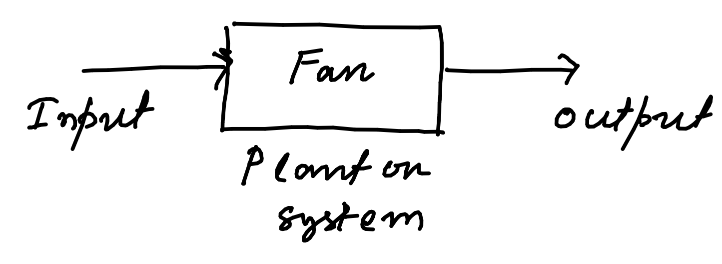
ELL 225

Lecture - 2

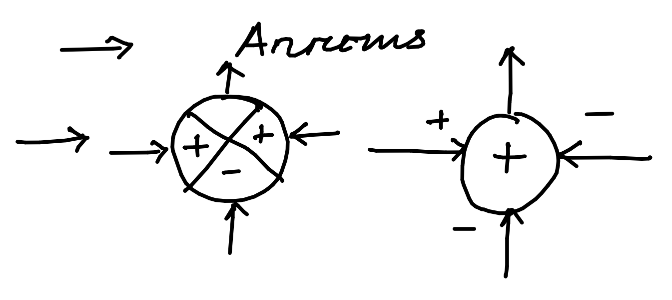
• Mathematical Modeling



Physical components → Rectangular blocks

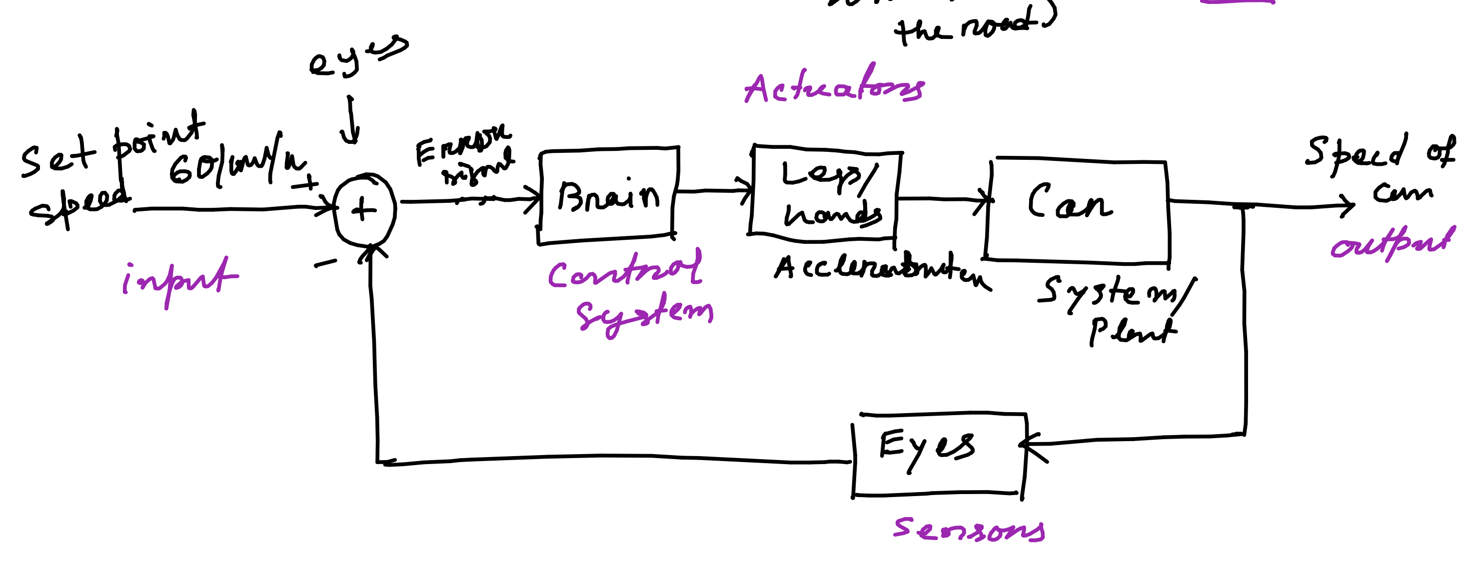
Signal flows → Arrows

Comparators →



- Driving a car

- specified speed of the car for a particular zone of the road } → 60 km/h



Basic block diagram of a feedback control system

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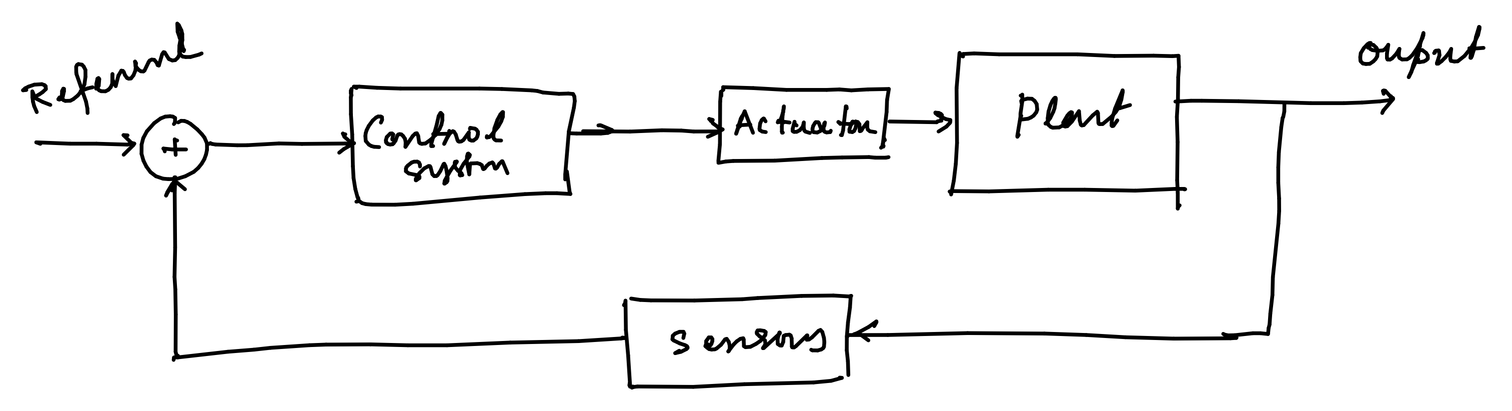
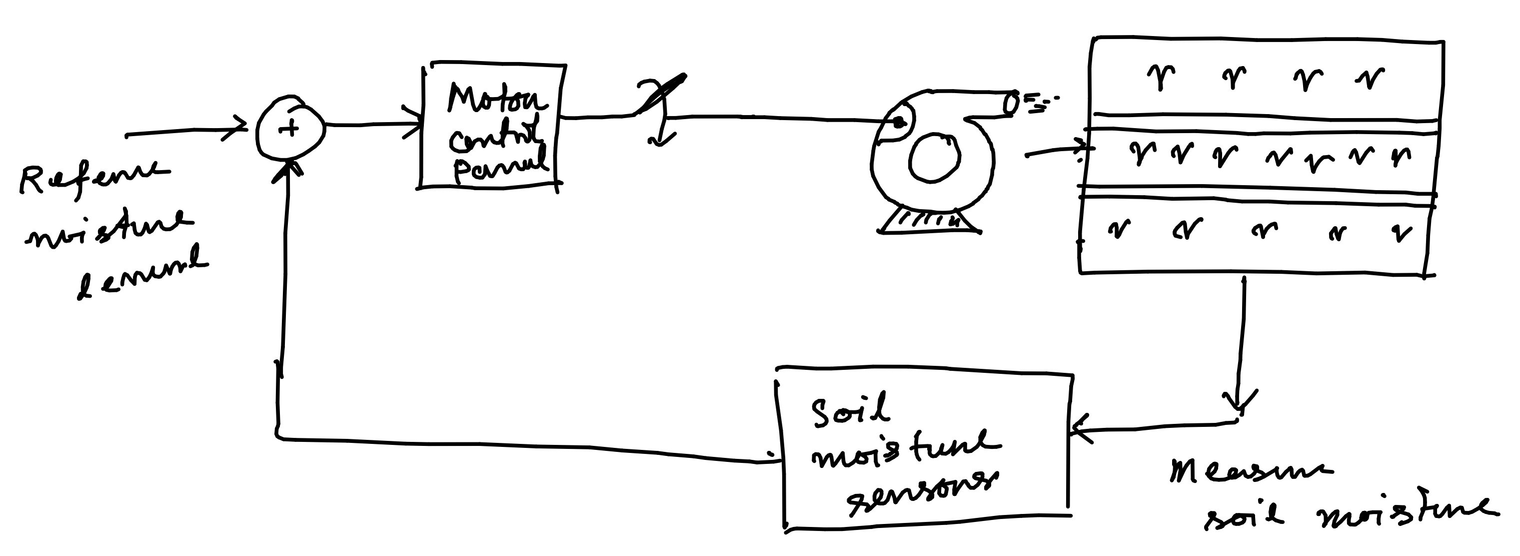


→ Automatic irrigation system for Agriculture



Schematic diagram

Closed-loop feedback control system

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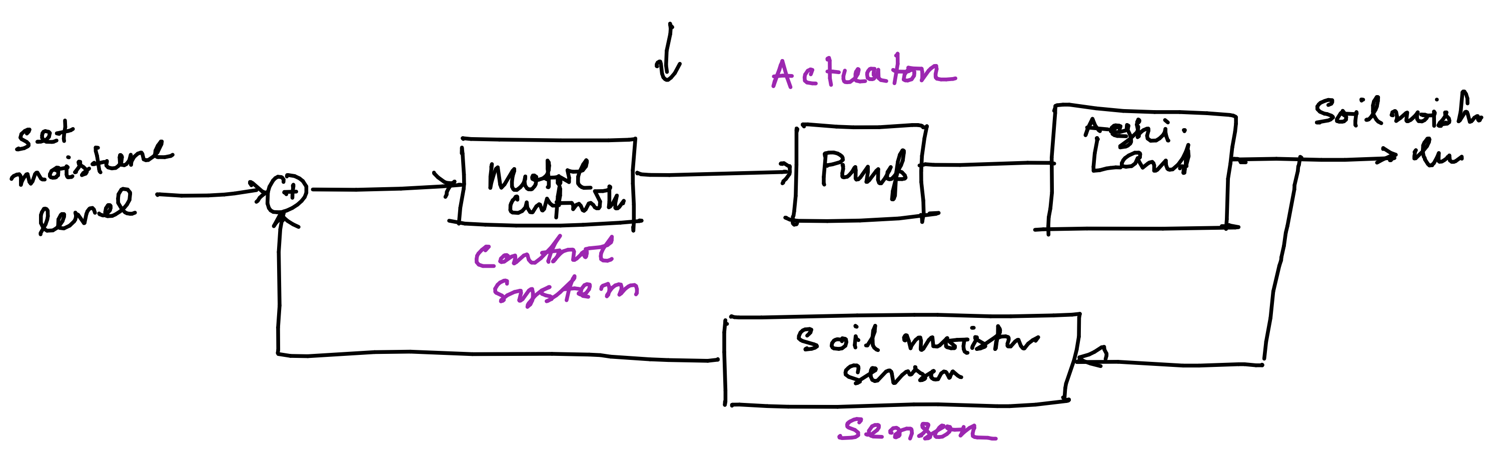
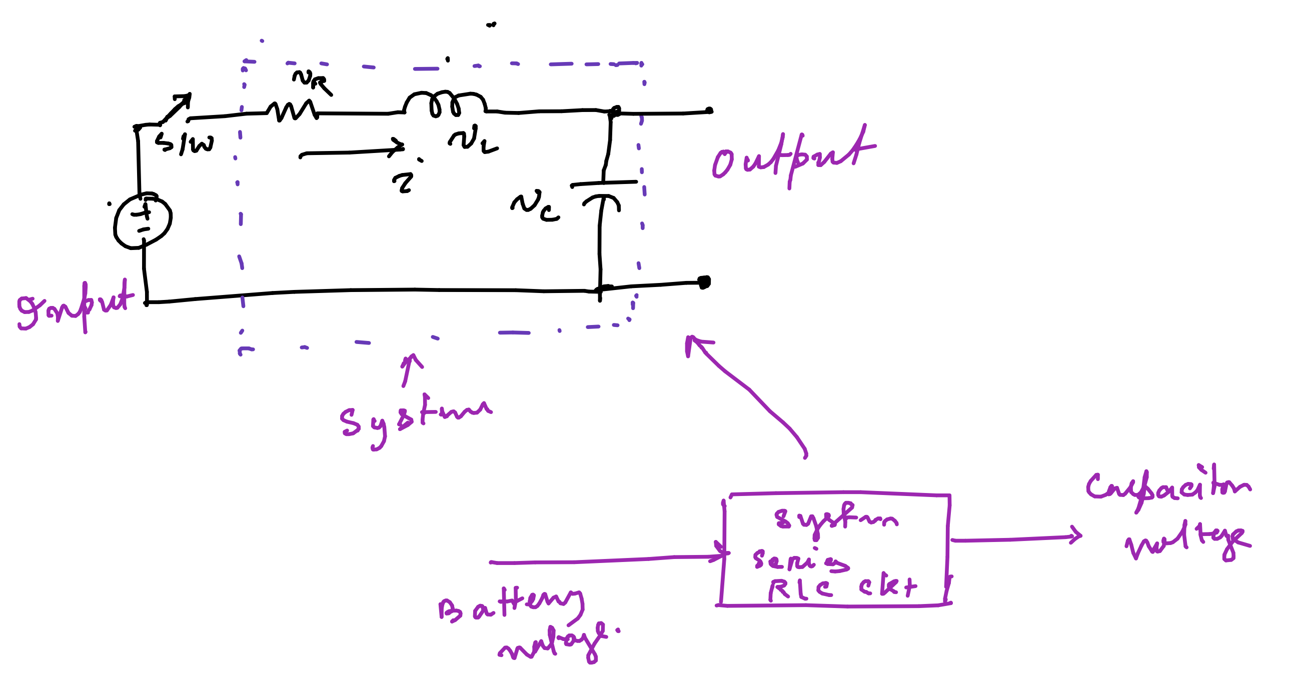


Block Diagram

- Run the pump for some time T.



Assume that at $t = 0$ we have closed the switch of the circuit.

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Then one can see that some of the parameters will evolve with respect to time

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- Current through the ckt
- Voltage across the capacitor
- voltage across the inductor
- voltage across the resistor.

$$\begin{cases} v_R = iR \\ v_L = L\dfrac{di}{dt} \\ \dfrac{dv_c}{dt} = \dfrac{1}{c} i \\ V = v_R + v_L + v_c \end{cases}$$

Differential equations + Algebraic equations

$$\left\{\begin{aligned} \frac{di}{dt} &= \frac{1}{L} v_L \\ \frac{dv_c}{dt} &= \frac{1}{c} i \\ 0 &= v_R - iR \\ 0 &= v_R + v_L + v_L - \underset{\text{input}}{V} \end{aligned}\right\}$$

output variable

Mathematical model of a system.

→ A mathematical model of a system is a set of mathematical equations (differential and/or algebraic) which bring relations between $\underbrace{\text{input variable}}_{V}$, $\underbrace{\text{output variables}}_{v_c}$ and some $\underbrace{\text{internal variables}}_{i}$.

- A mathematical model describes the dynamic behaviour of a physical system.

→ After eliminating the algebraic equations:

$$\begin{cases} \dfrac{di}{dt} = -\dfrac{R}{L} i - \dfrac{1}{L} v_c + \dfrac{1}{L} V \\ \dfrac{dv_c}{dt} = \dfrac{1}{c} i \end{cases}$$

$$\underbrace{\begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{dv_c}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -\dfrac{R}{L} & -\dfrac{1}{L} \\ \dfrac{1}{c} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i \\ v_c \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}}_{B} V$$

$$\underbrace{v_c}_{y} = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} i \\ v_c \end{bmatrix}}_{x}$$

State-space model of a system.

State : the parameters that evolve with respect to time

i & v_c

$$x = \begin{bmatrix} i \\ v_c \end{bmatrix}$$

input → u

output → y

State-space representation

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $x = \begin{bmatrix} 0.2 \\ 4 \end{bmatrix}$

$x = \begin{bmatrix} * \\ * \\ * \end{bmatrix}$ ← 3-D space

The state variable x belongs to the Euclidean space.