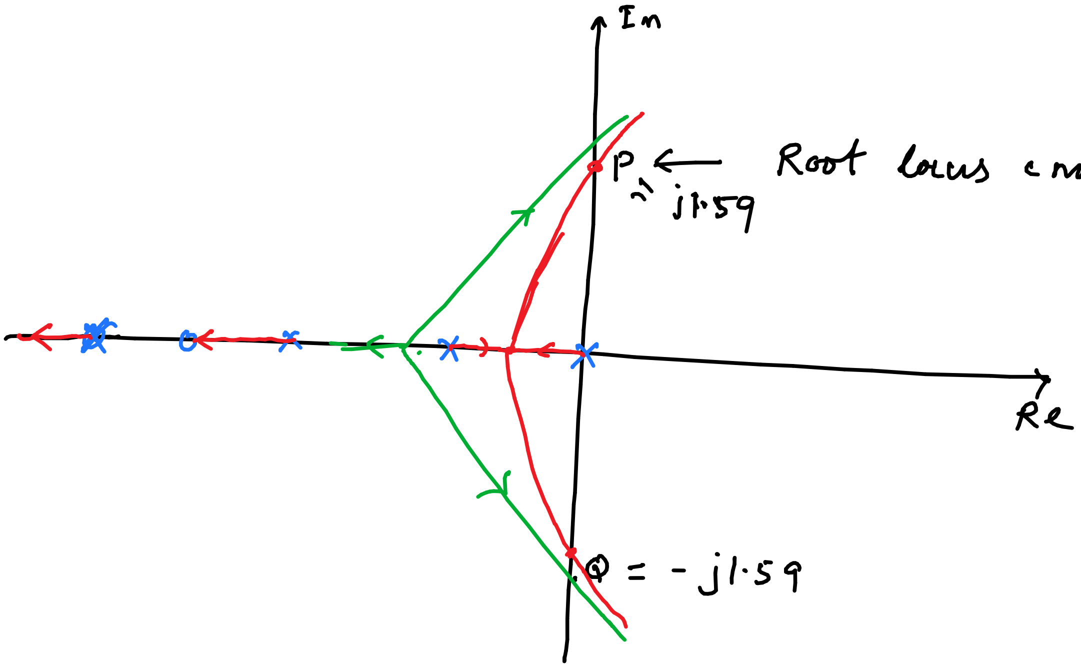
ELL225

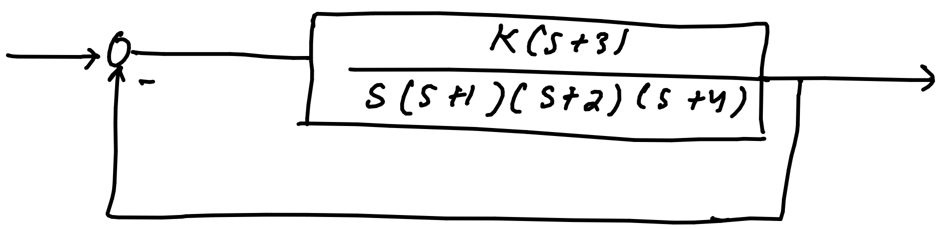
Lecture-32

$$G(s)H(s) = \frac{(s+3)}{s(s+1)(s+2)(s+4)}$$



→ Imaginary axis crossing of Root locus :

How to compute P & Q point ?



Closed loop T.F. : $\dfrac{K(s+3)}{s^4 + 7s^3 + 14s^2 + (8+K)s + 3K}$

obtain Routh table.

→ In Routh-Hurwitz criterion

- If there is a zero row, then there is an even polynomial which is factor of original polynomial.
- The roots of even polynomial are symmetric to the origin.

s^4	1	14	$3K$
s^3	7	$8+K$	0
s^2	$90-K$	$21K$	0
s^1	$\dfrac{-K^2-65K+720}{90-K}$	0	0 ← Enforce this row to zero to get an even polynomial
s^0	$21K$		

$$-K^2-65K+720=0$$

$0 \leq K < \infty$

$$\boxed{\Rightarrow K = 9.65}$$

Use for constructing the even polynomial

$$(90-K)s^2 + 21K = 80.35s^2 + 202.7$$

To find the points P & Q

compute the roots of the computed even polynomial.

$$80.35 s^2 + 202.7 = 0$$

$$\Rightarrow \quad s = \pm j\,1.59$$

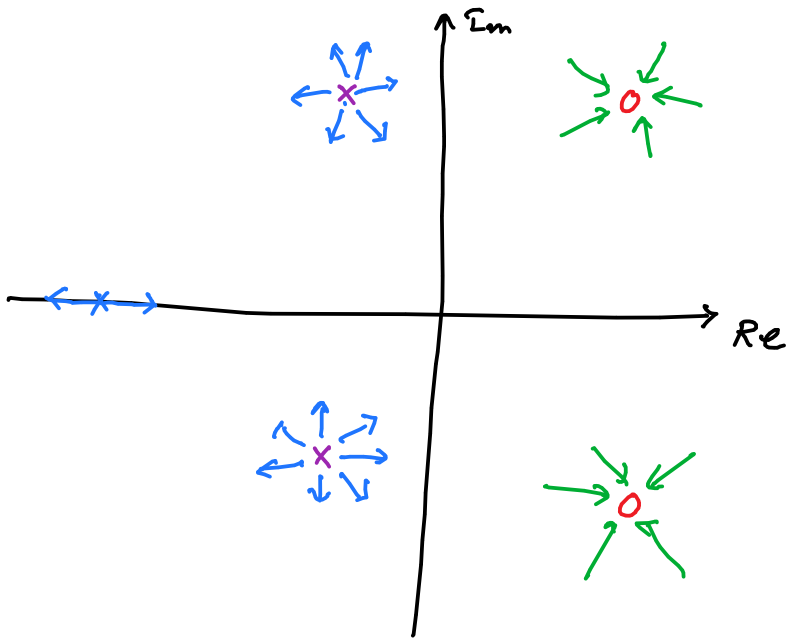
$P = j1.59$

$Q = -j1.59$

- For the $0 \le K < 9.65$, all the branches of root-locus are in open left half of complex plane i.e. the closed loop poles are in open left half of $\mathbb{C}$ $\Rightarrow$ the closed loop system is (BIBO) stable for the above range of K

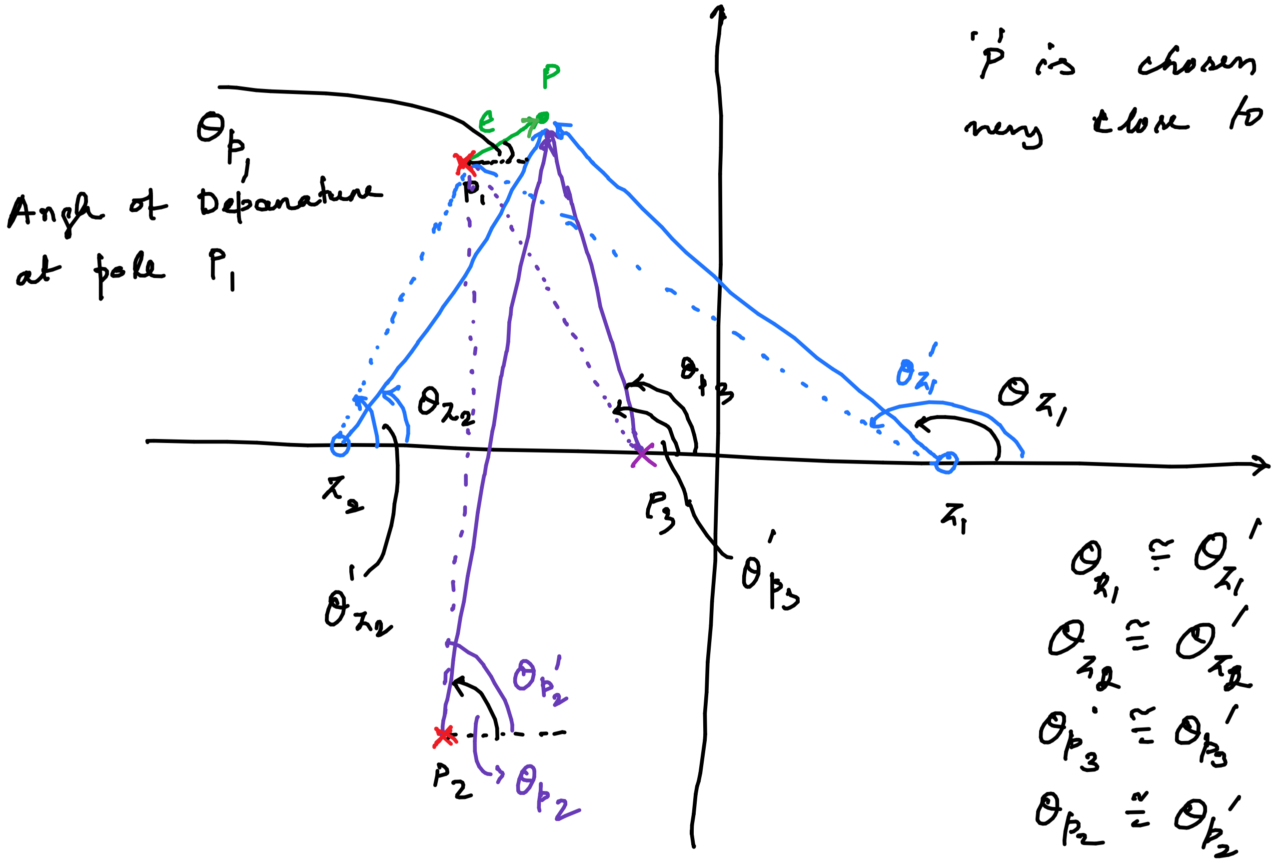
- $K \geq 9.65 \leftarrow$ Not BIBO stable.

→ Angles of Arrival & Departure :



Angle of Departure : The angle at which the root locus depart from a pair of complex conjugate poles.

Angle of Arrival : The angle at which the root locus arrive at a pair of complex conjugate zeros



'P' is chosen very close to P_1

$\theta_{z_1} \simeq \theta'_{z_1}$

$\theta_{z_2} \simeq \theta'_{z_2}$

$\theta_{p_3} \simeq \theta'_{p_3}$

$\theta_{p_2} \simeq \theta'_{p_2}$

At point P the total angle due to the poles & zeros

$$\theta_p = \theta_{z_1} + \theta_{z_2} - \theta_{p_1} - \theta_{p_2} - \theta_{p_3} \leftarrow \text{At point } P$$

$$\simeq \theta'_{z_1} + \theta'_{z_2} - \theta_{p_1} - \theta'_{p_2} - \theta'_{p_3}$$

Since all θ'_{z_i} & θ'_{p_2} θ'_{p_3} are known

$$\theta_p = \theta_{z_1} + \theta_{z_2} - \theta_{p_1} - \theta_{p_2} - \theta_{p_3}$$

$$\cong \underbrace{\theta_{z_1}' + \theta_{z_2}' - \overset{?}{\theta_{p_1}} - \theta_{p_2}' - \theta_{p_3}'}$$

known quantities, since P_1 location is exactly known.

Since point P is on the root locus.

$\Rightarrow$ at point $P \rightarrow \theta_p = (2k+1)180^0$

$$\Rightarrow \quad \theta_{p_1} = \theta_{z_1}' + \theta_{z_2}' - \theta_{p_2}' - \theta_{p_3}' - (2k+1)180^0$$

Angle of departure at pole P_1.

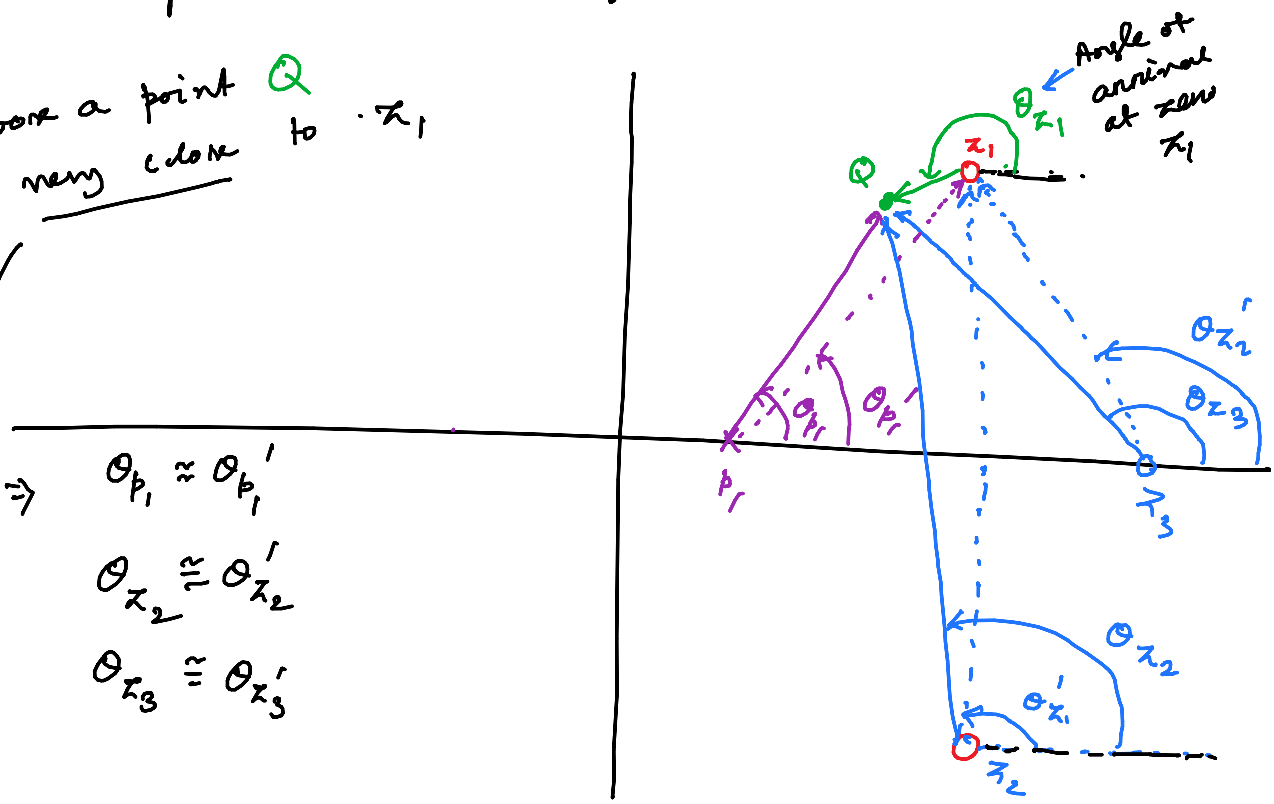
$\rightarrow$ Similar procedure can be followed to compute the angle of arrival.

Choose a point Q very close to z_1

$\Rightarrow \theta_{p_1} \approx \theta_{p_1}'$

$\theta_{z_2} \cong \theta_{z_2}'$

$\theta_{z_3} \cong \theta_{z_3}'$



$$\theta_z = \theta_{z_1} + \theta_{z_2} + \theta_{z_3} - \theta_{p_1}$$

$$\approx \underset{?}{\theta_{z_1}} + \underbrace{\theta'_{z_2} + \theta'_{z_3} - \theta'_{p_1}}_{\text{known}}$$

Since the point Q is on the root locus we have the following angle condition at Q :

- Total angle at point Q due to all poles & zeros is $(2k+1)\,180^\circ$.

$$\Downarrow$$

$$\theta_z = (2k+1)\,180^\circ$$

$$\Downarrow$$

$$(2k+1)180^\circ \simeq \theta_{z_1} + \theta'_{z_2} + \theta'_{z_3} - \theta'_{p_1}$$

$$\Rightarrow \theta_{z_1} = (2k+1)180^\circ - \theta'_{z_2} - \theta'_{z_3} + \theta'_{p_1}$$

Angle of arrival at zero z_1.