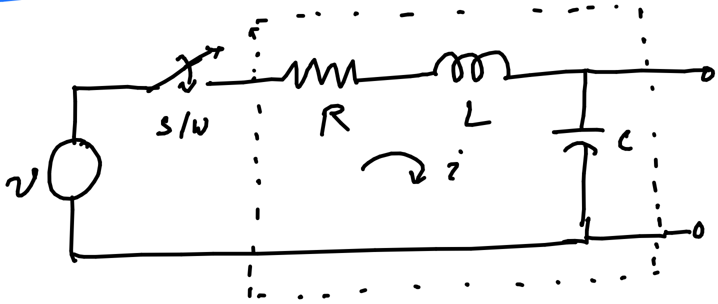
Lecture - 3



$$\frac{di}{dt} = -\frac{R}{L} i - \frac{1}{L} v_c + \frac{1}{L} v$$

$$\frac{dv_c}{dt} = \frac{1}{C} i$$

$$\underbrace{\begin{bmatrix} \frac{di}{dt} \\ \frac{dv_c}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i \\ v_c \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B} v$$

$\dot{x}$ state variable

v ← input to the system u

$$\underbrace{v_c}_{y} = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} i \\ v_c \end{bmatrix}}_{x}$$

output y

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

← State space representation

The state $x \in \mathbb{R}^n$

$$\begin{bmatrix} * \\ * \\ \vdots \\ * \end{bmatrix} \in \mathbb{R}^n$$

$$\frac{di}{dt} = \frac{1}{L} v_L \xrightarrow[\text{transform}]{\text{Laplace}} s I(s) - i(0_-) = \frac{1}{L} V_L(s)$$

$$\frac{dv_c}{dt} = \frac{1}{C} i \longrightarrow s V_c(s) - v_c(0_-) = \frac{1}{C} I(s)$$

$v(t) \rightarrow V(s)$

$i(t) \rightarrow I(s)$

$i(0_-)$: initial condition → The current through the inductor just before the closing of switch.

$v_c(0_-)$: initial condition → The voltage accross the capacitor just before the closing of switch.

- Assumption

$\boxed{\text{All the initial conditions are set to 0.}}$ $v_c(0_-) = 0$, $i(0_-) = 0$

$$V_L(s) = sL\, I(s)$$

$$V_c(s) = \frac{1}{sC} I(s)$$

$$V_R(s) = R I(s)$$

$$v = v_R + v_L + v_c$$

$$V(s) = V_R(s) + V_L(s) + V_c(s)$$

$$= \left[sL + \frac{1}{sC} + R \right] I(s)$$

$$V(s) = \frac{s^2LC + sRC + 1}{sC} I(s)$$

$$\frac{I(s)}{V(s)} = \frac{sC}{s^2LC + sRC + 1}$$

$$I(s) = sC\, V_c(s)$$

output

input

$$\boxed{\frac{V_c(s)}{V(s)} = \frac{1}{s^2LC + sRC + 1}}$$

Transfer function of a system

To obtain a transfer funct^n of a system.

- Write down the differential eq^n
- Take Laplace transformat
- Set all initial conditions to 0.
- Obtain the ratio between input variable and output variable.

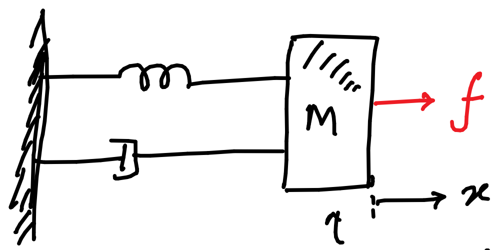
Mathematical model

State-space Representation
- input variable
- output variable
- state variable

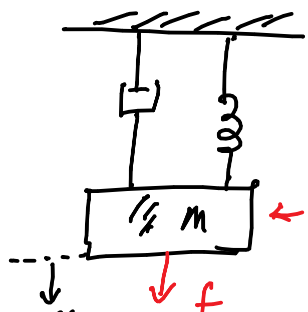
Transfer function Representation
- Input variable
- Output variable.

State-space Rep. is given $\longrightarrow$ Transfer fun^n reprs.
$\longleftarrow$

→ **Mechanical System**



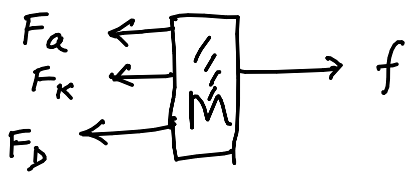
May be considered as a Railway bogie



One wheel of a car

Input is external force f

Output : position of the mass x



$$f = F_a + F_K + F_D$$
$$= M\frac{d^2x}{dt^2} + K_s x + K_D\frac{dx}{dt}$$

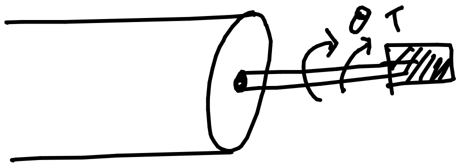
$$f = M\frac{d^2x}{dt^2} + K_D\frac{dx}{dt} + K_s x$$

Laplace transform by setting all initial conditions to 0

$$F(s) = s^2 M X(s) + K_D s X(s) + K_s X(s)$$

$$\Rightarrow \boxed{\frac{X(s)}{F(s)} = \frac{1}{s^2 M + s K_D + K_s}}$$

Rotational system



The input torque is T

$\theta \rightarrow$ Angular position of the shaft

- Inertia torque
- Torsion
- Viscous / Damping

$$T = T_{iner} + T_{tor} + T_{vis}$$

$$= J\frac{d^2\theta}{dt^2} + K_s\theta + K_D\frac{d\theta}{dt}$$

$\downarrow$ Laplace transform set all initial conditions to 0

$$\boxed{\frac{\Theta(s)}{T(s)} = \frac{1}{s^2 J + s K_D + K_s}}$$

Transfer funct?.

$$T = J\frac{d^2\theta}{dt^2} + K_D\frac{d\theta}{dt} + K_s\theta$$

Assume that $\quad x_1 = \theta$

$\qquad\qquad\qquad\quad x_2 = \dot{\theta}$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{\theta} = -\frac{K_D}{J}\dot{\theta} - \frac{K_s}{J}\theta + \frac{T}{J}$$

$$= -\frac{K_D}{J}x_2 - \frac{K_s}{J}x_1 + \frac{T}{J}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K_s}{J} & -\frac{K_D}{J}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J}\end{bmatrix}T$$

$$x_1 = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

state-space representation