# ELL225

## Lecture - 36

- Frequency domain specifications:
  - → Phase margin $\Phi_m$ (P.M)
  - → Gain margin (G.M.)

→ To design compensators for achieving desired P.M. & G.M. we use Bode magnitude & phase plots.

→ Controller Design for achieving desired Phase margin $\bar{\Phi}_M$.

Consider the previous example (Bode plots)

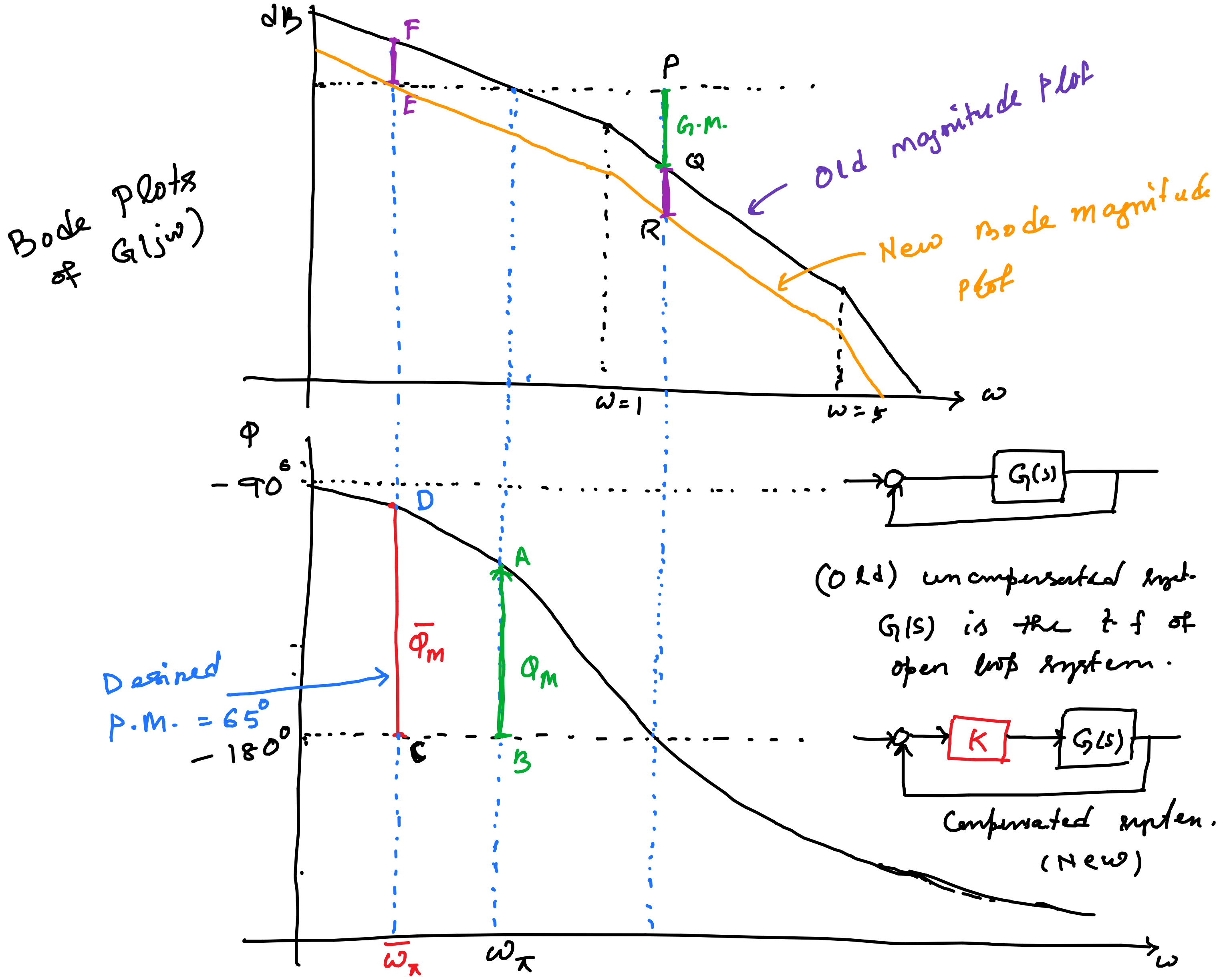
$$G(j\omega) = \frac{5}{j\omega(1+j\omega)(5+j\omega)}$$

$$= \frac{1}{j\omega(1+j\omega)(1+j0.2\omega)}$$

Let us assume that we need to design a controller to achieve $65^\circ$ P.M.

For this example we had calculated (see Lecture 28 on Bode plots) that P.M = $43^\circ$.

& G.M = 15.6




(uncompensated)

- The <u>old</u> P.M $\varphi_m$ of the system is AB & old G.M is PQ.

- The phase cross-over frequency (Gain = 1) $\omega_\pi$ is for the uncompensated system (old).

→ We need to first find the New phase cross-over frequency $\bar{\omega}_\pi$ from Bode phase plot, where we have the desired P.M. $\bar{\varphi}_m$.

→ Then, we need the gain of the compensated system (New) at $\bar{\omega}_x$ to be one, that is, the Bode magnitude plot must pass through 0 dB line at frequency $\bar{\omega}_x$.

→ By changing the gain K we can make the Bode magnitude plot to move <u>up</u> & <u>down</u>.

⇓

We <u>change gain K</u>

by an amount EF s.t. the point E will pass thru 0 dB line (at $\bar{\omega}_x$). (Push down the Bode plot by an amount EF)

↑

The change of K has no effect on the Phase plot.

- Note that by changing the gain the G.M. which was <u>PQ</u>, now changed to <u>PR</u> value. It has also increased.

- For second order system, there are some relations between frequency domain specifications with time domain specifications (see N. Nise Book or similar)

For instance,

P.M. $\Phi_m$ is related with the damping ratio ($\xi$) or % overshoot.

- Here, one can also translate some time domain specificat$^n$ to frequency domain, & then use Bode Plots for G.M. & P.M. adjustments.

↑

These translated relations are valid only for second order systems.

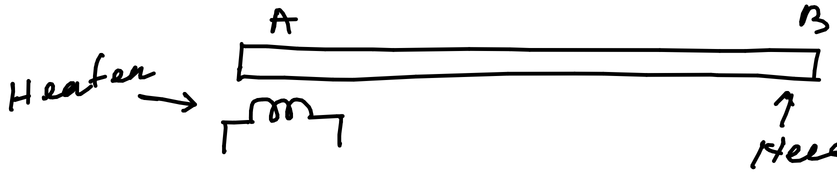
⇓

- If the specifications are given in time domain, for instance S.S.E elimination, damping ratio, settling time, then use "Root locus" approach to design different controllers.

- If the specifications are given in frequency domain, such as P.M. & G.M. adjustment, phase & gain compensations at some desired frequency level, then use "Bode plots" for controllers design.

[Recall from Nyquist plot that the P.M and G.M are defined for unity negative feedback closed loop systems by looking at the Nyquist plot of open loop t.f. $G(s)H(s)$. Hence for P.M. and G.M. adjustments through Bode plots, one needs to plot the magnitude and phase plots of open loop t.f. $G(j\omega)H(j\omega)$, and then change $K$ to achieve desired P.M/G.M. of closed loop unity feedback system.]

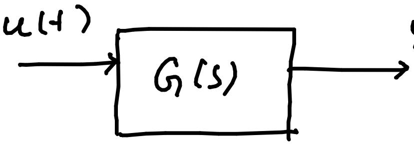
→ Systems with time delay / transport lag:



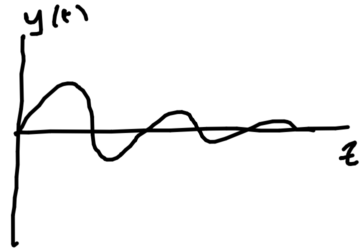

Need temp at this point $45^\circ C$

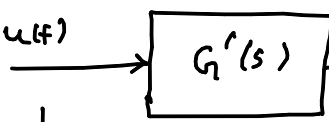
After switch on the heater, one can not get $45^\circ C$. temp at point B, immediately. It needs some time to reach the point B at $45^\circ C$.

(this effect can be considered as "delay" in the system response.




System without delay

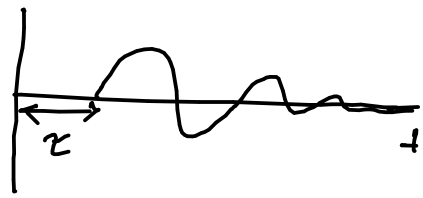



$y'(t) = y(t-\tau)$

System with delay

delay time




Laplace transform

$$Y(s) = G(s)\,U(s)$$

$$Y'(s) = e^{-s\tau}\,Y(s)$$

$$Y'(s) = G'(s)\,U(s)$$

$$\Rightarrow e^{-s\tau}\,Y(s) = G'(s)\,U(s)$$

$$\Rightarrow e^{-s\tau}\,G(s)\,U(s) = G'(s)\,U(s)$$

$$\Rightarrow \boxed{G'(s) = e^{-s\tau}\,G(s)}$$

A system with delay component has a factor $e^{-s\tau}$ in the transfer funct<sup></sup>.

- In frequency response analysis

$$s \to j\omega$$

$$G'(j\omega) = e^{-j\omega\tau} G(j\omega)$$

$$e^{-j\omega\tau} = 1 \angle -\omega\tau$$

$$\downarrow$$

$$\cos\omega\tau - j\sin\omega\tau$$

$$|G'(j\omega)| = |e^{-j\omega\tau} G(j\omega)|$$

$$= |G(j\omega)|$$

$$\angle G'(j\omega) = \angle e^{-j\omega\tau} G(j\omega)$$

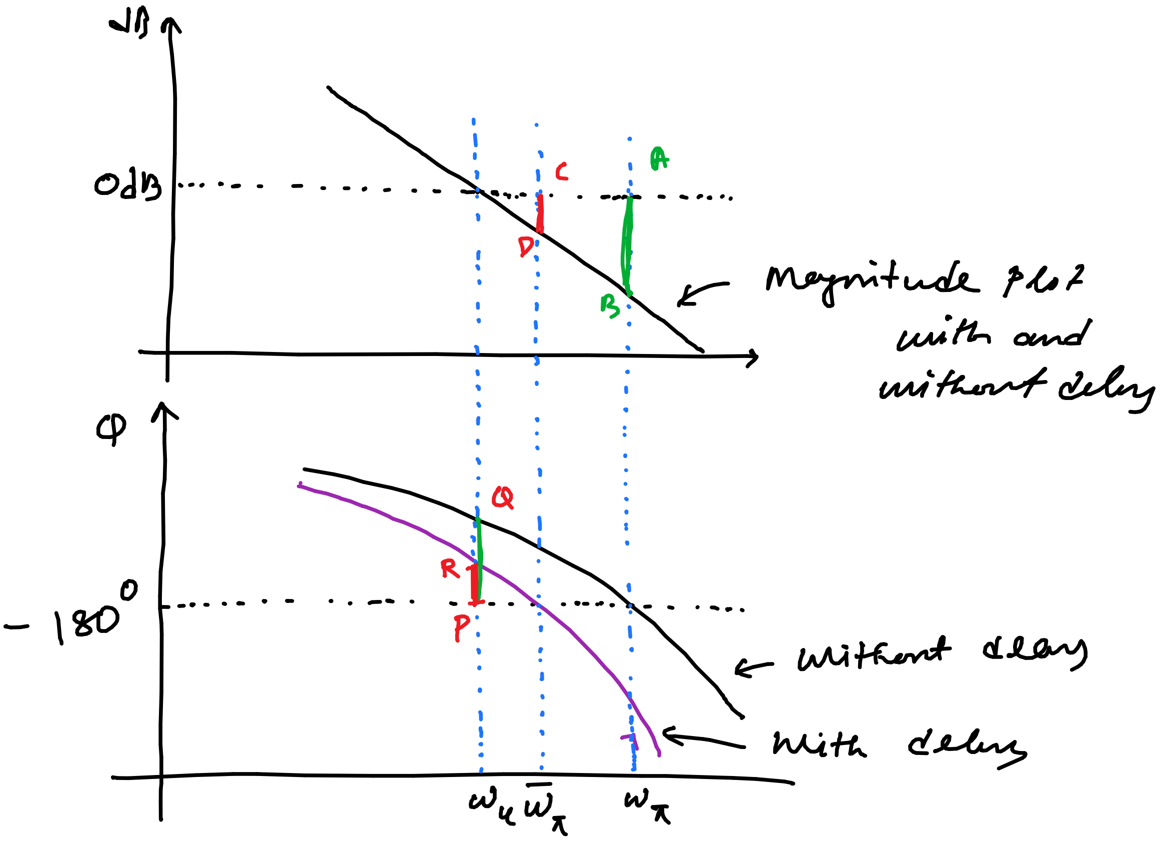
$$= \angle G(j\omega) - \omega\tau$$

$$\Downarrow$$

- The presence of delay does not affect the magnitude plot of $G(j\omega)$, however, reduce the phase by $\omega\tau$, and hence change the phase plot.

With increase in frequency, the quantity $\omega\tau$ will increase.

Bode Plots




- Bode phase and magnitud plots for both: system without delay and same system with delay are presented above.

- Due to the presence of delay the phase plot has changed without affecting the magnitude plot.

$\Downarrow$

- For system without delay, the phase cross-over frequency was $\omega_\pi$ & the corresponding G.M. was AB.

$\downarrow$

- Due to delay the phase cross-over frequency has changed to $\bar{\omega}_\pi$, leading to G.M. CD.

=> The G.M. has now reduced.

- Since the gain cross-over frequency remained same $\omega_u$, the P.M. margin has reduced from PQ to PR.

$\Downarrow$

Reduction in both PM & GM makes the system moving towards instability.

$\Downarrow$

∴ One can use the previously discussed method to improve P.M. & G.M. by appropriate compensation (proportional controller)