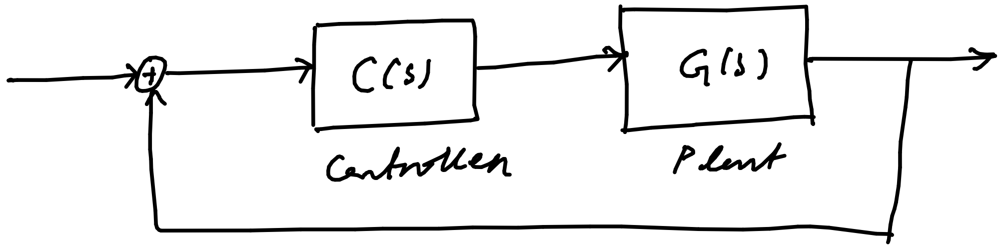
ELL 225

Lecture - 34



- Eliminate S.S.E (Use PI controller)

$$C(s) = K_p + \frac{K_i}{s}$$

- Reduce the S.S.E. (Use Lag compensator)

$$C(s) = \frac{K(s+z_c)}{(s+p_c)}$$

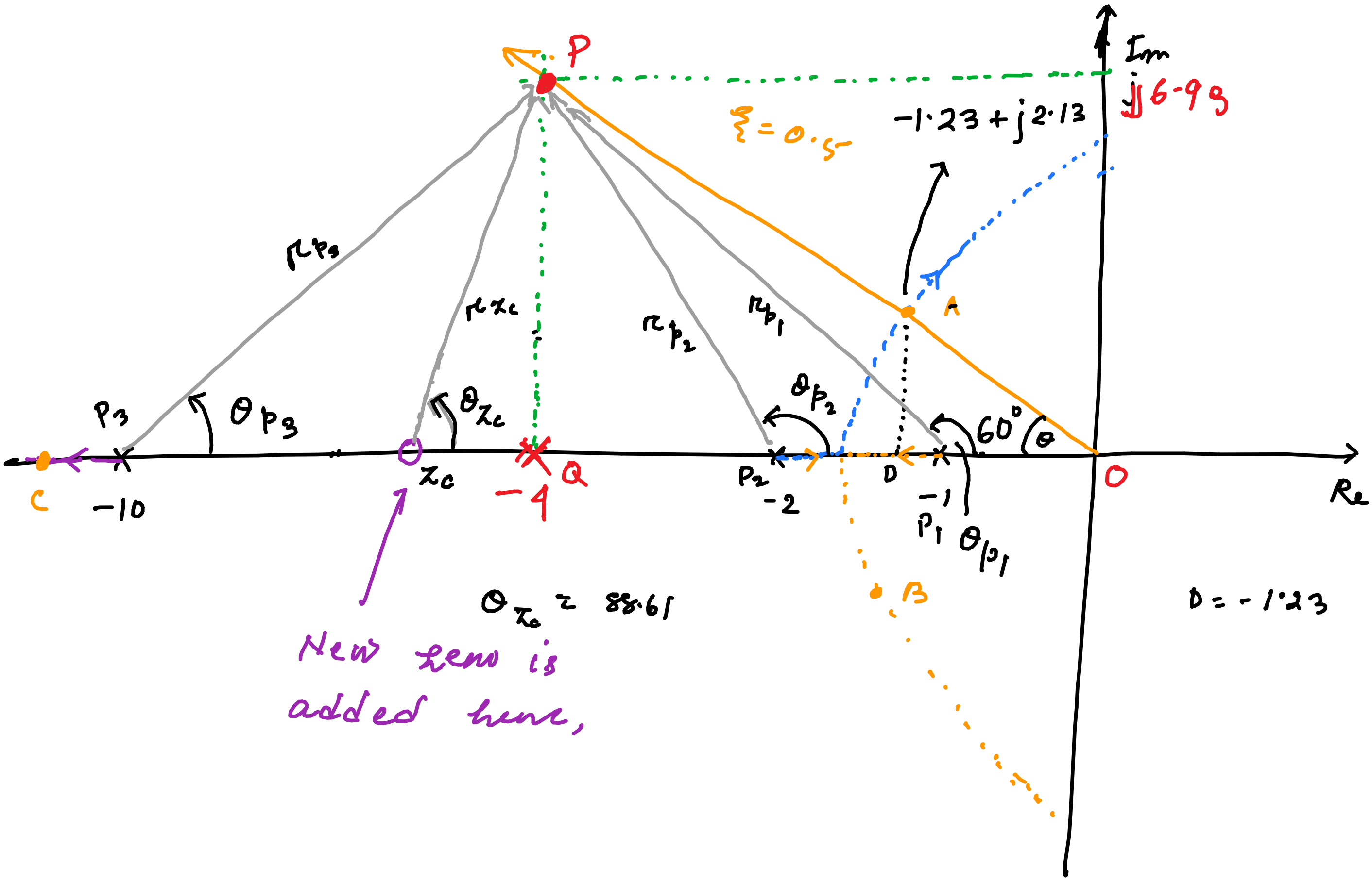
→ Achieving / Improving Transient behaviour :

Specifications : • Damping ratio
• Settling time

Ex : Plant $G(s) = \dfrac{1}{(s+1)(s+2)(s+10)}$

Design requirements:

- Damping ratio $\xi = 0.5$
- Settling time $T_s = 1$ sec.



Damping ratio 0.5 $\longrightarrow$ $\cos\theta = 0.5$

$\Rightarrow \theta = 60^\circ$

<u>Change gain K</u> s.t. the closed loop poles are at loc^n (A, B, C)

$\Downarrow$

The system response, which is dominated by a pair of complex conjugate poles (A, B), will satisfy the damping ratio $\xi = 0.5$.

- With respect to the new locations of closed loop poles (at A, B, C), the effect of pole at C is very small on the output response in comparison to the dominant complex conjugate pair at A, B.

At (A, B, C) pt.s of closed loop poles

the settling time :

$$T_s = \frac{4}{\xi \omega_n} \approx \frac{4}{\sigma_d} \approx \frac{4}{1.23} = 3.25 \text{ sec.}$$

- If we use <u>Only proportional controller</u>

then we have achieved appropriate $\xi = 0.5$

but Not achieved specified settling time T_s.

We need $T_s = 1$ sec.

$$\Rightarrow \quad T_s \simeq \frac{4}{\sigma_d} = 1$$

$$\Rightarrow \quad \sigma_d = 4 \quad \Downarrow$$

We need a pair of complex conjugate closed loop poles, whose real part

$$\sigma_d = -4$$

$OQ = 4$ $\quad$ $PQ = OQ \tan 60^0$

$$= 6.93$$

- To achieve $\xi = 0.5$ & $T_s = 1$ sec. we need our root locus to pass through the point P.

→ To make the root locus pass through pt. P we must ensure the angle criteria.

Since in the uncompensated system the Root locus does not pass the P

⇓

$$-\theta_{p_1} - \theta_{p_2} - \theta_{p_3} \neq (2k+1)180^0$$

$$\theta_{p_3} = \tan^{-1}\left(\frac{PQ}{QC}\right) = \tan^{-1}\left(\frac{6.93}{6}\right) = 49.11^0$$

$$\theta_{p_1} = 180^0 - \tan^{-1}\left(\frac{6.93}{3}\right) = 113.4^0$$

$$\theta_{p_2} = 180^0 - \tan^{-1}\left(\frac{6.93}{2}\right) = 106.1^0$$

• At point P

$$-\theta_{p_1} - \theta_{p_2} - \theta_{p_3} = -268.61 \neq (2k+1)180^0$$

Now let us introduce a zero at z_c (exact loc" will be computed)

At point P :

$$\theta = \theta_{z_c} - \theta_{p_1} - \theta_{p_2} - \theta_{p_3} = -180^0$$

$$\Rightarrow \theta_{z_c} = -180^0 + 268.61 = 88.61^0.$$

$$\frac{6.93}{z_c - 4} = \tan 88.61$$

$$\frac{PQ}{Qx_c} = \tan 88.61 \quad \Rightarrow \quad \frac{6.93}{x_c - 4} = \tan 88.61$$

$$\Rightarrow \quad x_c \cong 4.16$$

$$\Downarrow$$

The zero locatⁿ will be at -4.16

$$z_c = -4.16$$

We implement a controller as follow

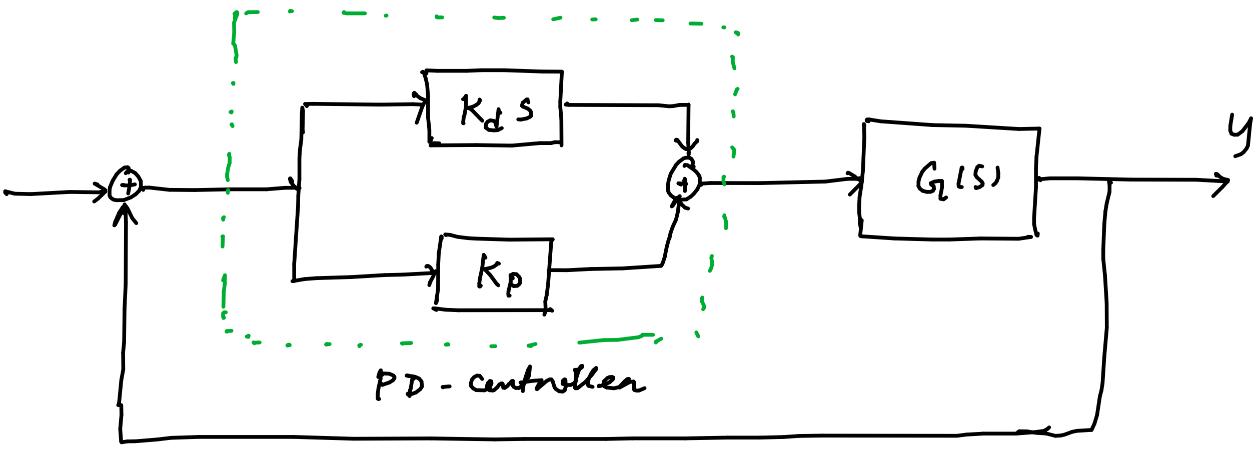
$$\boxed{\begin{aligned} C(s) &= K_p + K_d s \\ &= K_d \left(s + \frac{K_p}{K_d} \right) \end{aligned}}$$

$$\downarrow$$

$$K_d\,(s + z_c)$$

Proportional-plus-derivative controller

PD - controller



At point P, the gain should be 1.

$$K = \frac{r_{p_1} r_{p_2} r_{p_3}}{r_{z_c}} = \frac{7.2 \times 7.55 \times 9.16}{6.932}$$

we need to change gain K s.t.

$$\frac{K\, r_{zc}}{r_{p_1} r_{p_2} r_{p_3}} = 1$$

$$= 71.93 = K_d$$

$$z_c = -4.16$$

$$\frac{K_p}{K_d} = 4.16$$

$$\Rightarrow K_p = 4.16 \times K_d$$

Controller $C(s) = K_d (s + z_c)$

$$= 71.93\left(s + (4.16 \times 71.93)\right)$$

$$C(s) = \underbrace{71.93}_{K_d}\left(s + \frac{4.16 \times 71.93}{71.93}\right)$$

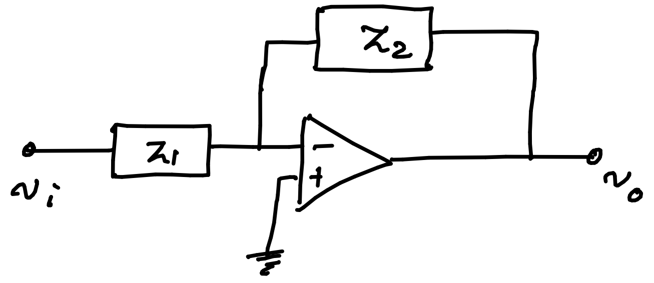
With this compensation, the root-locus will pass thru point P. & hence in the closed loop we will have

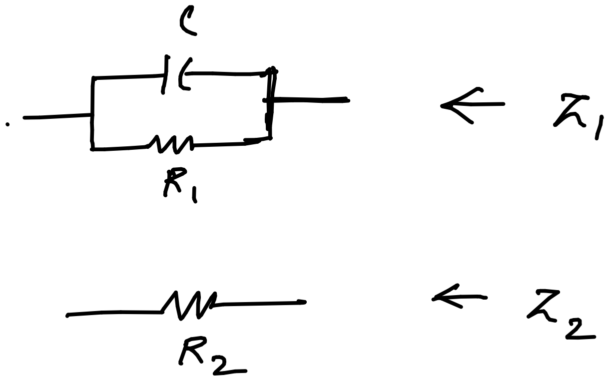
$$\begin{cases} \xi = 0.5 \\ T_s = 1 \text{ sec.} \end{cases}$$

* With the PD controller, we can (approximately) achieve the specified transient response, however, the steady state error may not go to zero.

Check the SSE for the considered example in "SISOtool" in MATLAB.

→ Circuit Realization for PD controller :





$$G_c(s) = -\frac{V_o(s)}{V_i(s)}$$

$$= -R_2 C \left(s + \frac{1}{R_1 C}\right)$$