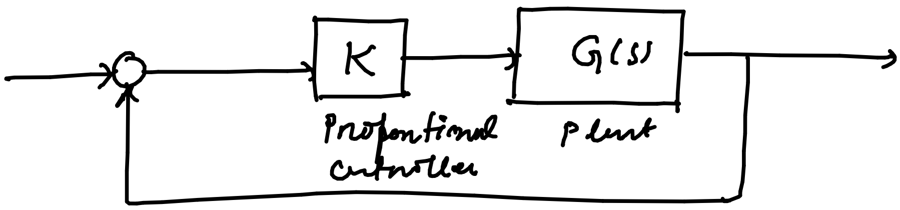
EL 225

Lecture - 33

- Controller / Compensator Design:



→ Performance Specifications (Closed loop)

- Stable system
- Elimination of Steady State Error (SSE)
- Transient performances → Damping ratio, Settling time

→ Compensator Design for SSE elimination:

Plant $G(s) = \dfrac{K}{(s+1)(s+2)(s+10)}$ ✓

Give step input to this system.

Steady state error:

$$e(\infty) = \frac{1}{1 + K_p}$$

$$K_p = \lim_{s \to 0} G(s)$$

Assume $K = 164.6$ $\qquad K_p = \frac{164.6}{20}$

$$e(\infty) = \frac{1}{1 + K_p} \neq 0$$

- S.S.E can not be eliminated by using only proportional controller.

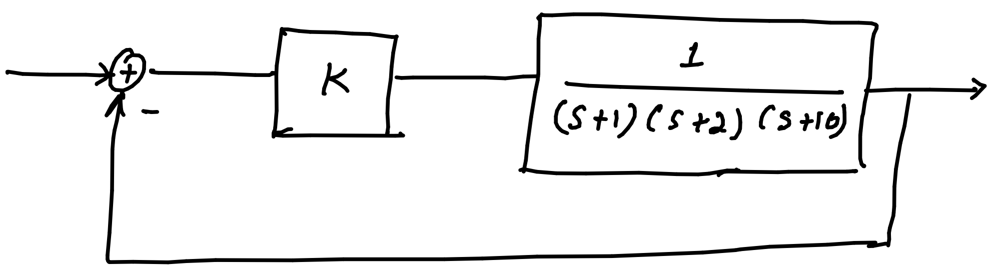
$$G(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

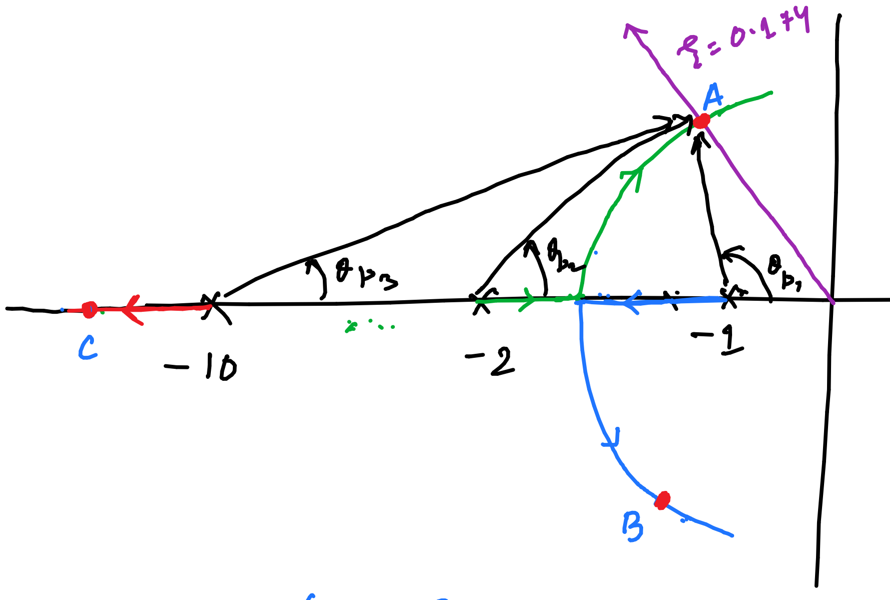
$\downarrow$ modify

$$G(s) = \frac{K(s+z_1)\cdots(s+z_m)}{\underline{\underline{s^N}}(s+p_1)\cdots(s+p_n)}$$

$N \geq 1$ to eliminate SSE for <u>step input</u>.

$\rightarrow$ Root locus of



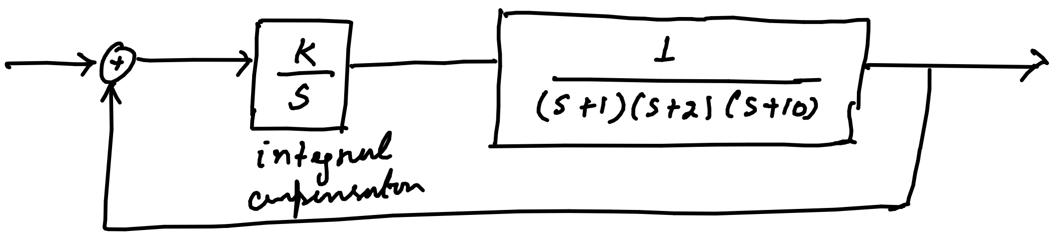


$$-\theta_{p_1} - \theta_{p_2} - \theta_{p_3} = (2k+1)\,180$$

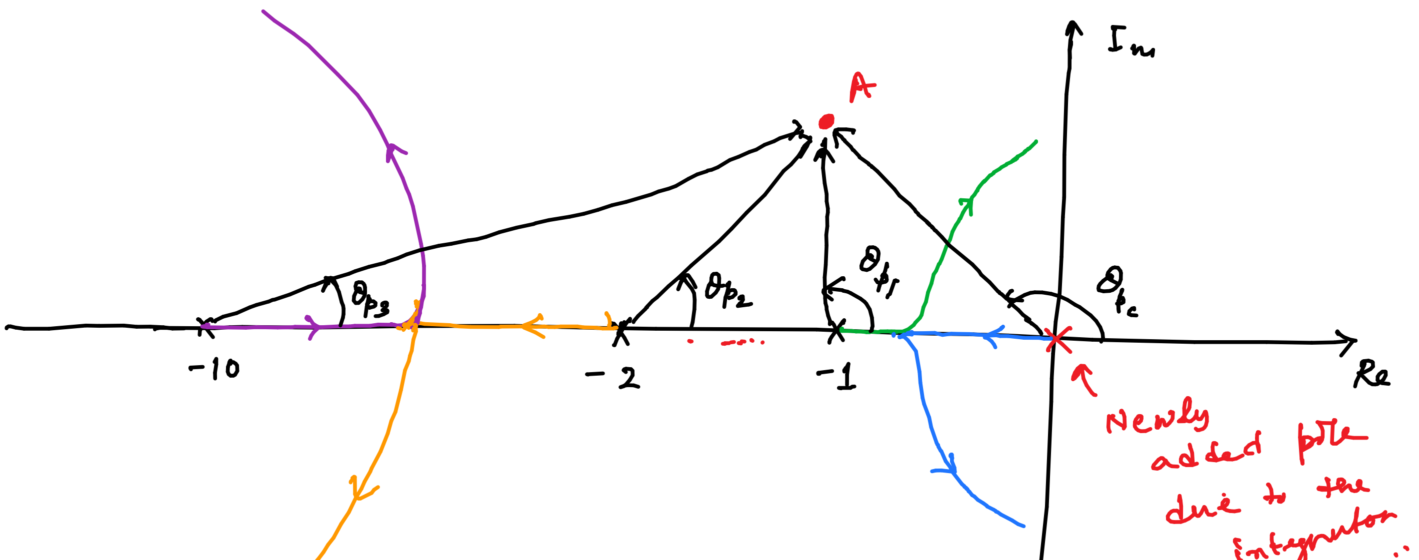
closed loop poles ● are at A, B & C for $K = 164.6$

We need $\xi = 0.174$

→ We need at least one integrator to eliminate SSE.



The closed loop is modified to make SSE to 0.



$$-\theta_{p_1} - \theta_{p_2} - \theta_{p_3} - \theta_{p_c} \neq (2k+1)\,180^\circ$$

* The root locus is now not passing through point A.

⇒ the transient specification i.e $\xi = 0.174$ will not meet.

→ By adding one pole at origin (i.e. introducing an integrator compensator), the transient performance of the closed loop systems has changed drastically i.e. $\zeta = 0.174$ will not meet ∴

↓

We do not want to disturb the transient performance specification.

|||

We need our root-locus to remain as it was originally (i.e. the root-locus pass through point A).

↓

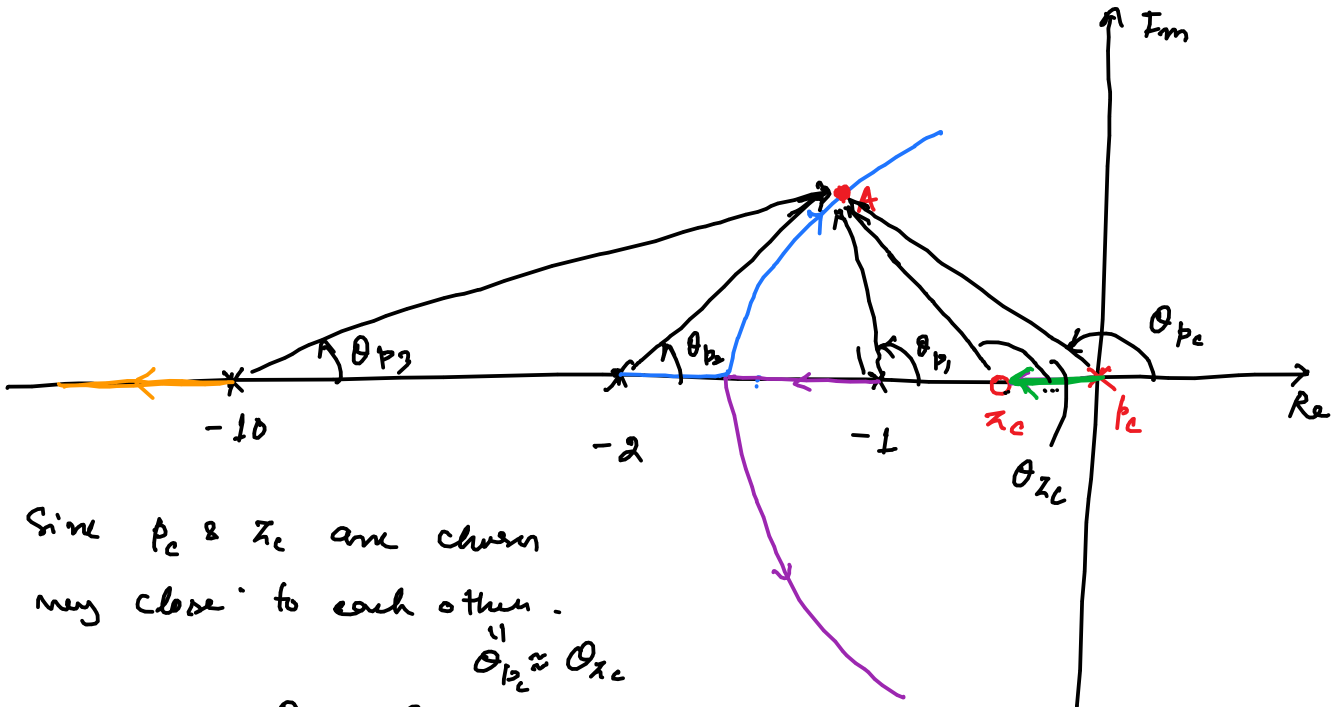
To get the root-locus in its original form

→ introduce a zero very close to the pole at origin.
z_c p_c

→ Modify the compensator as follows

$$\frac{K(s+z_c)}{s}$$

z_c is chosen very close to $p_c = 0$.



Since p_c & z_c are chosen very close to each other.

$$\underbrace{-\theta_{p_1} - \theta_{p_2} - \theta_{p_3}}_{=(2k+1)180^0} + \underbrace{\theta_{z_c} - \theta_{p_c}}_{\approx 0}$$

$\theta_{p_c} \approx \theta_{z_c}$

The vector lengths due to p_c & z_c at point A is almost equal & hence they will cancel.

By introducing a pole at origin & a zero, very close to pole at origin

⇓

- the angular contribution $\theta_{p_c} \approx \theta_{z_c}$

So the original angle will be preserved at point A

- The magnitude will also remain same,

since $r_{p_c} \approx r_{z_c}$.

$\Downarrow$

We can now preserve the original root locus.

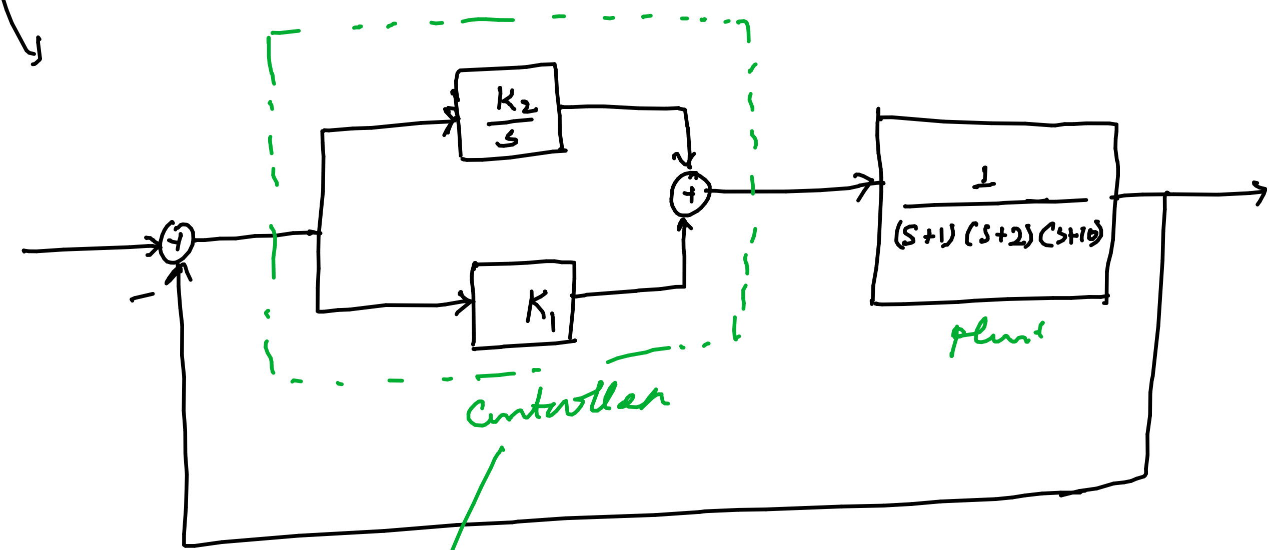
$\Rightarrow$ the transient behavior will remain same.

$\rightarrow$ The compensator is of the following form

$$C(s) = K_1 + \frac{K_2}{s}$$

$$= \frac{K_1 (s + K_2/K_1)}{s}$$

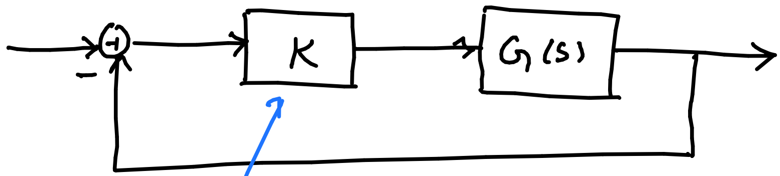
Choose K_2/K_1 s.th. it is very close to origin (in left side of pole at origin)



Proportional-plus-integral controller (PI-controller)

→ We now use a different compensator to reduce the S.S.E.

The uncompensated system:



Proportional controller.

Let $G(s) = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$, $n \geq m$

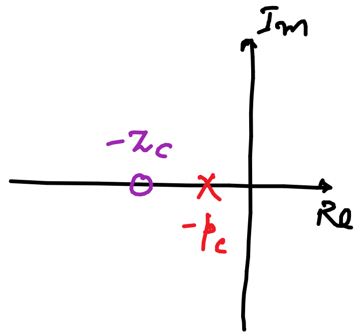
Position error const.

$$K_{p_0} = \lim_{s \to 0} KG(s) = \frac{K \cdot z_1 z_2 \cdots z_m}{p_1 p_2 \cdots p_n}$$

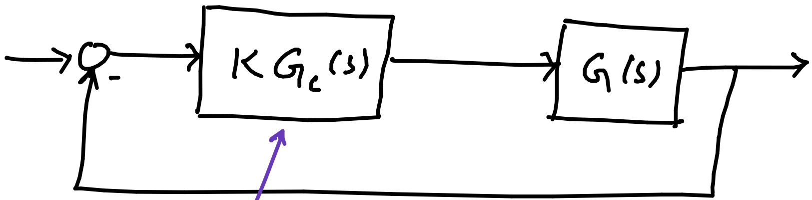
Uncompensated / old position error const.

→ We use following new compensator/controller

$$\boxed{G_c(s) = \frac{s+z_c}{s+p_c}}$$



↓ closed loop



New controller

- New position error const:

$$K_{p_n} = \lim_{s \to 0} K G_c(s) G(s) = \left[\frac{K z_1 z_2 \cdots z_m}{p_1 p_2 \cdots p_n}\right]\left[\frac{z_c}{p_c}\right]$$

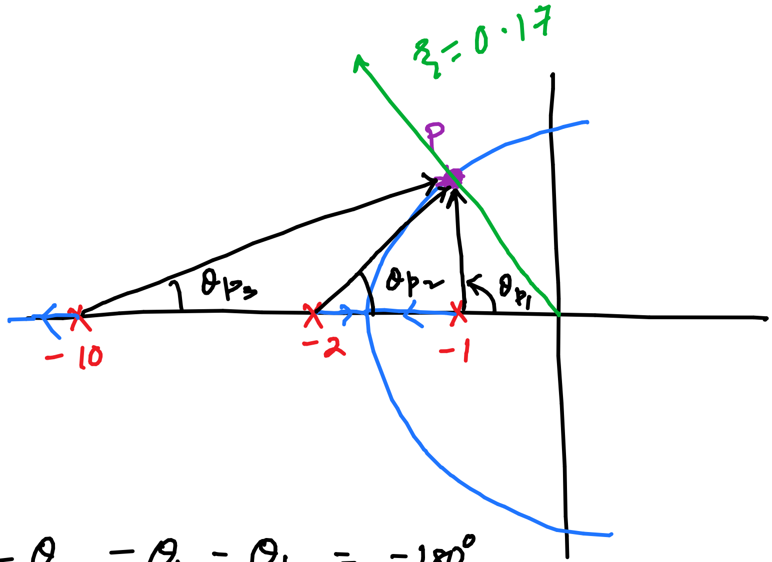
$$= K_{p_0}\left(\frac{z_c}{p_c}\right)$$

$$\boxed{\Rightarrow \quad K_{p_n} = K_{p_0}\left(\frac{z_c}{p_c}\right)}$$

$$e(\infty) = \frac{1}{1+K_p}$$

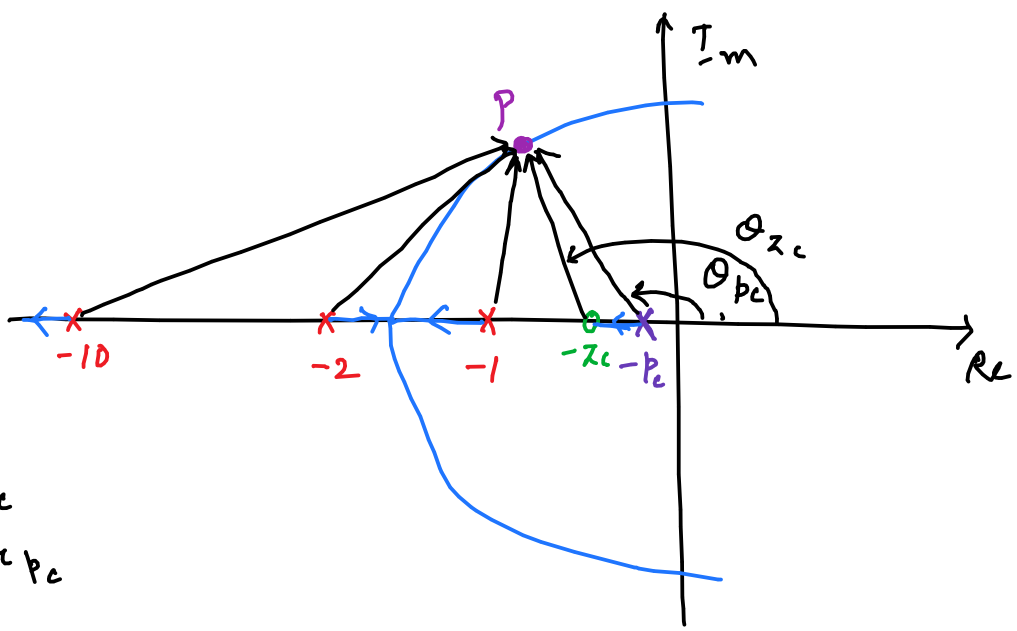
$\downarrow$

K_{p_n} or K_{p_0}



Uncompensated closed loop

$$-\theta_{p_1} - \theta_{p_2} - \theta_{p_3} = -180^\circ$$



$\theta_{p_c} \approx \theta_{z_c}$

$r_{z_c} \approx r_{p_c}$

Compensated closed loop

- To reduce the S.S.E, we need K_{p_n} to be larger than K_{p_0}.

- We can chose z_c & p_c very close to each other (so that the root locus of original system will not change much) (so the transient performance specification will meet)

Let say $z_c = -4.1$
$p_c = -4$

$$\frac{z_c}{p_c} = 0.1$$

$$K_{p_n} = K_{p_0} \times 0.1$$

$\Rightarrow$ S-S-E will not improve.

What choice we must make for z_c & p_c s.t. there will be significant improvement in K_{p_n}.

Let us choose

$$z_c = -0.01$$

$$p_c = -0.001$$

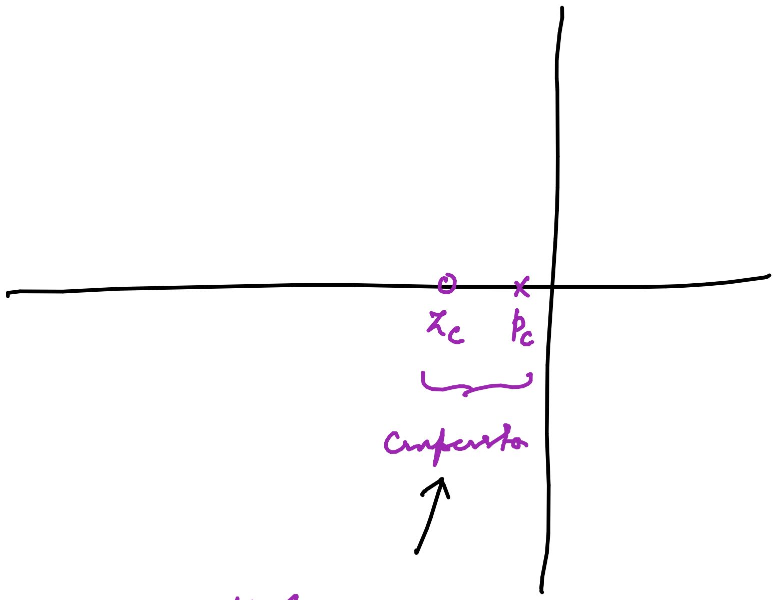
$$\frac{z_c}{p_c} = 10 \quad \Rightarrow \quad K_{p_n} = 10 K_{p_o} \leftarrow$$

there is 10 time improvement in K_{p_n} & hence the S.S.E will also be improved 10 times.

$$z_c = -0.01$$

$$p_c = -0.001$$

$$\frac{z_c}{p_c} = 100 \quad \leftarrow \text{100 times improvement.}$$



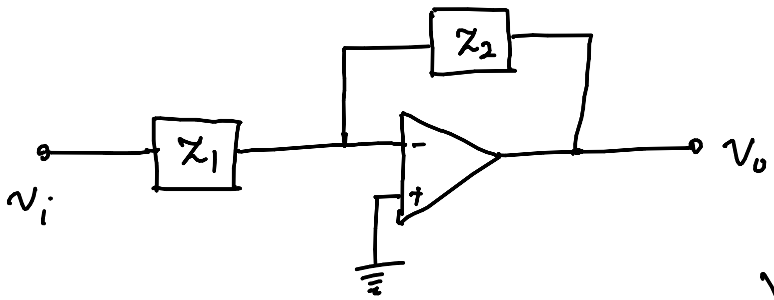
$$C(s) = \frac{K(s+z_c)}{(s+p_c)} \leftarrow \underline{\underline{\text{Lag compensator}}}$$

$\downarrow$

S.S.E can be <u>improved</u> but <u>can not be</u> completely eliminated.

- Multiple lag compensator exists for a given error improvement specification.

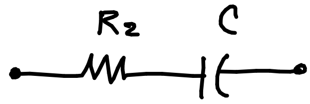
→ Circuit Realization of PI Controller



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

$Z_1 \rightarrow$

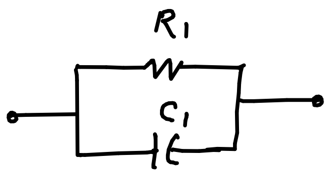
$Z_2 \rightarrow$



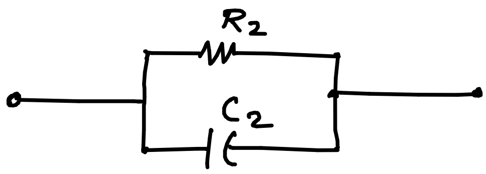
$$C(s) = \frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1}\frac{\left(s + \frac{1}{R_2 C}\right)}{s}$$

→ Lag Controller

$Z_1 \rightarrow$

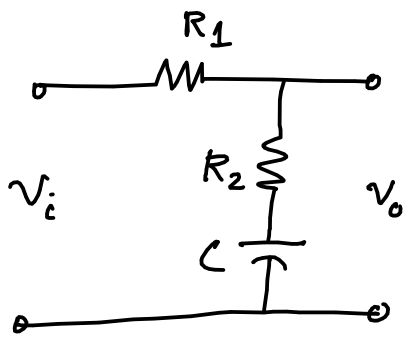


$Z_2 \rightarrow$



$$C(s) = \frac{V_o(s)}{V_i(s)} = -\frac{C_1}{C_2}\frac{\left(s + \frac{1}{R_1 C_1}\right)}{\left(s + \frac{1}{R_2 C_2}\right)} \qquad R_2 C_2 > R_1 C_1$$

→ One can also use passive network to realize Lag Controller:



$$G_c(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1+R_2}\,\frac{\left(s+\frac{1}{R_2 C}\right)}{\left(s+\frac{1}{(R_1+R_2)C}\right)}$$