ELL225

Lecture - 29

$$F(s) = \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Evaluate $F(s)$ at point $s = s^*$

$$F(s^*) = v = m e^{i\theta}$$

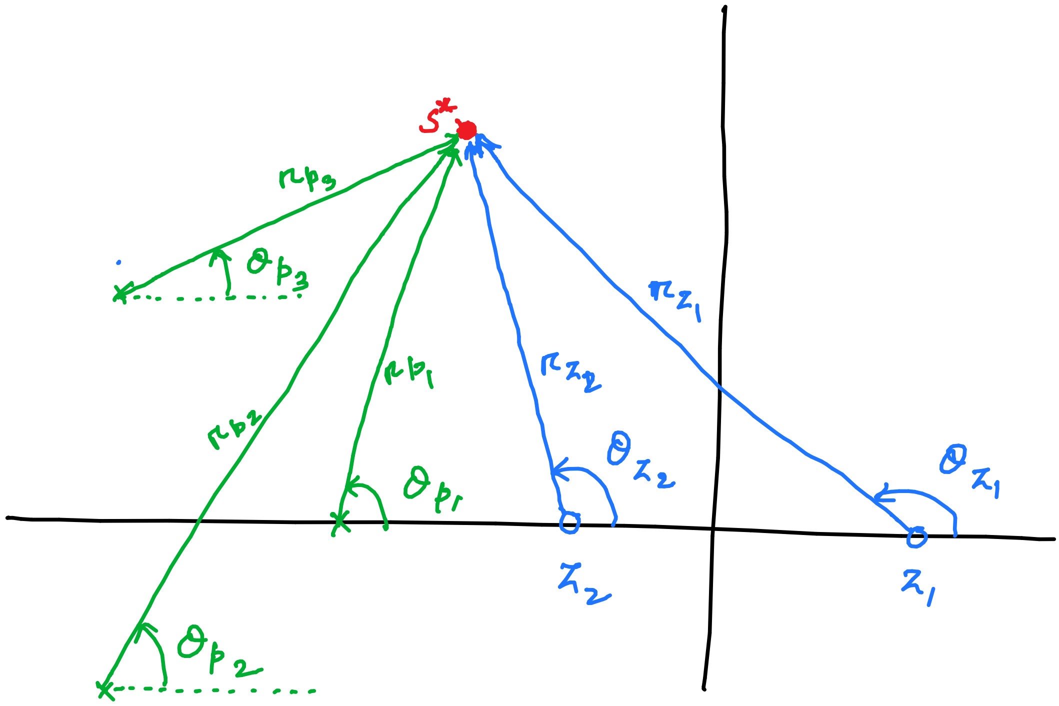
where $m = |F(s^*)|$

$\theta = \angle F(s^*)$

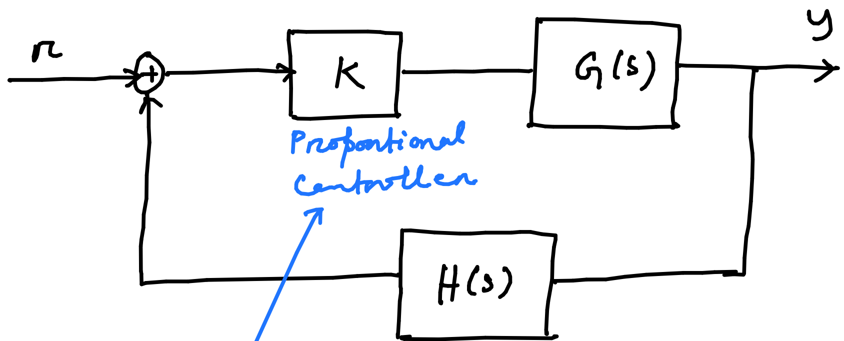
$$|F(s^*)| = \frac{r_{z_1} r_{z_2} \cdots r_{z_m}}{r_{p_1} r_{p_2} \cdots r_{p_n}}$$

$$r_{z_i} = |s^* - z_i| \qquad r_{p_i} = |s^* - p_i|$$

$$\theta = (\theta_{z_1} + \theta_{z_2} + \cdots + \theta_{z_m}) - (\theta_{p_1} + \theta_{p_2} + \cdots + \theta_{p_n})$$



→ Root-Locus

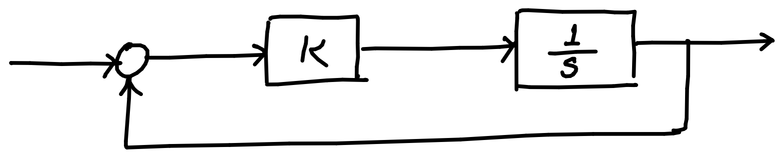


which will be changed within following range $0 \leq K < \infty$

Root Locus: It is the locus of the closed loop poles with respect to change in the gain parameter K.

T.F. of closed loop system : $\boxed{T(s) = \dfrac{K\,G(s)}{1 + K\,G(s)\,H(s)}}$

For instance:



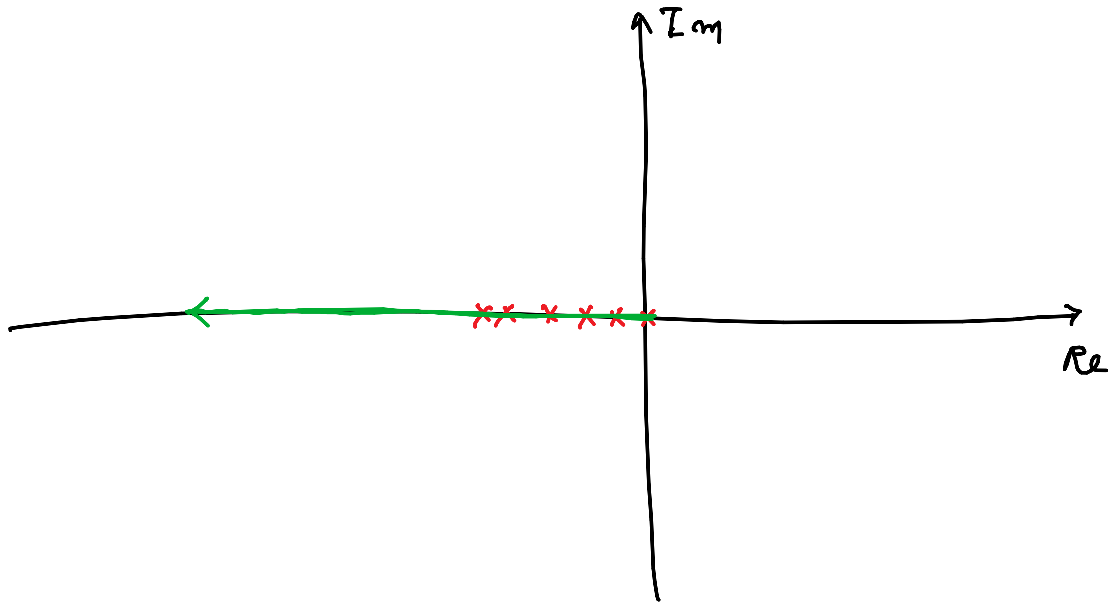
The closed loop characteristic polynomial

$$\alpha(s) = s + k$$

$$\Downarrow$$

By changing the values of k, we will get a set of closed loop poles.

For $0 \leq k < \infty$



$$G(s) = \frac{n_g(s)}{d_g(s)} \qquad H(s) = \frac{n_h(s)}{d_h(s)}$$

$$T(s) = \frac{K\, n_g/d_g}{1 + K\frac{n_g n_h}{d_g d_h}} = \frac{K n_g d_h}{d_g d_h + K n_g n_h}$$

closed loop characteristic polynomial, whose roots change with change in k.

For what value of 's', it will become a closed loop pole?

↓

→ Any 's' that satisfies $1 + KG(s)H(s) = 0$ is a closed loop pole.

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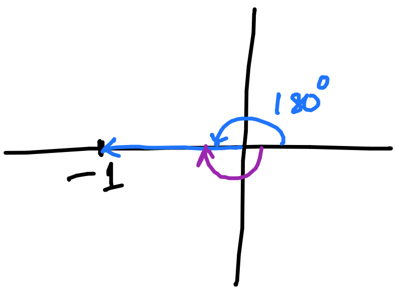
Any 's' that satisfies $\boxed{KG(s)H(s) = -1}$ . . . (*) is a pole of closed loop system.



$$-1 \longrightarrow |1|\angle(2l+1)180^\circ$$

$$l = 0, \pm 1, \pm 2 \dots$$

$$\boxed{K\,G(s)\,H(s) = -1}$$

$$\Updownarrow$$

Any s that satisfies these eq^ns will become a closed loop pole.

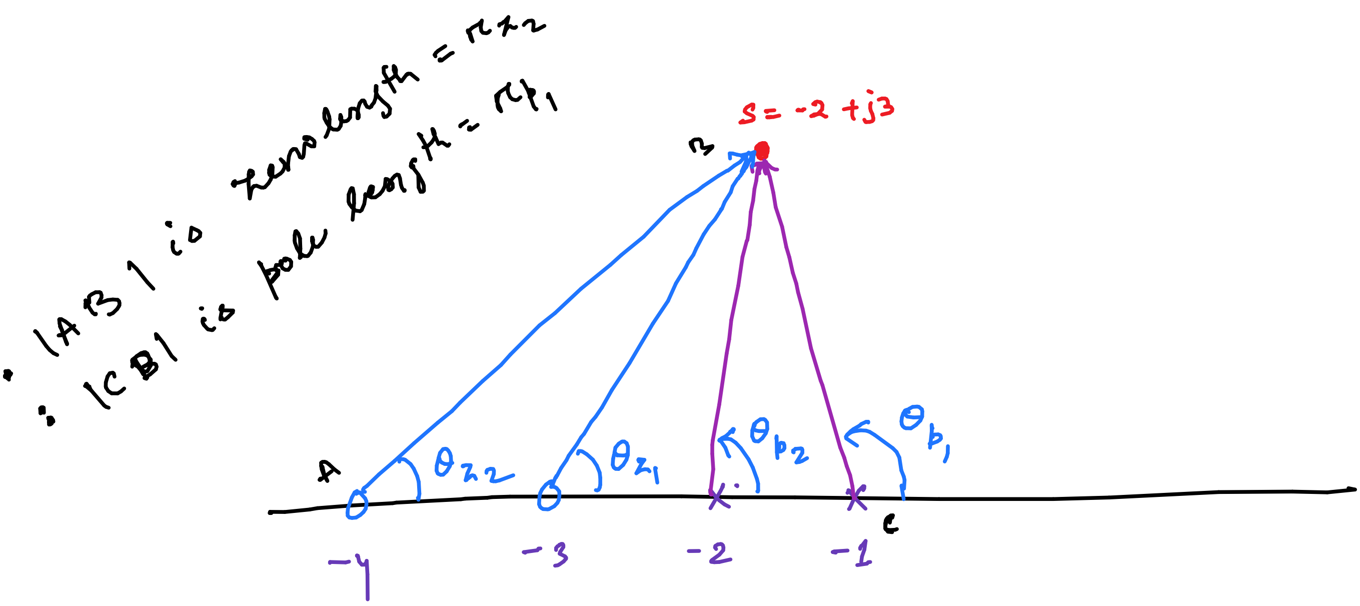
$$\boxed{\begin{aligned} &|K\,G(s)\,H(s)| = 1 \\ &\angle K\,G(s)\,H(s) = (2l+1)\,180^\circ \quad , \quad l = 0, \pm 1, \pm 2 \ldots \end{aligned}}$$

Ex :

$$K\,G(s)\,H(s) = \frac{K(s+3)(s+4)}{(s+1)(s+2)}$$

Test point $s = -2 + j3$, we are interested to know if this test point is a closed loop pole

$$\theta = \angle K\,G(s)\,H(s) = \;?$$



$$\theta = \theta_{z_1} + \theta_{z_2} - \theta_{p_1} - \theta_{p_2} = 56.31 + 71.57 - 90^\circ - 108.4^\circ$$

$$= -70.55 \Rightarrow \text{Not a closed loop pole.}$$

Test point: $s = -2 + j(1/\sqrt{2})$

θ is an odd multiple of 180^0

$\Downarrow$

s is a closed loop pole for some value of gain K.

For what value of K, $s = -2 + j(1/\sqrt{2})$ is a closed loop pole?

Since we need $|K\,G(s)\,H(s)| = 1$

$$\Rightarrow \quad K = \frac{1}{|G(s)\,H(s)|}$$

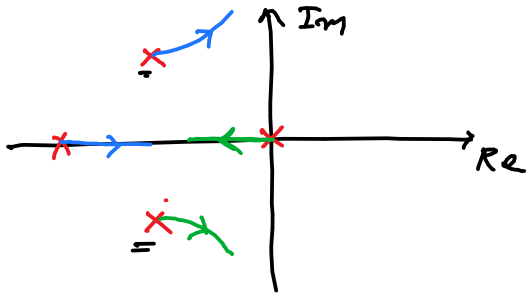
$$= \frac{r_{p_1}\, r_{p_2} \cdots r_{p_n}}{r_{z_1}\, r_{z_2} \cdots r_{z_m}}$$

$$= \frac{\Pi \text{ pole lengths}}{\Pi \text{ zero lengths}}$$

→ Sketching Root-locus

(1) Number of branches (locus): This is equal to the number of closed loop poles.

$\alpha(s) = s^2 + 2s + K$



(2) Symmetry : Usually we deal with systems where the closed loop characteristic polynomial has <u>real</u> coefficients.

⇓

If $s = \sigma + j\omega$ is a root of the above polynomial then $\bar{s} = \sigma - j\omega$ is also a root.

- The root locus is symmetric about real axis.