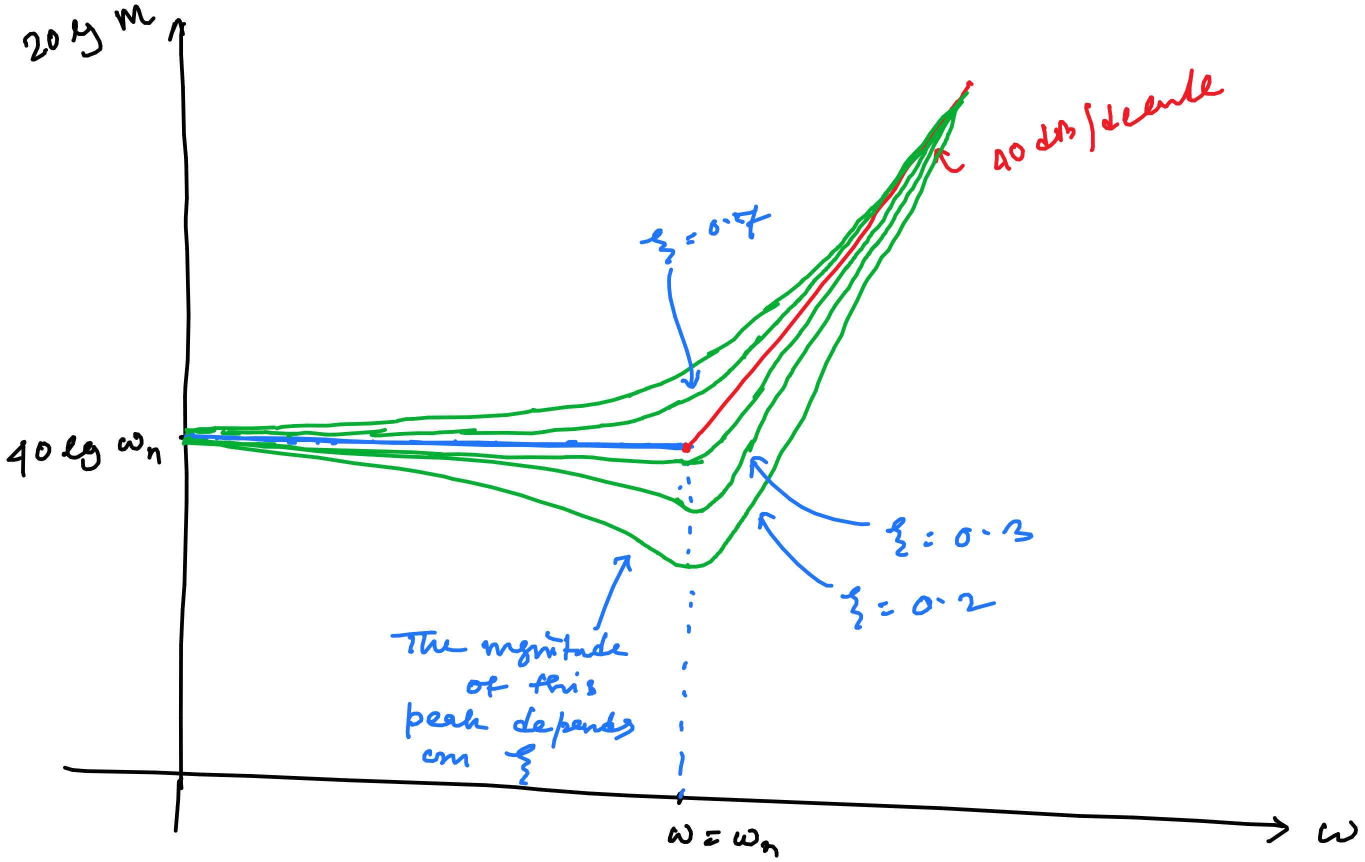
- The effect of ξ will be reflected near the natural frequency ω_n.

- For $\omega >> \omega_n$

$$M \approx \omega^2$$

$$20 \log M \approx 40 \log \omega$$ ← Not dependent on ξ

↑ straight line with 40 dB/decade slope.

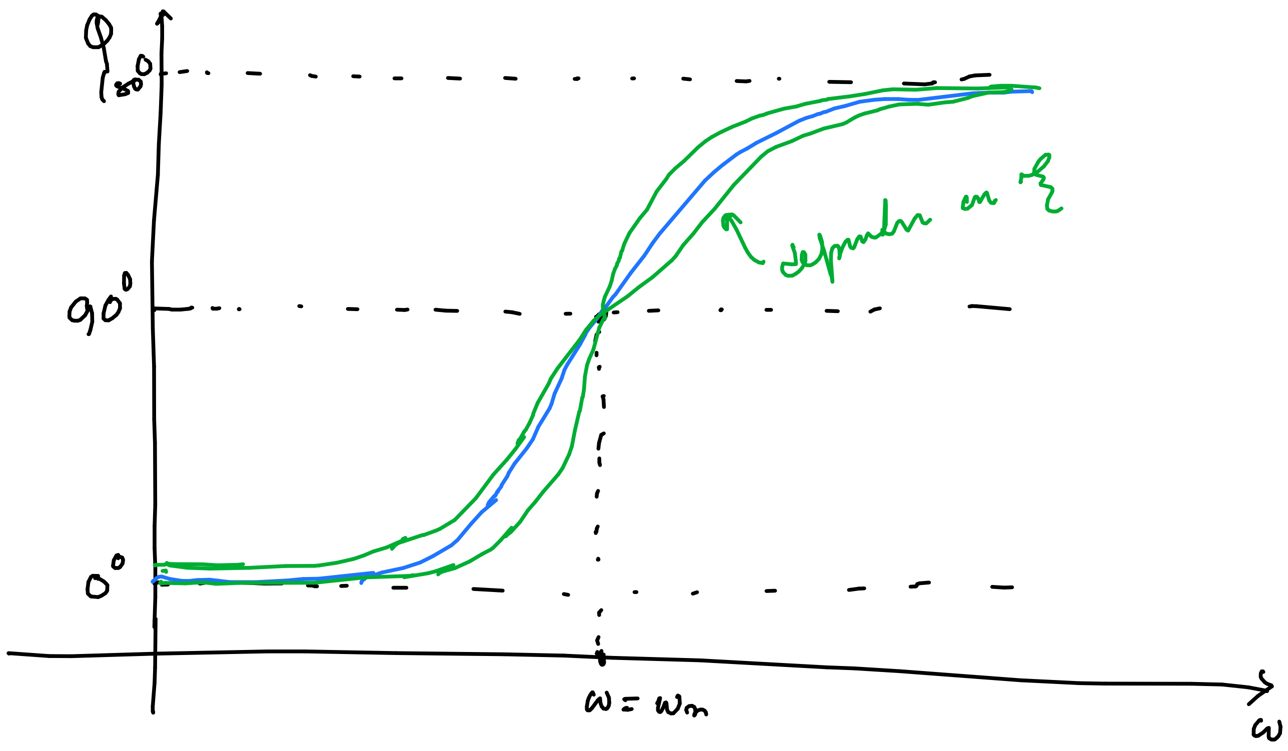


$\omega << \omega_n \qquad \phi \approx 0^\circ$

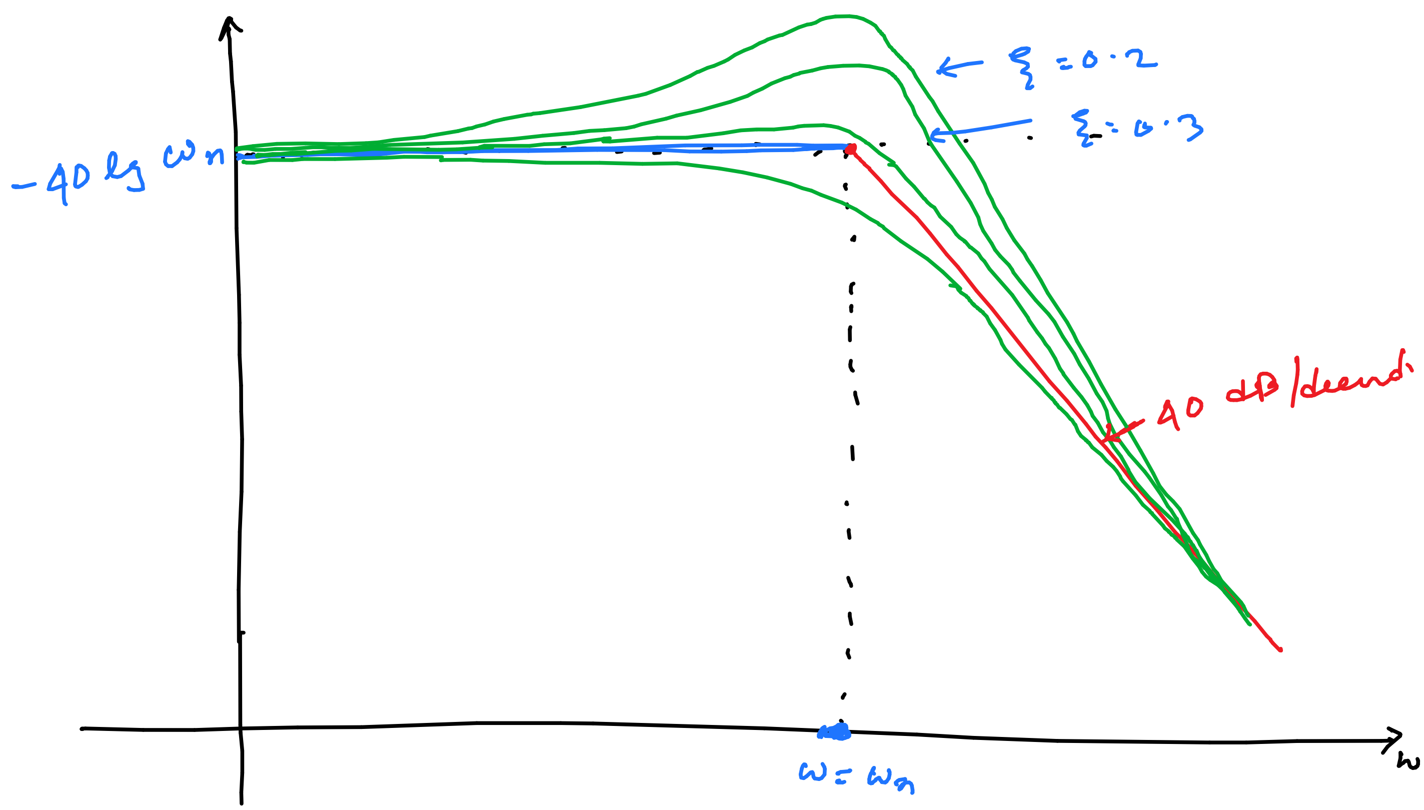
$\omega = \omega_n \qquad \phi = 90^\circ$

$\omega >> \omega_n \qquad \phi \approx 180^\circ$

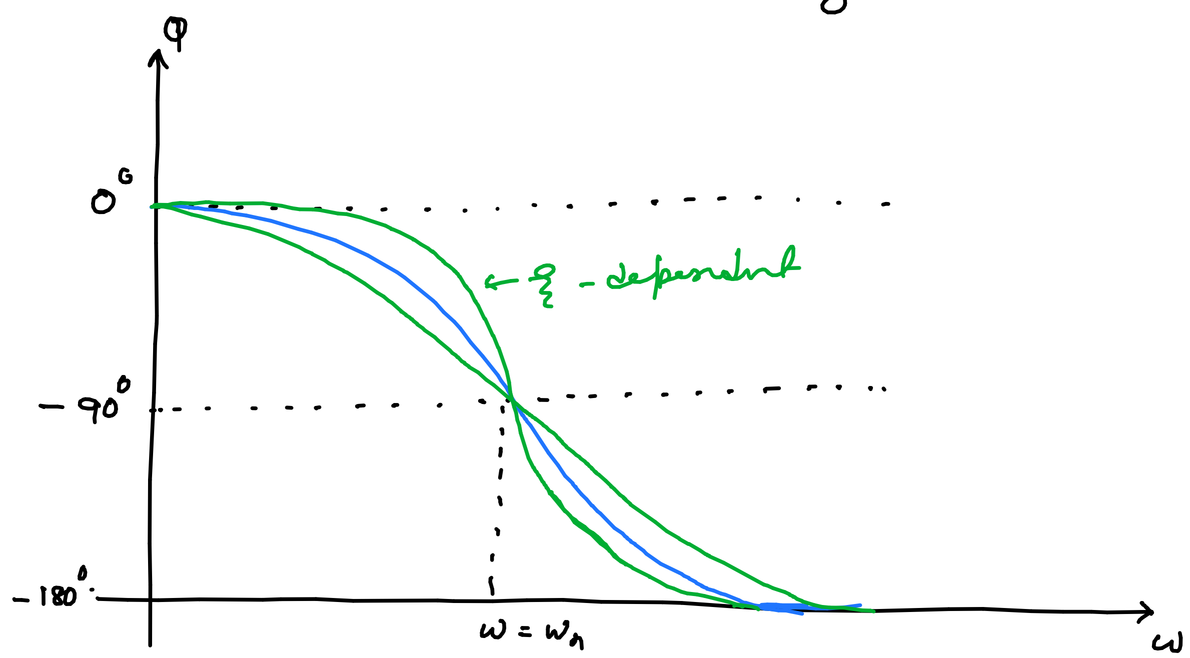
$$\boxed{\phi = \tan^{-1}\left(\frac{2\xi\omega_n\omega}{\omega_n^2 - \omega^2}\right)}$$



$$\frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



Magnitude plot



- For $\xi > 1$, the quadratic term can be expressed as a product of two first order terms.