ELL 225

Lecture - 35

- Design of a PD - compensator for improving the transient behaviour.
  - ↳ Damping ratio
  - ↳ settling time
  - ↳ Natural frequency

PD Compensator $C(s) = K_p + K_d s$

K_p & K_d are gains which needs to design appropriately

→ We use a compensator of the following form

$$C(s) = \frac{K(s+z_c)}{(s+p_c)}$$

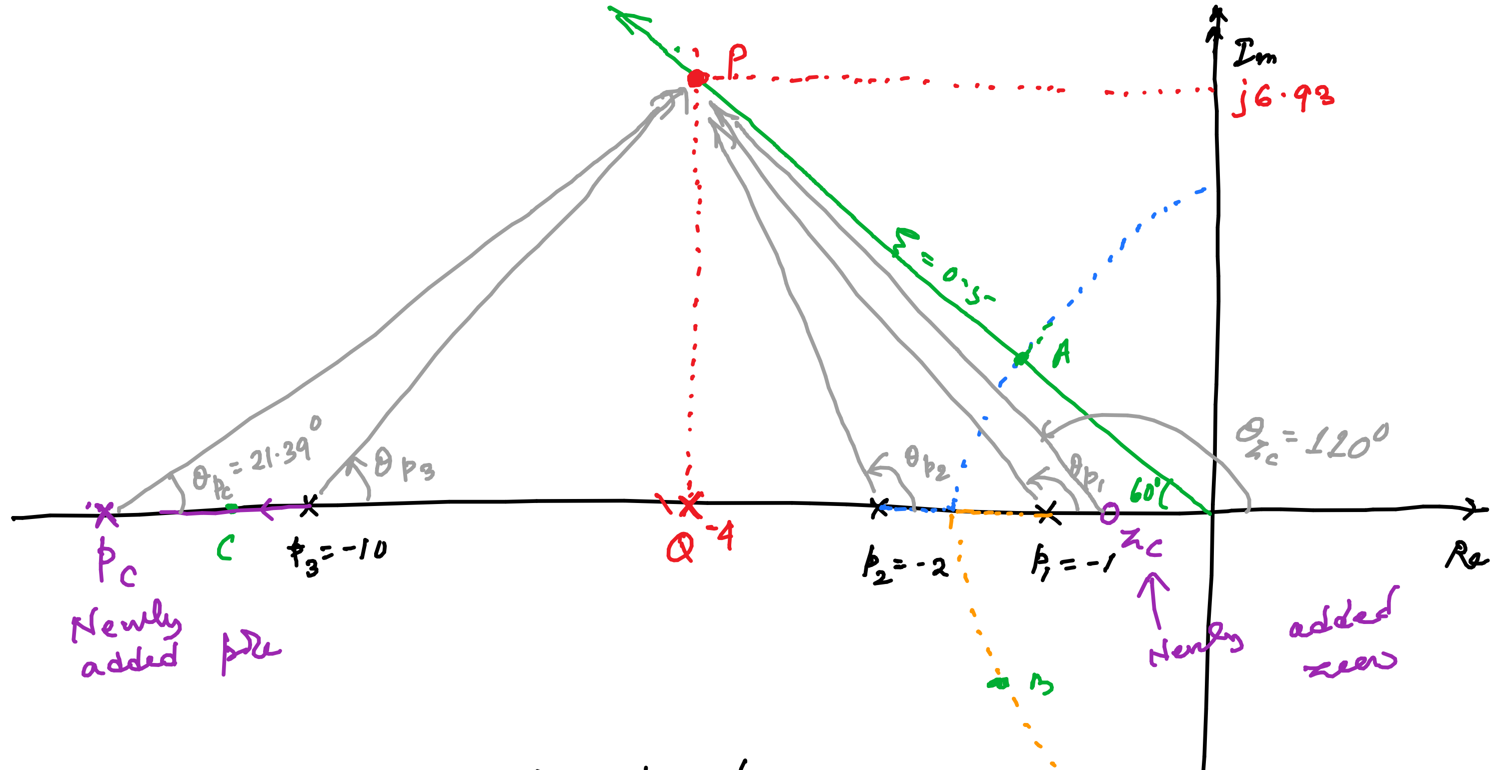
to achieve desired transient behaviour.

$$G(s) = \frac{1}{(s+1)(s+2)(s+10)}$$

Given desired specifications:

Damping ratio $\xi = 0.5$

Settling time $T_s = 1$ sec.



- Proceed similar to PD compensator design to obtain Point P

By change K we can only achieve damping ratio (A, B, C) closed loop poles.

To achieve desired setting time, the root-locus has to pass thru the point $P = -4 + j6.93$

→ Once we get the desired point P, we evaluate the angle at point P due to all poles & zeros.

At point P

$$\theta = -\theta_{p_1} - \theta_{p_2} - \theta_{p_3} = -268.61^\circ$$

To make the root locus pass thru the point P, we need total angular contribution due to all poles (including p_c) & zero (z_c) to be $(2k+1)180^\circ$.

Compensator $C(s) = \dfrac{K(s+z_c)}{(s+p_c)}$

$$\theta_c - 268.61^\circ = -180^\circ$$

$$\Rightarrow \theta_c = 88.61$$

$$\underbrace{\theta_{z_c} - \theta_{p_c}}_{\theta_c = 88.61} - \underbrace{\theta_{p_1} - \theta_{p_2} - \theta_{p_3}}_{-268.61} = -180^\circ \leftarrow$$

So we need $\theta_{z_c} - \theta_{p_c} = 88.61^\circ$.

So we have flexibility to choose

θ_{z_c} & θ_{p_c}.

$$\Downarrow$$

We have multiple compensators to satisfy the angle $\theta_{z_c} - \theta_{p_c} = 88.61$

Let us choose $\theta_{z_c} = 110^\circ$

$$\Rightarrow \theta_{p_c} = 110 - 88.61 = 21.39^\circ$$

Consider the right angle Δ

$$PQz_c \rightarrow \frac{PQ}{Qz_c} = \tan(180^\circ - 110^\circ)$$

$$\Rightarrow \frac{6.93}{4 - z_c} = \tan 70^\circ$$

$$\Rightarrow z_c = 1.47$$

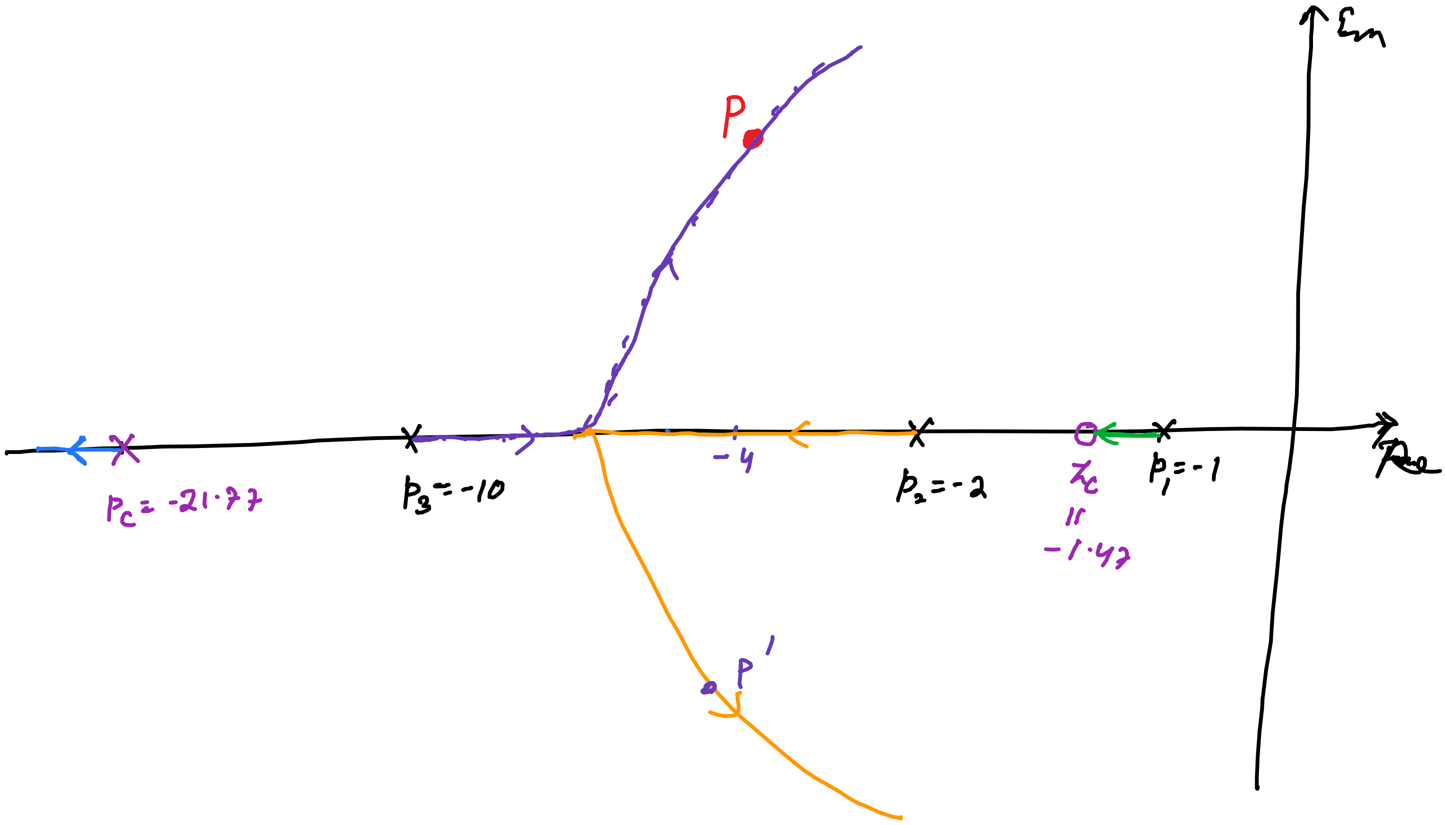
So the z_c is at -1.47

Consider the right only Δ

$$PQP_c \longrightarrow \frac{6\cdot93}{p_c - 4} = \tan 21\cdot39$$

$$\Rightarrow p_c = 21\cdot77$$

$\Rightarrow$ the pole p_c is at $-21\cdot77$.



- The zero z_c at -1.47 will app. cancel with pole at -1.

- The pole at $p_c = -21.77$ have very small contribution on the response.

- The closed loop response will approximately behave similar to second order system & the transient behavior is determined by the poles at P & P'.

$$C(s) = \frac{K(s + z_c)}{(s + p_c)}$$

- Using z_c & p_c we have achieved desired angle criteria $(2k+1)180^\circ$.

- We also have to ensure that the gain at point P due to all poles & zeros is 1 (including K)

The gain at point P

$$\boxed{\frac{K\, r_{z_c}}{r_{p_1} r_{p_2} r_{p_3} r_{p_c}} = 1} \quad \ldots \quad (*)$$

- Once the position of z_c & p_c are known, r_{p_c} & r_{z_c} can be computed.

$\Downarrow$

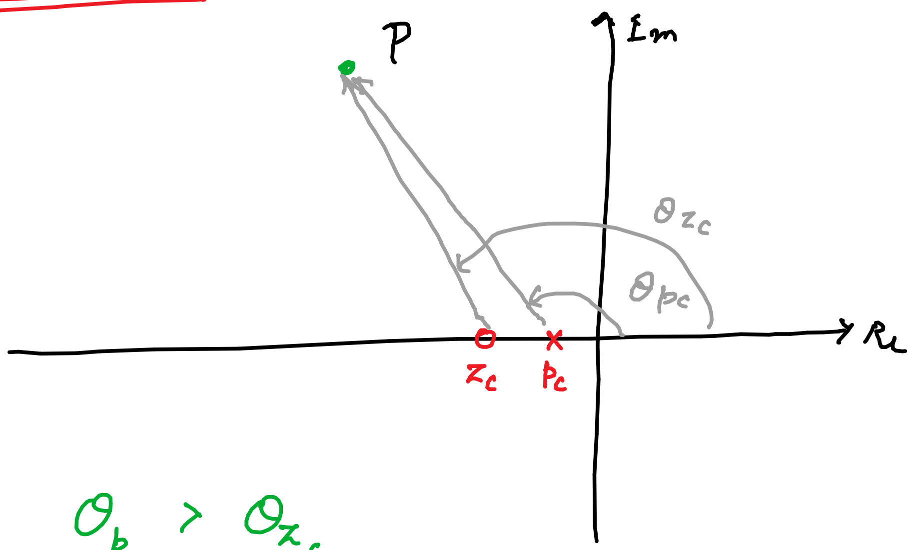
- Then we only need to adjust K s.t. the relation (*) holds.

The compensator $C(s) = \dfrac{K(s+z_c)}{(s+p_c)}$ that we have just designed is called

"**Lead Compensator**".

Lag Compensator

$$C_{Lag}(s) = \frac{K(s+z_c)}{(s+p_c)}$$

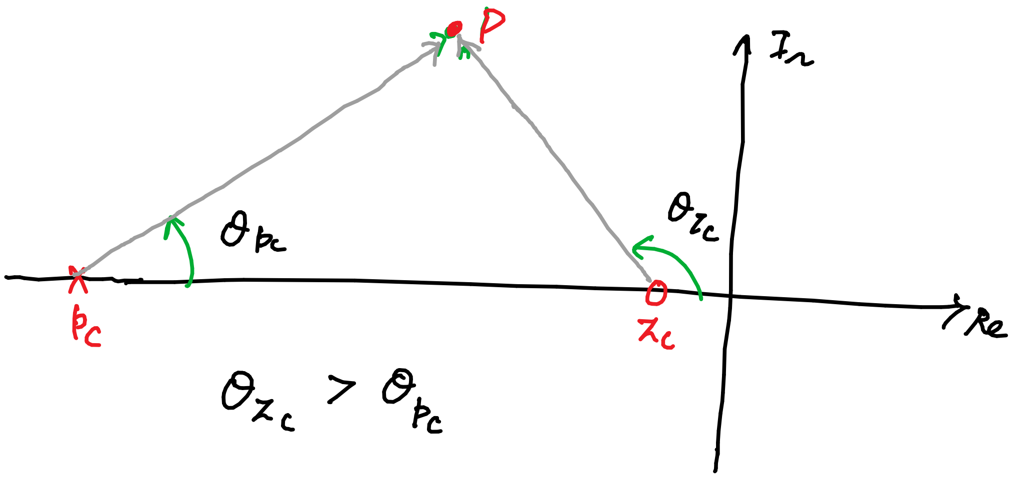


$$\theta_{p_c} > \theta_{z_c}$$

$$\theta_{c_{lag}} = \theta_{z_c} - \theta_{p_c} = -ve$$

- ## Lead Compensator

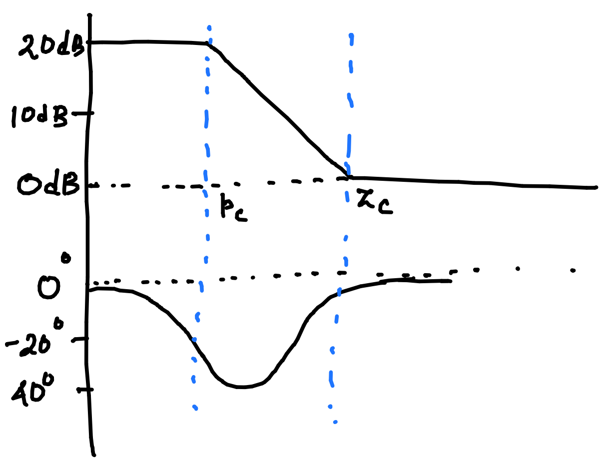
$$C_{lead}(s) = \frac{K(s+z_c)}{(s+p_c)}$$



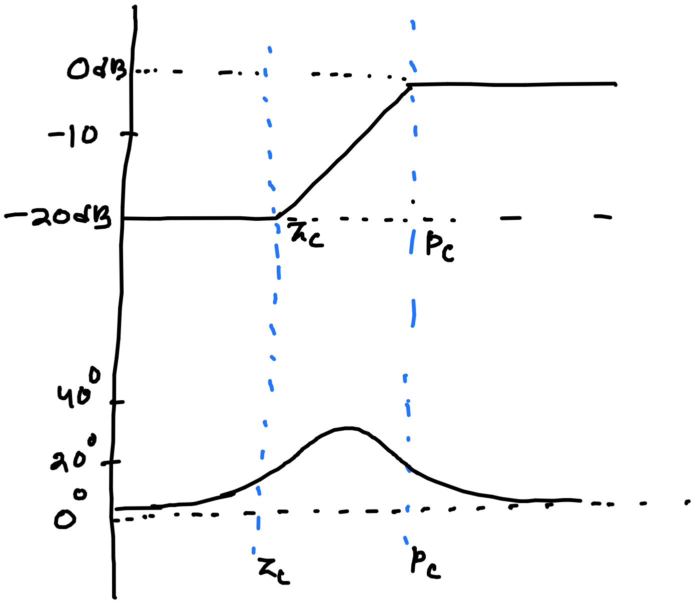
$$\theta_{z_c} > \theta_{p_c}$$

$$\Rightarrow \quad \theta_{c_{lead}} = \theta_{z_c} - \theta_{p_c} = +ve$$

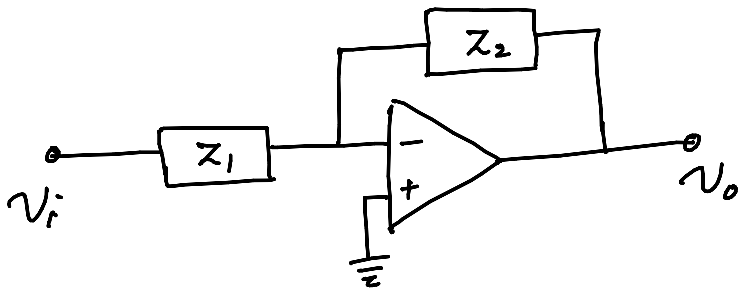
- Bode Plots for Lag Compensator.



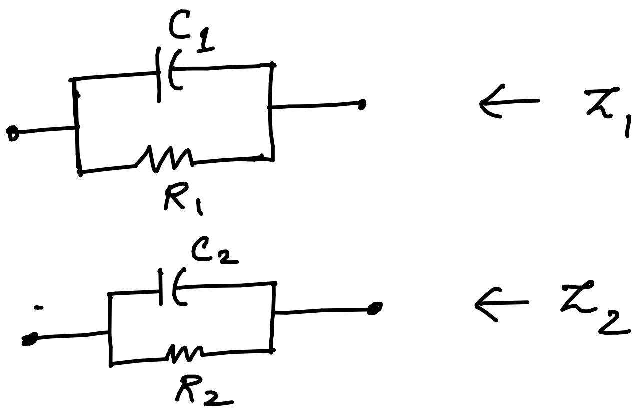
- Bode Plots for Lead Compensator



→ Circuit Realizations



- Lead Controller



$$G_c(s) = -\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)} \qquad R_1C_1 > R_2C_2$$

→ PI Compensator

→ SSE elimination

↓

In the design procedure, we placed a pole at origin & a zero very close to the pole at 0 → to ensure that the root locus does not modify (it remains as it was originally)

→ PD : Used for achieving transient behavior

↓

We designed $C(s) = K(s + z_c)$ s.t. the Root locus passes th. the desired point.

→ If we have specification (Design)

- Eliminate S.S.E.
- achieve transient behavior.

↳ We use

"Proportional - plus - integral - plus - derivative Controller"

$$\boxed{\begin{aligned} G(s) &= K_p + \frac{K_i}{s} + K_d s \\ &= \frac{K_p s + K_i + K_d s^2}{s} \end{aligned}}$$ PID Compensator

Design K_p, K_i & K_d to achieve S-S.E elimination & transient behavior.

↳ How to proceed:

1. Design PD compensator (i.e. by placing a zero at appropriate level?) to reshape the root locus s.t. transient specifications are met.

2. Design PI compensator (which will not change the shape of root locus) s.t. the S.S.E is eliminated.

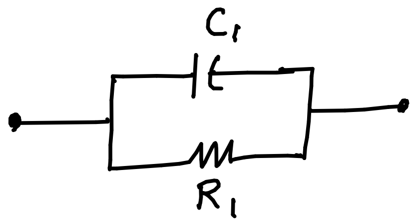
→ <u>Lag-Lead Compensator</u>:

(1) Design Lead compensator to achieve transient specificat., it will reshape the root locus.

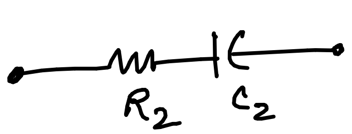
(2) Design Lag compensator to improve the S.S.E. (reduction in S.S.E) This design will not reshape the root locus.

→ Circuit Realizations

- PID Controller



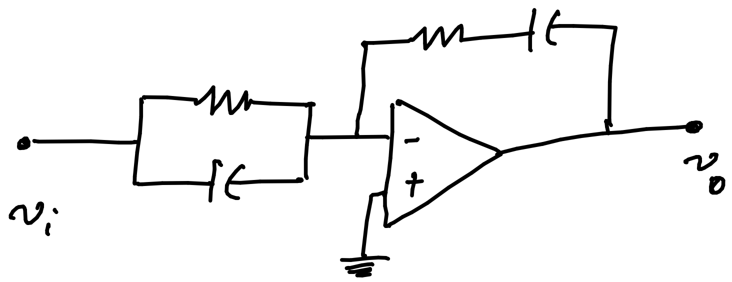
← Z_1



← Z_2

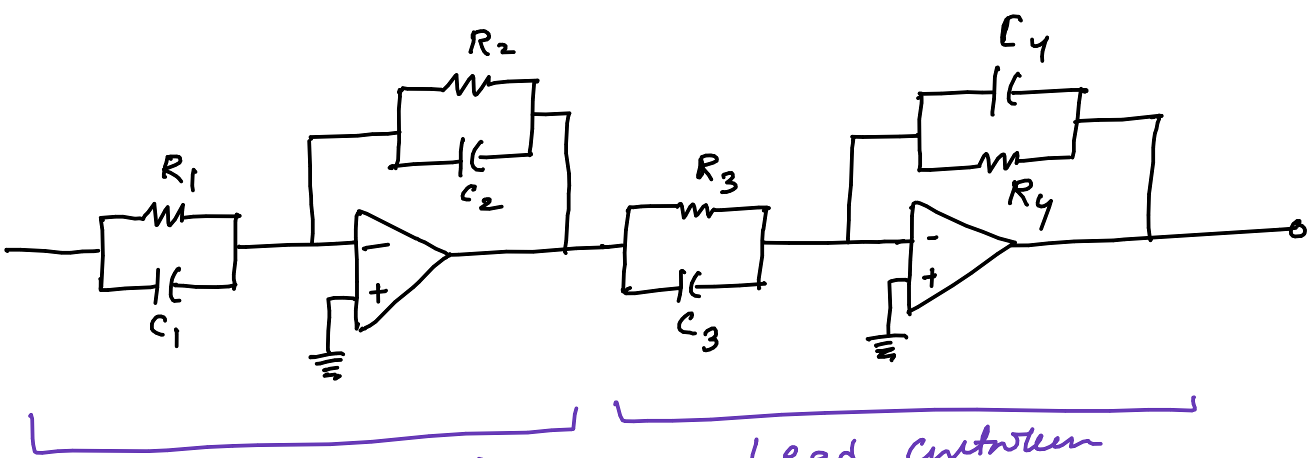
Controller transfer funct?

$$G_c(s) = -\left[\left(\frac{R_2}{R_1} + \frac{C_1}{C_2}\right) + R_2 C_1 s + \frac{1/R_1 C_2}{s}\right]$$



PID Controller

- Lag-Lead Controller

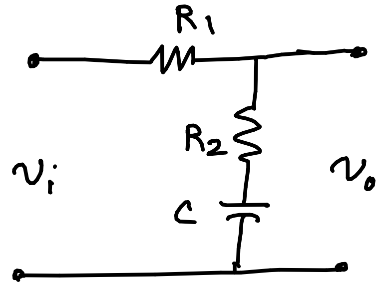


Lag Controller

Lead Controller

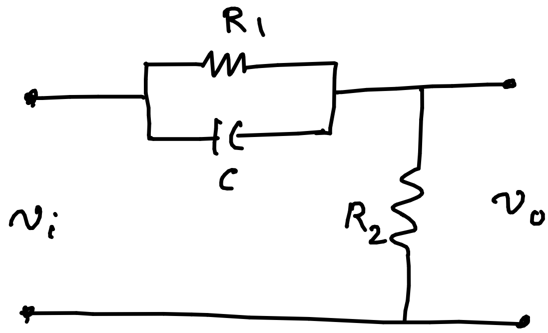
→ Passive Circuits for Compensators :

- Lag compensator



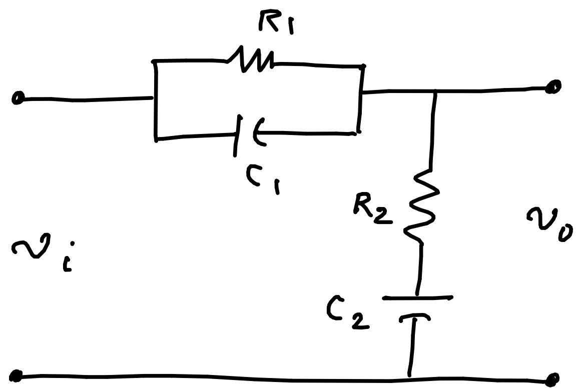
$$\frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2} \frac{\left(s + \frac{1}{R_2 C}\right)}{\left(s + \frac{1}{(R_1 + R_2) C}\right)}$$

- Lead compensator



$$\frac{v_o}{v_i} = \frac{s + \frac{1}{R_1 C}}{s + \frac{1}{R_1 C} + \frac{1}{R_2 C}}$$

- Lag-Lead compensator



$$\frac{v_o}{v_i} = \frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$