ELL225

Lecture - 28

Bode Plot for combined terms that appear in $G(j\omega)$.

$\downarrow$

- The combined plot drawing is easier if we write the individual terms in "Normalized form", as follows:

$\rightarrow$ $s+z \rightarrow \left(\frac{1}{z}s+1\right)$ or $\underline{\underline{Ts+1}}$

where $T = \frac{1}{z}$

$\downarrow s \to j\omega$

$$\boxed{1 + j\omega T}$$

$$\boxed{|G(j\omega)| = M = \sqrt{1 + (\omega T)^2}} \qquad \boxed{\phi = \tan^{-1} \omega T}$$

- For $\omega T << 1 \equiv \omega << \frac{1}{T} \equiv \omega << z$

$M \approx 1$ $\qquad$ $\phi \approx 0^{\circ}$

$\Rightarrow 20 \log M \approx 0$ ✓

- For $\omega T >> 1 \equiv \omega >> \frac{1}{T} \equiv \omega >> z$

$M \approx \omega T$ $\qquad$ $\phi \approx 90^{\circ}$

$$\boxed{20 \log M \approx 20 \log \omega T} \leftarrow \text{written } y = 20x \text{ for}$$

when $\boxed{\begin{array}{l} y = 20 \log M \\ x = \log \omega T \end{array}}$

$\uparrow$

straight line with slope 20

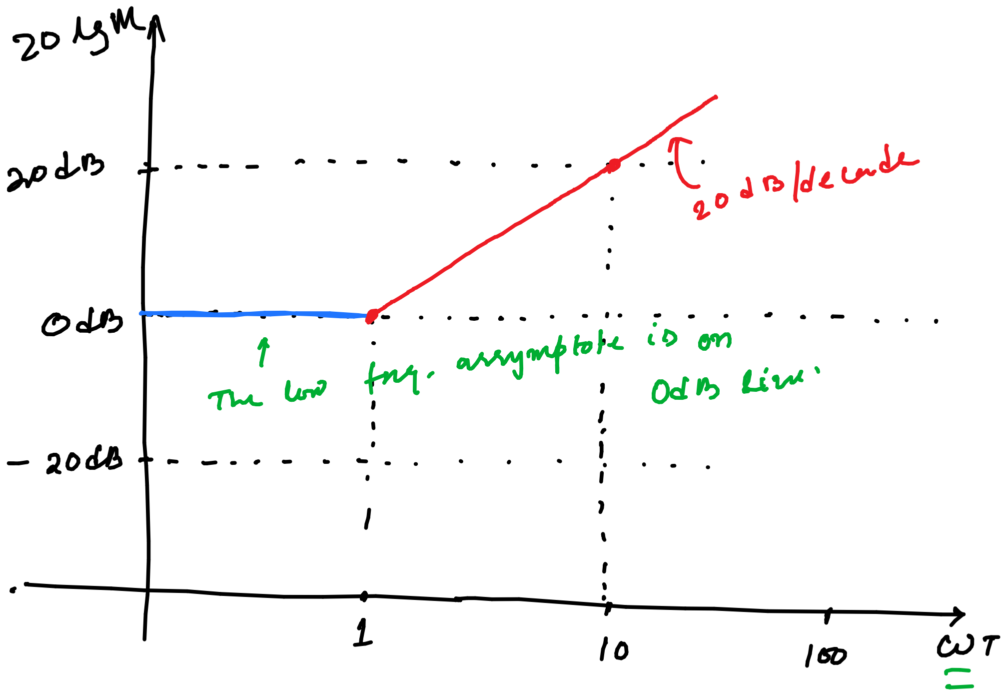
Then for $\omega = \frac{1}{T} \equiv \omega T = 1 \equiv \omega = \tau$

- Break freq. or Corner freq. is $\frac{1}{T} = \tau$

$$20 \log M \approx 20 \log 1 = 0 \qquad \varphi = 45^\circ$$

for $\omega = \frac{10}{T} \equiv \omega T = 10$

$$20 \log M \approx 20 \log 10 = 20$$



Magnitude plot

- Phase plot will remain same what we had done in previous class
  ↓
  but horizontal axis will be ωT

→ Similarly for

$$\frac{1}{s+p} \xrightarrow{\text{Normalized}} \frac{1}{Ts+1}$$

form (skipped the associated gain) will be included later

↓

where $T = \frac{1}{p}$

$$\frac{1}{Ts+1} \longrightarrow \frac{1}{1+j\omega T}$$

$$\boxed{M = \sqrt{\left(\frac{1}{1+(\omega T)^2}\right)^2 + \left(\frac{\omega T}{1+(\omega T)^2}\right)^2}} \qquad \boxed{\varphi = -\tan^{-1}\omega T}$$

- For $\omega T << 1 \equiv \omega << \frac{1}{T} \equiv \underline{\omega << p}$

$M \approx 1$

$\Rightarrow 20 \log M \approx \underline{0 \text{ dB}}$

$\varphi \approx 0^\circ$

- For $\omega T >> 1 \equiv \omega >> \frac{1}{T} \equiv \omega >> p$

$$M \approx \sqrt{\frac{(\omega T)^2}{(\omega T)^4}} = \frac{1}{\omega T}$$

$$\boxed{\Rightarrow 20 \log M \approx -20 \log \omega T}$$

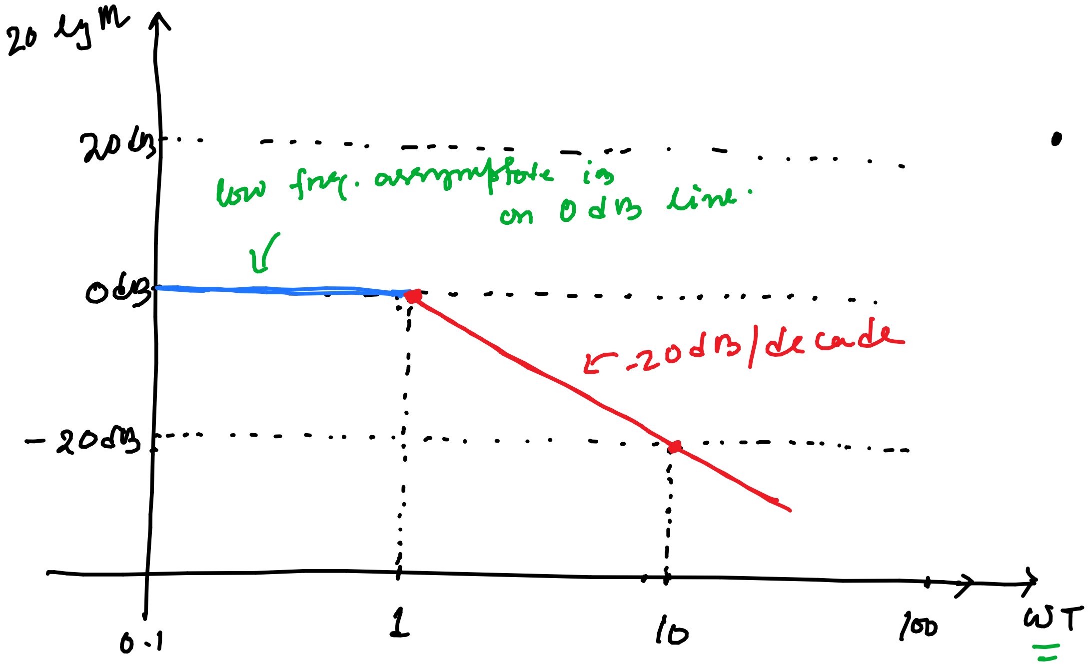
straight line with slope -20

For $\omega = \frac{1}{T}$ ← Break frequency

$20 \log M \approx 0$ dB

$\omega = \frac{10}{T}$

$20 \log M = -20$ dB



- Phase plot will remain same horizontal axis is in ωT

- $$s^2 + 2\xi\omega_n s + \omega_n^2 \xrightarrow[\text{form.}]{\text{Normalized}} 1 + j2\xi\left(\frac{\omega}{\omega_n}\right) - \left(\frac{\omega}{\omega_n}\right)^2$$

$$= \left(1 - \frac{\omega^2}{\omega_n^2}\right) + j2\xi\left(\frac{\omega}{\omega_n}\right)$$

$$\boxed{M = \sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$0 < \xi < 1$

Complex conjugate factors.

For $\frac{\omega}{\omega_n} << 1 \equiv \omega << \omega_n$

$$M \approx 1$$

$$\Rightarrow 20 \log M \approx 0 \text{ dB}$$

For $\frac{\omega}{\omega_n} >> 1 \equiv \omega >> \omega_n$

$$M \approx \left(\frac{\omega}{\omega_n}\right)^2$$

$$\boxed{\Rightarrow 20 \log M \approx 40 \log\left(\frac{\omega}{\omega_n}\right)}$$

$y = 40x$

where $\boxed{\begin{array}{l} y = 20 \log M \\ x = \log\left(\frac{\omega}{\omega_n}\right) \end{array}}$

Straight line with slope 40

- For $\omega = \omega_n$ Exact value $\rightarrow$ $\boxed{M = 2\xi}$

ω_n is corner frequency

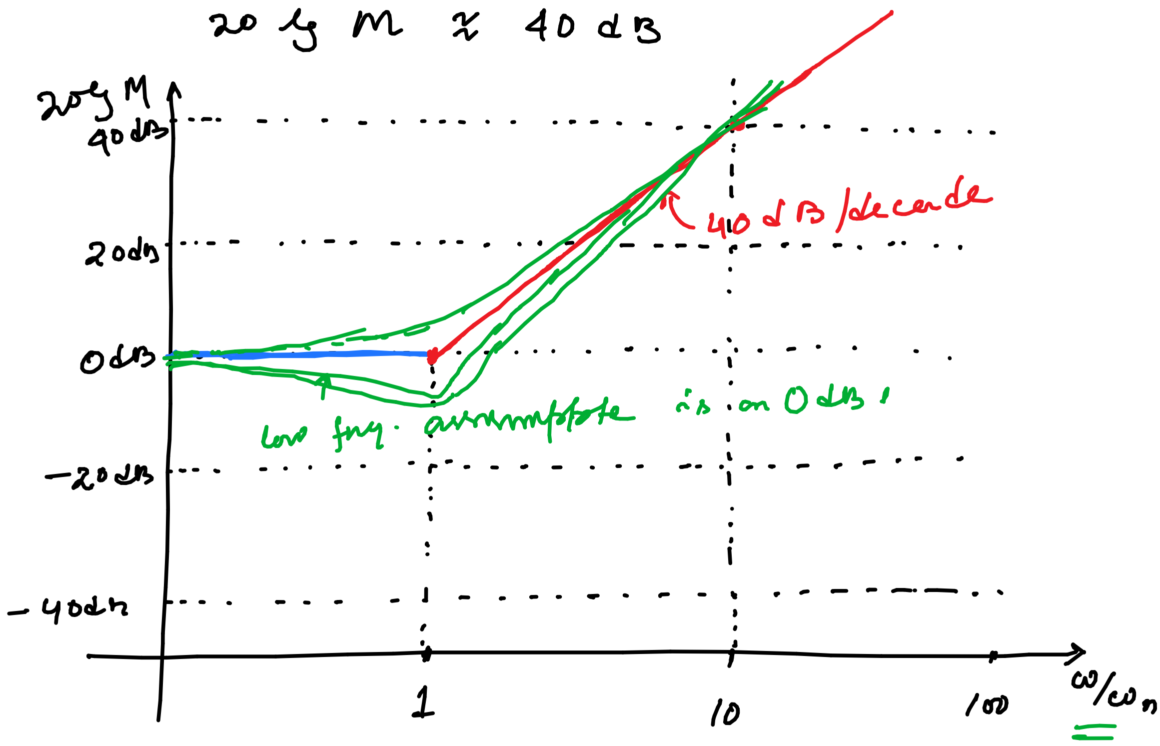
$$20 \log M \approx 0 \text{ dB}$$

$20 \log M = 20 \log(2\xi)$

dependent on ξ

For $\omega = 10\omega_n$

$$20 \log M \approx 40 \text{ dB}$$



Similarly it can be shown for

$$\frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \xrightarrow[\text{fn}]{\text{Normalized}} \frac{1}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + j\,2\xi\left(\frac{\omega}{\omega_n}\right)}$$

For $\frac{\omega}{\omega_n} << 1$

$M \approx 1$

$20 \log M \approx 0$ dB

For $\frac{\omega}{\omega_n} >> 1$

$M \approx 1/\left(\frac{\omega}{\omega_n}\right)^2$

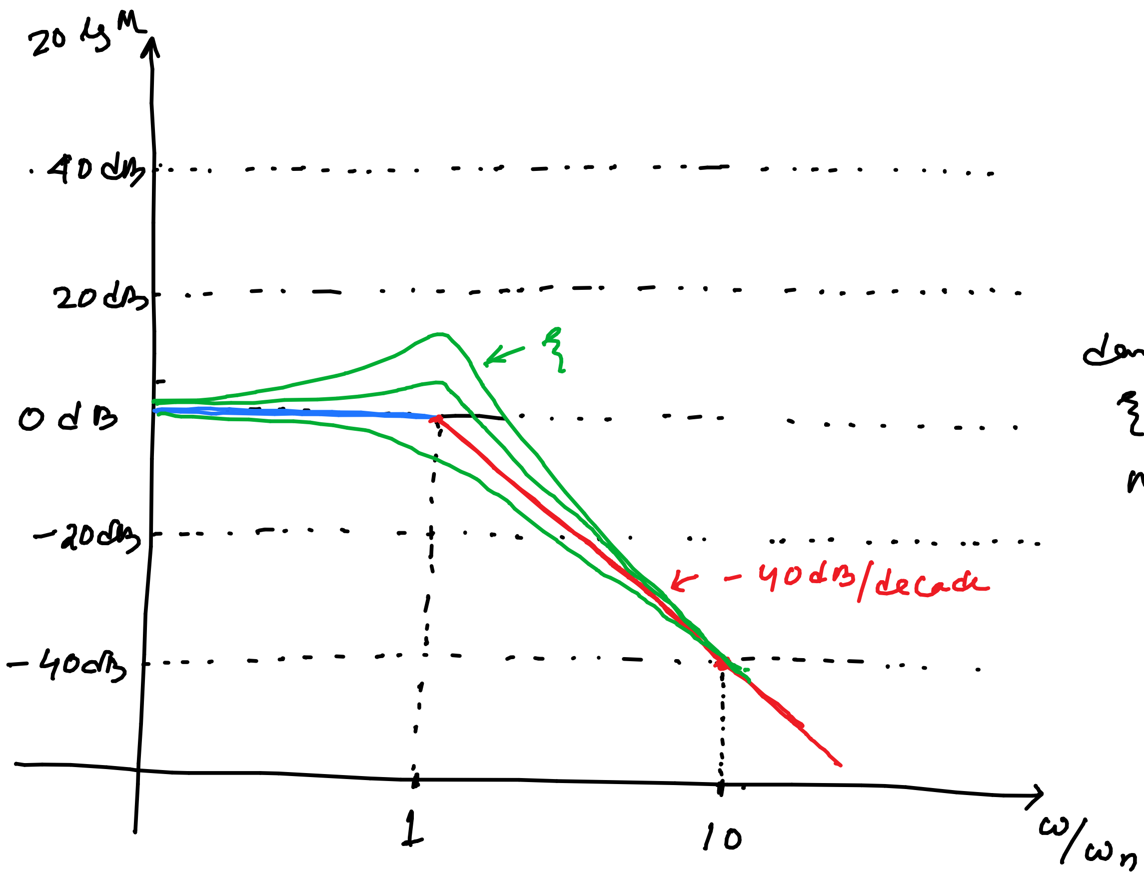
$\Rightarrow 20 \log M \approx -40 \log\left(\frac{\omega}{\omega_n}\right)$

slope -40

$\boxed{M = \frac{1}{2\xi}}$ ← For $\frac{\omega}{\omega_n} = 1$ $20 \log M \approx 0$ dB

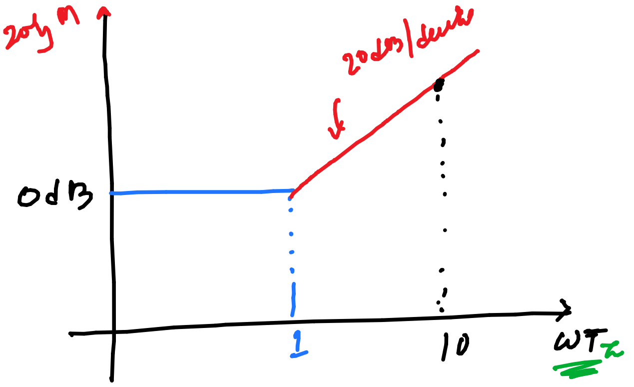
dependent on ξ

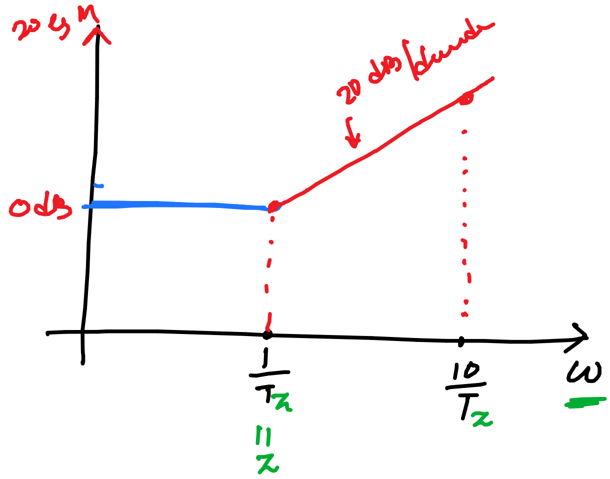
$\frac{\omega}{\omega_n} = 10$ $20 \log M \approx -40$ dB

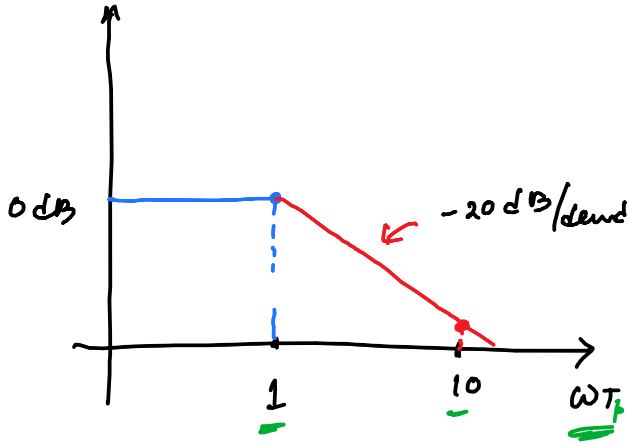


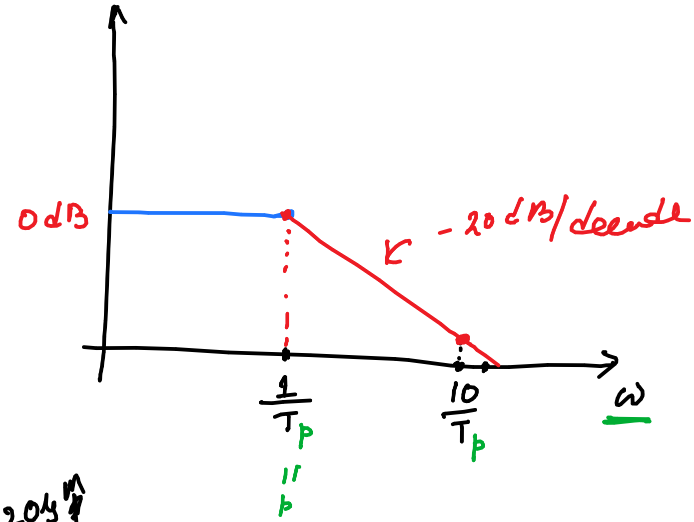
damping factor ξ will reflect at 1

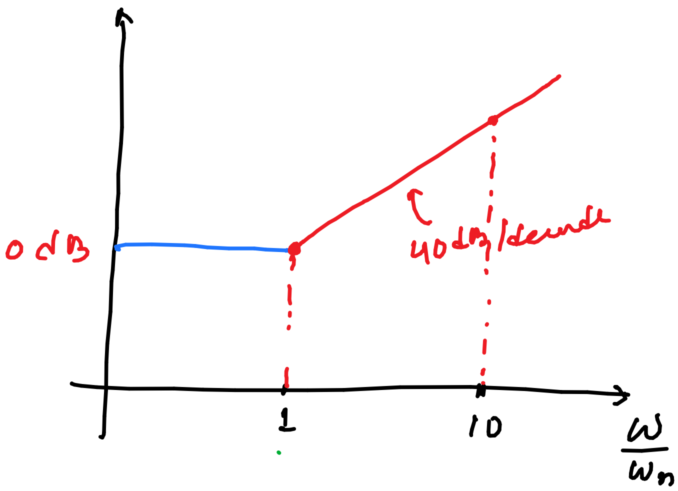
- The phase angle will remain same.

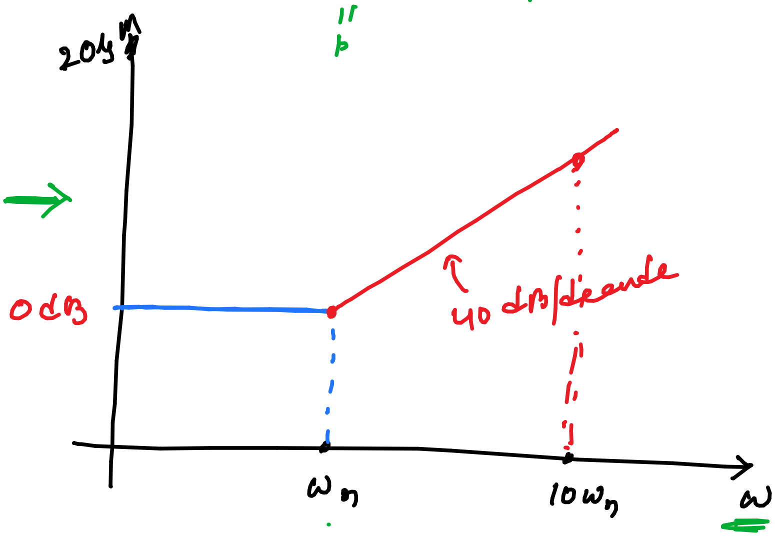

20 log M
20 dB/decade
0 dB
1
10
ωTz

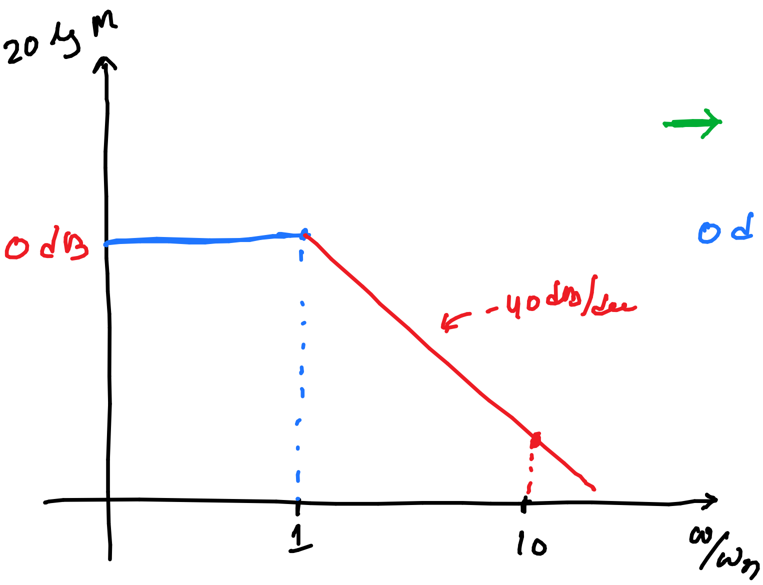

20 log M
20 dB/decade
0 dB
1/Tz
10/Tz
ω
z

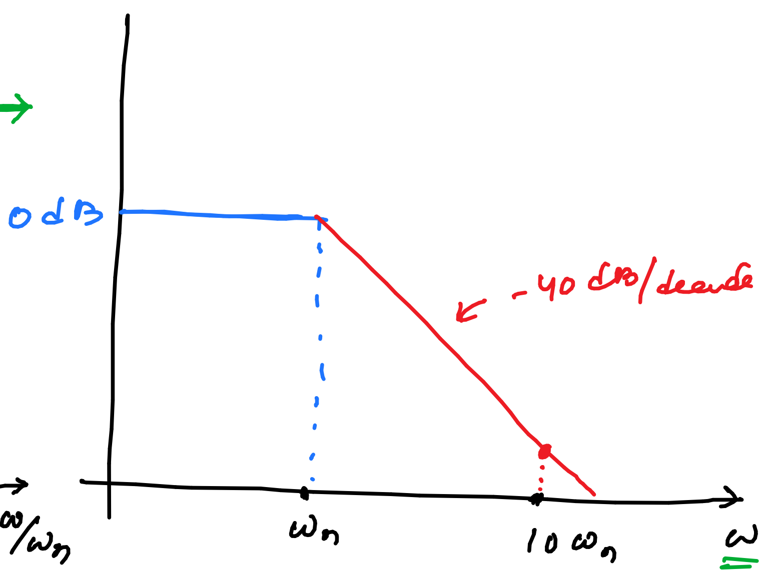

M
0 dB
-20 dB/decade
1
10
ωTp


0 dB
-20 dB/decade
1/Tp
10/Tp
ω
p


0 dB
40 dB/decade
1
10
ω/ωn


20 log M
0 dB
40 dB/decade
ωn
10ωn
ω


20 log M
0 dB
-40 dB/dec
1
10
ω/ωn

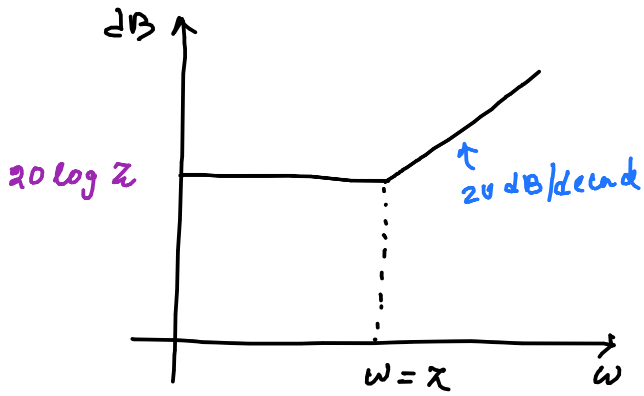

0 dB
-40 dB/decade
ωn
10 ωn
ω

Magnitude Plots

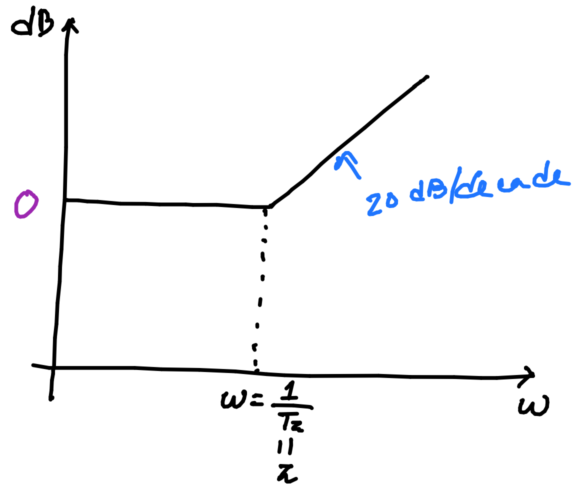
Non-normalized

Normalized

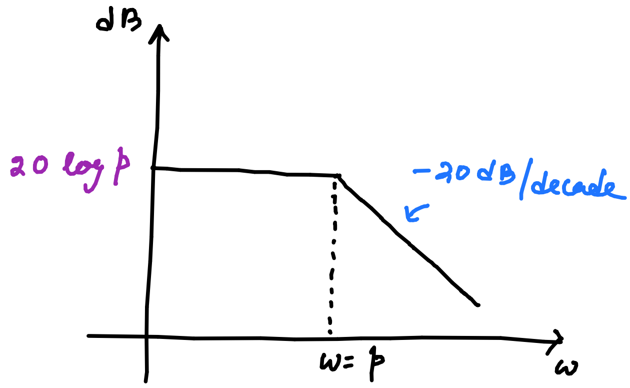
- $z + j\omega$ $\quad (s + z)$



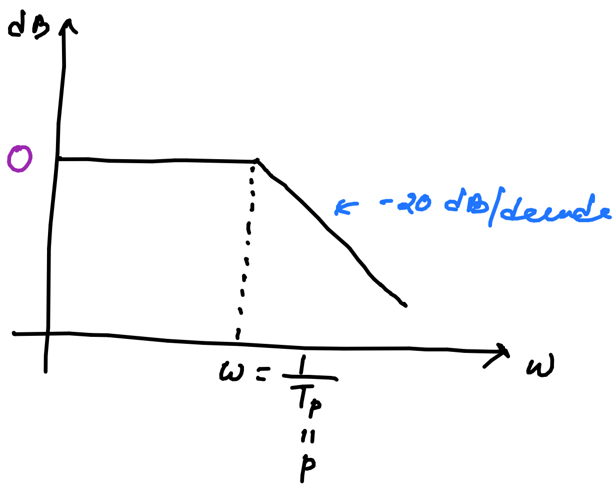
- $1 + j\omega T_z$ when $T_z = \frac{1}{z}$



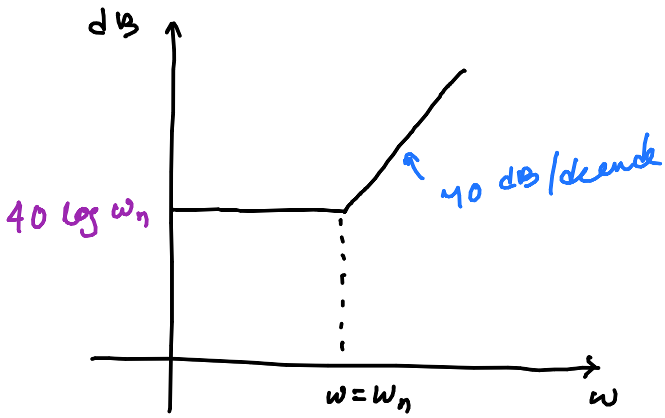
- $\frac{1}{p + j\omega}$ $\quad \left(\frac{1}{s+p}\right)$



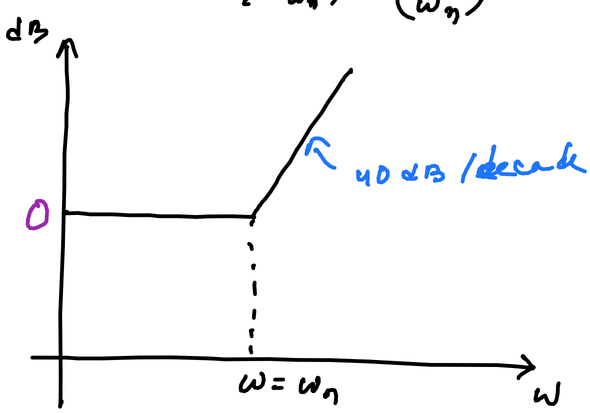
- $\frac{1}{1 + j\omega T_p}$



- $\omega_n^2 + j2\xi\omega_n\omega - \omega^2$



- $1 + j2\xi\left(\frac{\omega}{\omega_n}\right) - \left(\frac{\omega}{\omega_n}\right)^2$



- In Normalized form all lower frequency curves are having 0 dB line, whereas in non-normalized, they are different.

We got following advantages:

① Low frequency assymptotes for all 1st order & second order terms are on 0 dB line.

② By looking at the slope change at corner frequencies, instead on 1 or 10, we have made the horizontal axis same for all type of terms.

<u>Denote</u>

- T_z : corresponds to the term $(s+z) \rightarrow (T_z s + 1)$
- T_p : corresponds to the term $\frac{1}{s+p} \rightarrow \frac{1}{T_p s + 1}$

→ Example

$$G(s) = \frac{7(s+4)}{(s+10)}$$

↓ Normalized form

$$\frac{7 \times 4 \left(\frac{1}{4} s + 1\right)}{10 \left(\frac{1}{10} s + 1\right)} = \frac{2.8 (T_z s + 1)}{(T_p s + 1)}$$

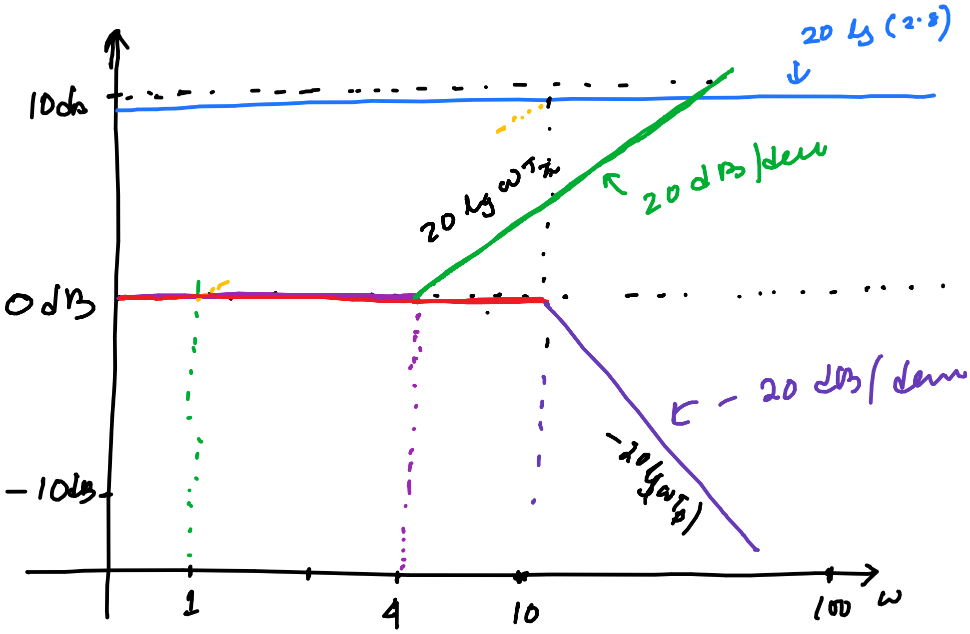
when $T_p = \frac{1}{10}$ $\quad T_z = \frac{1}{4}$

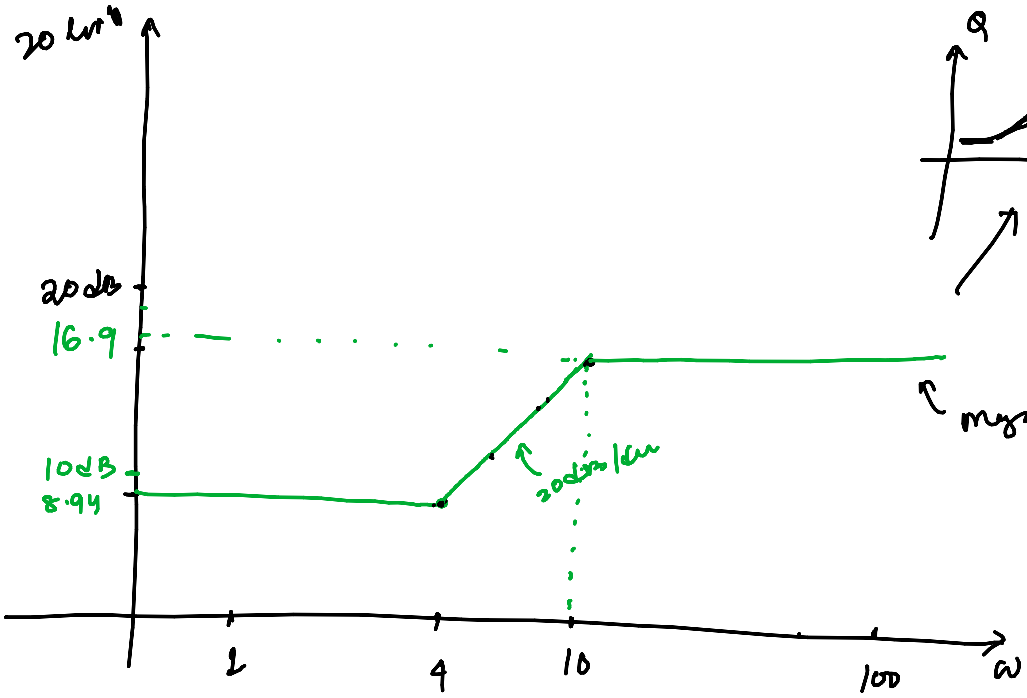
The corner / Break frequencies $\omega_p = 10$ $\quad \omega_z = 4$

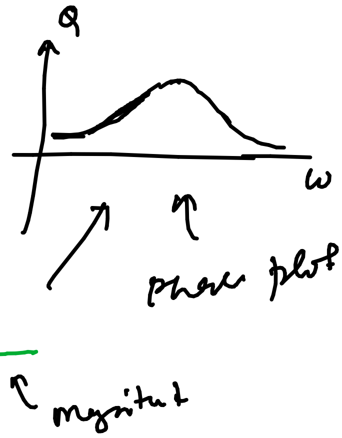
We have following three terms

$\underline{2.8}$, $(1 + j\omega T_z)$, $\left(\frac{1}{1 + j\omega T_p}\right)$

$20 \lg(2.8) = 8.94$







Upto $w = 4$, we have $8.94 + 0$ dB

From $w = 4$ to $w = 10$, we have $y \doteq mx + c \rightarrow \underline{\underline{20 \lg \omega T_z + 8.9}}$

$\underline{\underline{\text{At } w=10}}$: $8.94 + 20 \lg \omega T_z + 0 \quad (T_z = \frac{1}{4})$

$= 8.94 + 20 \lg \left(\frac{10}{4}\right)$

$= 8.94 + 7.95$

$= 16.89$ dB

After $\omega = 10$

$$8.9 + 20\log(\omega T_z) - 20\log(\omega T_p)$$

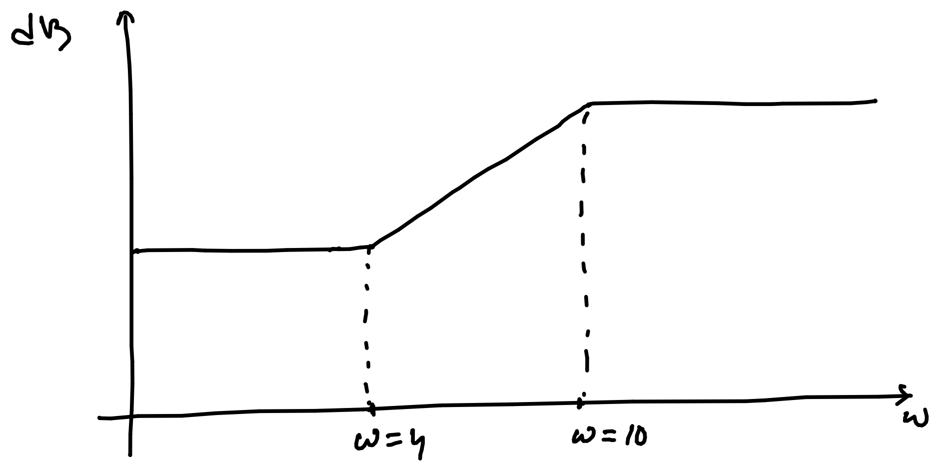
$$= 8.9 + 20\log\left(\frac{\omega}{4}\right) - 20\log\left(\frac{\omega}{10}\right)$$

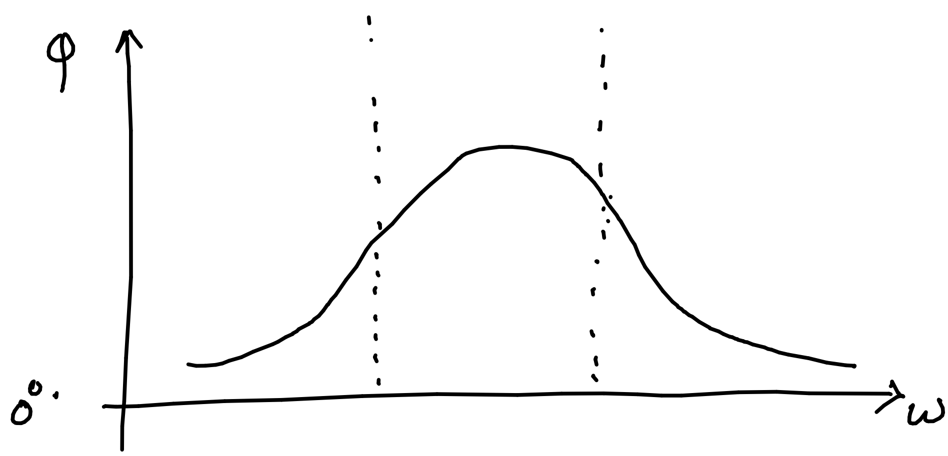
$$= 8.9 + \cancel{20\log\omega} - 20\log(4) - \cancel{20\log\omega} + 20\log 10$$

$$= 16.9\ dB$$

Phase angle $\varphi(\omega) = \tan^{-1}(\omega T_z) - \tan^{-1}(\omega T_p)$

$\omega \rightarrow$	1	4	7	10	50
$\varphi(\omega) \rightarrow$	8.31°	23.2°	25.25°	23.2°	6.79°

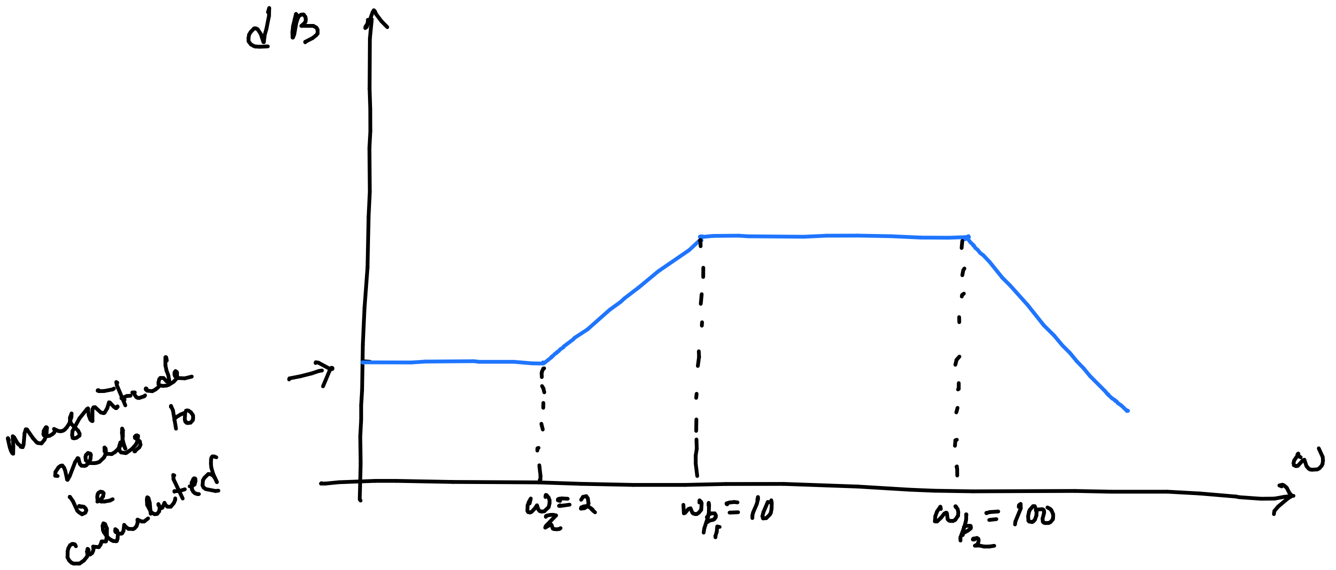




- By looking at the transfer funct^n one can roughly plot the Bode magnitude plot.

For instance $\quad G(s) = \dfrac{4(s+2)}{(s+10)(s+100)}$

Break frequencies : $\omega_z = 2 \quad \omega_{p_1} = 10 \quad \omega_{p_2} = 100$



- Similarly by looking at the Bode magnitude plot, one can construct the transfer function of the system.

$$G_N(j\omega) = \frac{K\,(1+j\omega T_{z_1})(1+j\omega T_{z_2})\cdots(1+j\omega T_{z_m})}{(j\omega)^N(1+j\omega T_{p_1})(1+j\omega T_{p_2})\cdots(1+j\omega T_{p_n})}$$

$$20\log|G_N(j\omega)| = 20\log K + 20\log|1+j\omega T_{z_1}| + \cdots + 20\log|1+j\omega T_{z_m}|$$
$$- 20\log|(j\omega)^N| - 20\log|1+j\omega T_{p_1}| - \cdots - 20\log|1+j\omega T_{p_n}|$$

$$\phi(\omega) = \tan^{-1}(\omega T_{z_1}) + \cdots + \tan^{-1}(\omega T_{z_m}) - N(90^0)$$
$$- \tan^{-1}(\omega T_{p_1}) - \cdots - \tan^{-1}(\omega T_{p_n})$$

- By experimentally performing the <u>frequency response analysis</u> (give different frequencies of sinusoidal signals to the physical system & then observe / plot the response (magnitude / phase) one can obtain the transfer funct of the system.

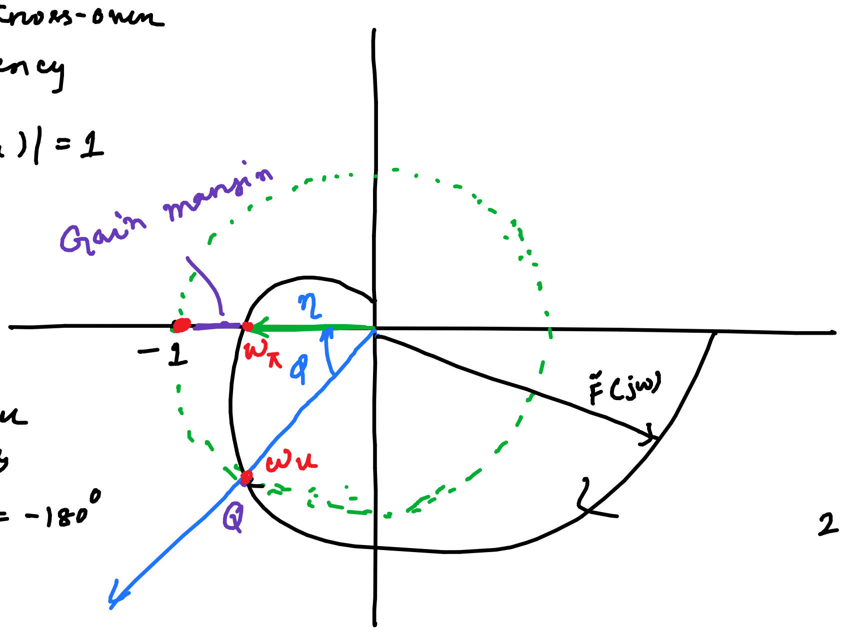
→ <u>Distance to instability</u> :

(i) Gain margin

(ii) Phase margin

ω_u : Gain cross-over frequency

$|\tilde{F}(j\omega_u)| = 1$



ω_π : Phase cross-over frequency

$\angle \tilde{F}(j\omega_\pi) = -180^0$

$PM = \varphi$

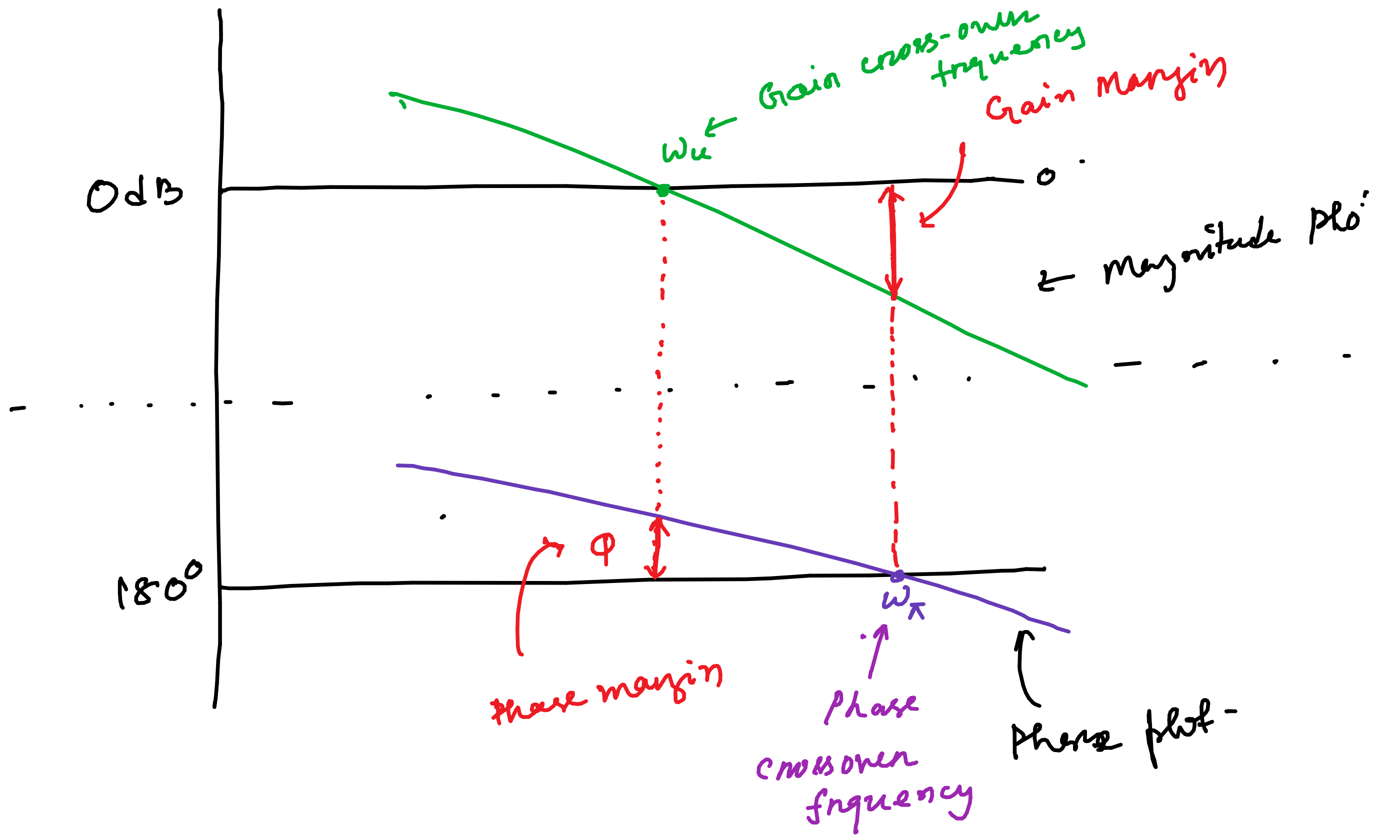
$GM = \frac{1}{\eta}$

$20 \log(\frac{1}{\eta})$

$= -20 \log \eta$ dB

- Gain margin : It is the change in open loop again, required at 180^0 phase to make the closed loop system unstable.

- Phase margin : It is the change in the open loop phase at unity gain to make the closed loop system unstable.



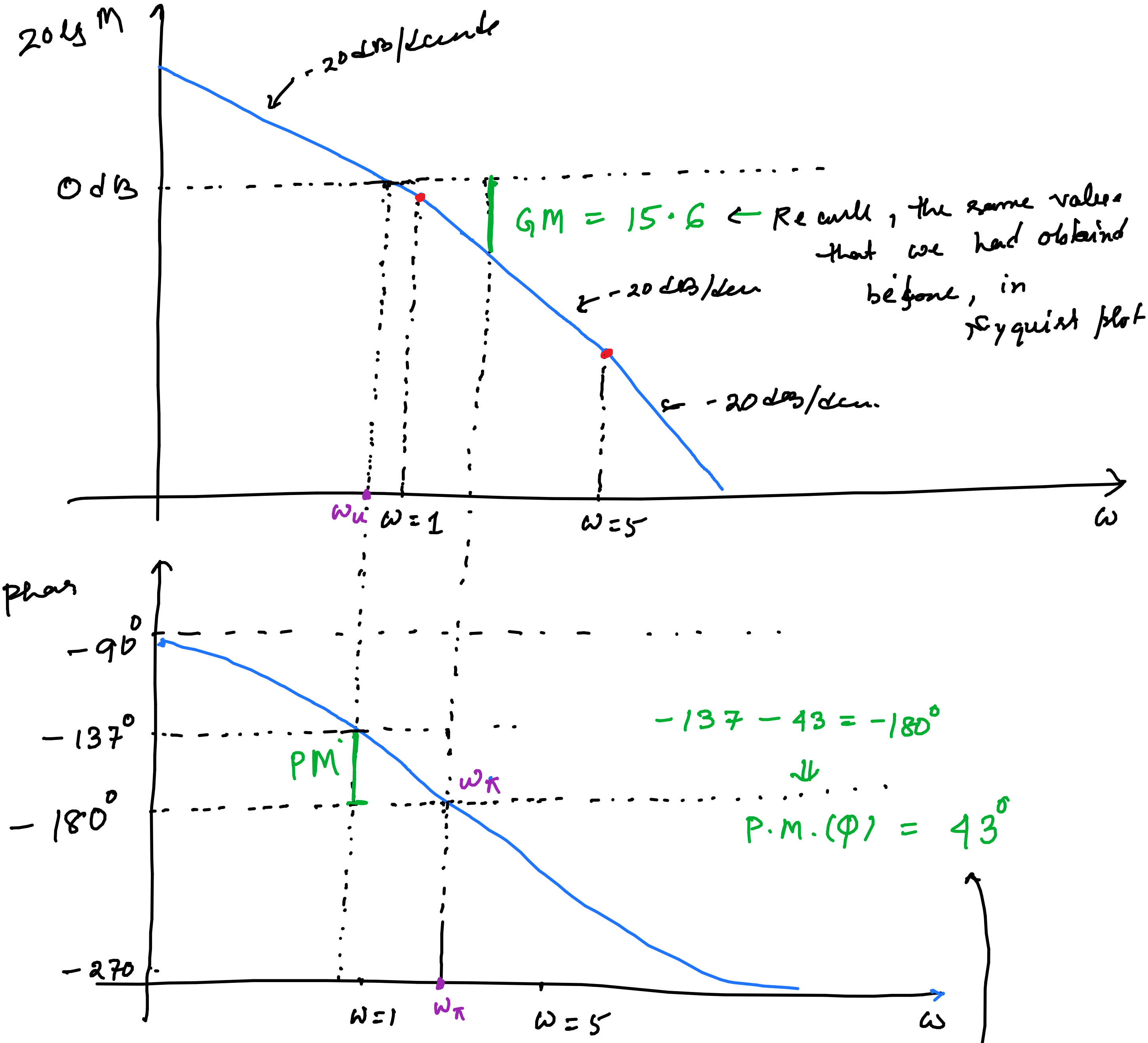
Example

$$G(j\omega) = \frac{5}{j\omega(j\omega+1)(j\omega+5)}$$

The Bode plot $= \dfrac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$

The terms are

$$\frac{1}{j\omega}, \quad \frac{1}{1+j\omega}, \quad \frac{1}{1+j0.2\omega}$$



Amount of phase lag required to make the system (BIBO) unstable.

(Recall the Nyquist plot before, where we had obtained this)