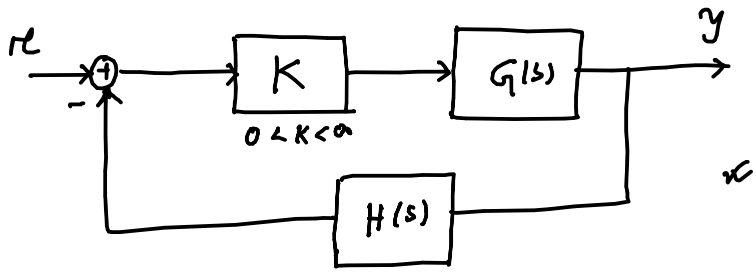
ELL 225

Lecture - 30



$$G(s) = \frac{n_g(s)}{d_g(s)}$$

$$H(s) = \frac{n_h(s)}{d_h(s)}$$

Closed loop T.F. $T(s) = \dfrac{K G(s)}{1 + K G(s) H(s)}$

$$= \frac{K\, n_g d_h}{d_g d_h + K\, n_g n_h}$$

Closed loop characteristic polynomial

$$\boxed{\sigma(s) = d_g(s) d_h(s) + K\, n_g(s)\, n_h(s)}$$

roots of $\sigma(s)$ are the closed loop poles

Root locus : Locus of roots of the polynomial $\sigma(s)$ with change in K within $0 \leq K < \infty$

→ Sketching of Root locus

the path traversed by a closed loop pole in $\mathbb{C}$.

① The number of branches in a root locus is equal to the number of closed loop poles.

② Since the polynomial $\sigma(s)$ has real-coefficients, the closed loop poles will appear with complex conjugate.

The root locus is symmetric about real axis.

- Starting & Ending points of root-locus:

$$\underline{\underline{0 \leq K < \infty}}$$

$$\sigma(s) = d_g(s)\, d_h(s) + K\, n_g(s)\, n_h(s)$$

For small value of K

$$\sigma(s) = d_g(s)\, d_h(s) + \epsilon \quad \leftarrow \text{small number.}$$

$$\approx d_g(s)\, d_h(s) \leftarrow \text{the } \underline{\text{poles of } G(s)H(s)}$$

For large value of K

$$\sigma(s) \approx K\, n_g(s)\, n_h(s)$$

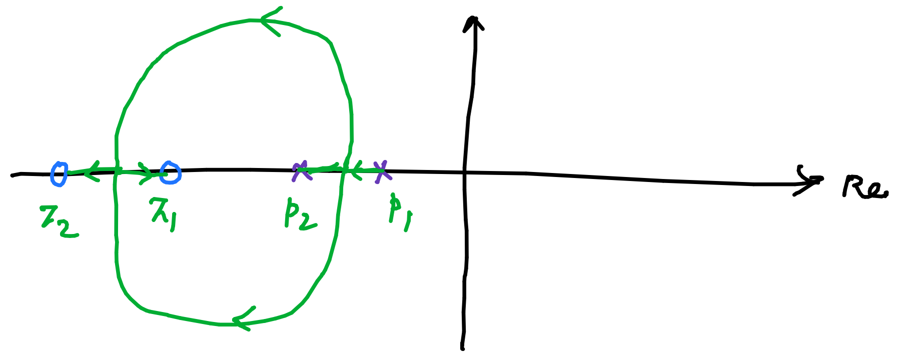
$\uparrow$

$\underline{\underline{\text{zeros of}}}$ $G(s)H(s)$

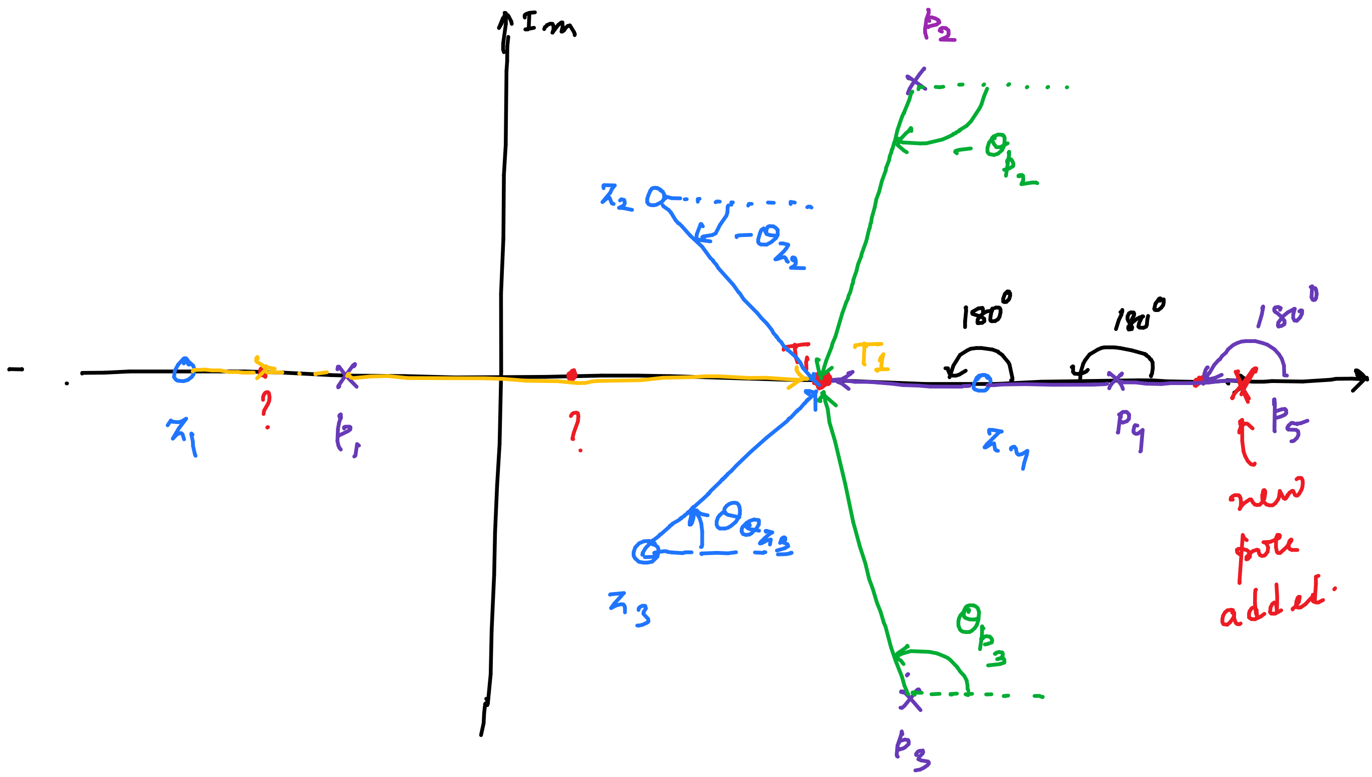
- The root locus begins at the finite & infinite poles of $G(s)H(s)$ & ends at the finite & infinite zeros of $G(s)H(s)$.

$$G(s)H(s) = \frac{(s+3)(s+4)}{(s+1)(s+2)} \leftarrow$$

It has finite number of poles & zeros.



→ Real axis segment



- A complex no. s is on the root locus

if
$$\begin{cases} |KG(s)H(s)| = 1 \\ \underbrace{\angle KG(s)H(s)}_{\theta} = (2l+1)180^\circ \qquad l = 0, \pm 1, \pm 2 \cdots \end{cases}$$

$$\theta = \theta_{z_1} + \theta_{z_2} + \theta_{z_3} + \theta_{z_4} - \theta_{p_1} - \theta_{p_2} - \theta_{p_3} - \theta_{p_4}$$

→ θ_{p_2} & θ_{p_3} are equal angle but opposite sign. So the net angular contribution to form θ due to θ_{p_2} & θ_{p_3} = 0°

→ θ_{z_2} & θ_{z_3} are equal angles but they have opposite sign. So net angular contribution to form θ due to θ_{z_2} & θ_{z_3} = 0°

- At each point on the real axis of $\mathbb{C}$, the angular contribution for θ due to a pair of complex conjugate poles or zeros is 0°.

→ The angular contribution at T_1 due to p_1 & z_1 is 0°.

- The angular contribution of poles & zeros of $G(s)H(s)$, which are on real axis and <u>left</u> to the test point, is 0°.

- The only contribution to form θ at test point T_1 is due to the poles & zeros of $G(s)H(s)$, which are on the real axis of $\mathbb{C}$ and to the <u>right</u> of test point T.

→ The angular contribution at T_1 is thus only due to z_4 & p_4

$$\theta = \theta_{z_4} - \theta_{p_4} = 180^\circ - 180^\circ = 0^\circ$$

$$\Downarrow$$

T_1 is not on the root locus

(we have <u>even</u> number of poles & zeros to the right of T_1)

By adding one extra pole to the right of T_1

(We have now odd number of poles & zeros

to the right of T_1)

$\Downarrow$

$$\theta = \theta_{z_4} - \theta_{p_4} - \theta_{p_5}$$

$$= 180^\circ - 180^\circ - 180^\circ = -180^\circ$$

$\Downarrow$

T_1 is on the root locus.

→ On the real axis, for $K > 0$ the root locus exists to the left of an <u>odd</u> number of real axis finite poles and/or zeros of $G(S)H(S)$.

→ <u>Behavior at infinity</u>

$$G(s) = \frac{n(s)}{d(s)}$$

For any values of s s.t $G(s) \to \infty$

then s is a pole of $G(s)$

For any value of s s.t $G(s) \to 0$,

the s is a zero of $G(s)$.

- A pole of $G(s)$ is at ∞ if

$$G(s) \to \infty \quad \text{as} \quad s \to \infty$$

$$\text{Ex :} \quad G(s) = s$$

- A zero of $G(s)$ is at ∞ if

$$G(s) \to 0 \quad \text{as} \quad s \to \infty$$

$$\text{Ex.} \quad G(s) = \frac{1}{s}$$

$$G(s) = \frac{(s+3)}{(s+1)(s+4)(s+5)}$$

Finite poles : $-1, -4, -5$

Finite zeros : -3

$$G(s) \cong \frac{1}{s \cdot s} \quad \text{For} \quad s \to \infty$$

It has 2 <u>zeros</u> at ∞.

It has no <u>poles</u> at ∞

- For every transfer funct $G(s)$, there are <u>equal</u> number of poles & zeros if we include the poles & zero at finite as well as infinite position.

$$G(s) = \frac{(s+2)(s+3)}{(s+2)}$$

It has No zeros at ∞

It has One pole at ∞

By changing K, the real roots of a quadratic polynomial, beams complex conjugate

$$K\,G(s) = \frac{K}{(s+1)(s+4)}$$

$$\downarrow$$

Closed loop ch. polynomial

$$\alpha(s) = s^2 + 5s + 4 + K$$

$s = -1, -4$ for $K = 0$

$s = -3.62, -1.38$ $k = 1$

$s = -3, -2$ $k = 2$

$s = -2.5 \pm j0.86$ $K = 3$

$s = -2.5 \pm j2.78$ $k = 10$