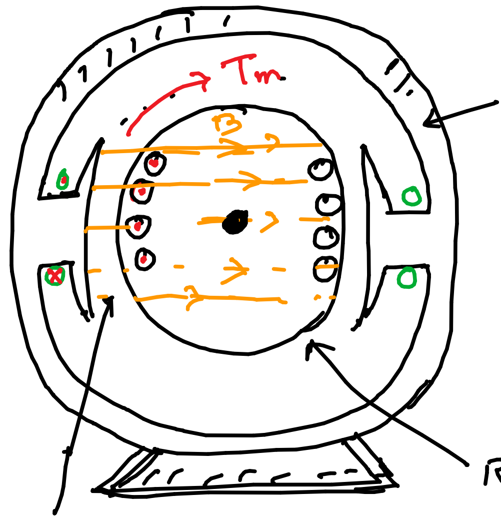
ELL 225

Lecture - 4

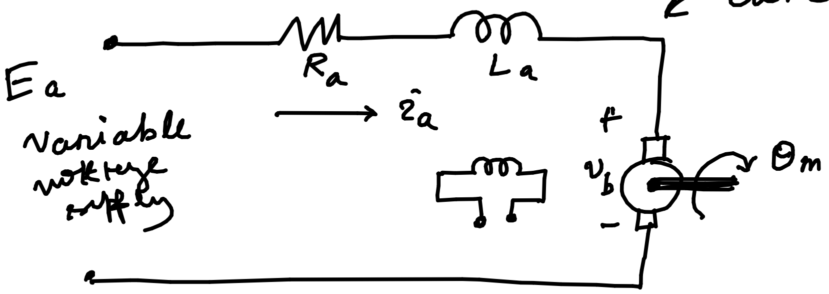
DC - Servomotor ← Used as actuators

- Robotic arms
- Flight 3-axis control
- Position control of Antenna



- Field winding placed in Stator
- Armature winding (placed in rotor)

← ckt diagram for armature winding



The speed of the servo motor can be changed using the variable input voltage to armature ckt.

↓

Armature controlled DC - servo motor

- Torque / Force experienced by the armature conductor $F = B l i_a$
- Torque is proportional to the armature current : $T_m = K_t i_a$ $\quad K_t$: const.

- Back e.m.f : $v_b = B l v$

$l \to$ length of the conductor
$v \to$ linear velocity of the conductor

$$v_b = K_b \frac{d\theta}{dt}$$

The armature ckt eq^n :

$$R_a i_a + L\frac{di}{dt} + v_b = E_a$$

$$J\frac{d^2\theta_m}{dt^2} + D\frac{d\theta_m}{dt} + K_s\theta_m = T_m$$

$$R_a I_a(s) + LsI(s) + \underset{\substack{\| \\ K_b s\theta_m(s)}}{V_b(s)} = E_a(s)$$

$$T_m(s) = K_t I(s)$$

$$\frac{(R_a + sL)\,T_m(s)}{K_t} + K_b s\,\theta_m(s) = E_a$$

The resistance of armature winding is usually very very greater than inductance L on armature winding

so $L \leftarrow$ can be neglected

$$\frac{R_a}{K_t}\left[s^2 J\theta_m(s) + sD\theta_m + K_s\theta_m\right] + K_b s\,\theta_m(s) = E_a(s)$$

$$\left[\frac{R_a J}{K_t}s^2 + \left(\frac{R_a D}{K_t} + K_b\right)s + \frac{R_a K_s}{K_t}\right]\theta_m(s) = E_a(s)$$

$$\Rightarrow \frac{\theta_m(s)}{E_a(s)} = \text{calculate}$$

$$\frac{\theta(s)}{E_a(s)} = \frac{b}{s^2 + \alpha_1 s + \alpha_0}$$

b, α_1, α_0 are constants

- <u>State space model</u>

$$R_a \frac{T_m}{K_t} + K_b \frac{d\theta_m}{dt} = E_a$$

$$\frac{R_a}{K_t}\left[J \frac{d^2\theta_m}{dt^2} + D\frac{d\theta_m}{dt} + K_s\,\theta_m\right] + K_b \frac{d\theta_m}{dt} = E_a$$

$$\Rightarrow \frac{R_a J}{K_t}\frac{d^2\theta_m}{dt^2} + \left(\frac{R_a D}{K_t} + K_b\right)\frac{d\theta_m}{dt} + \frac{R_a K_s}{K_t}\theta_m = E_a$$

$$\Rightarrow \frac{d^2\theta_m}{dt^2} = -\frac{K_t}{R_a J}\left(\frac{R_a D}{K_t} + K_b\right)\frac{d\theta_m}{dt} - \frac{K_t}{R_a J}\cdot\frac{R_a K_s}{K_t}\theta_m + \frac{K_t}{R_a J}E_a$$

Define a variable $x_1 = \theta_m$

$x_2 = \dot{\theta}_m$

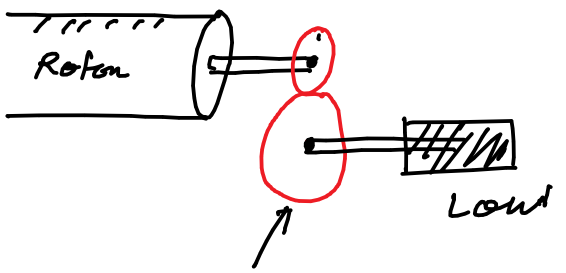
$$\dot{x}_1 = \dot{\theta}_m = x_2$$

$$\dot{x}_2 = \ddot{\theta}_m =$$

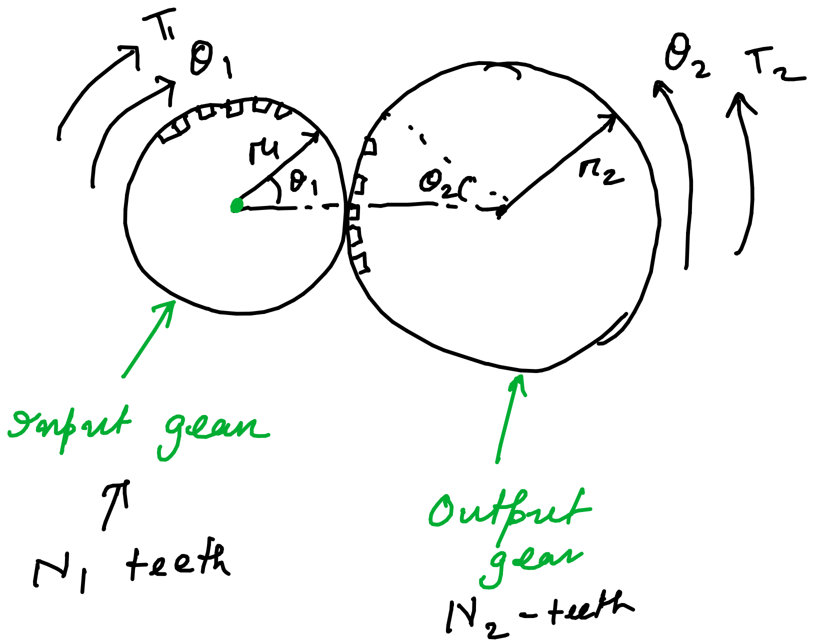
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ * & *\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{K_t}{R_a J}\end{bmatrix}E_a$$

$$\theta_m = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

DC servomotor ← Electro-mechanical system



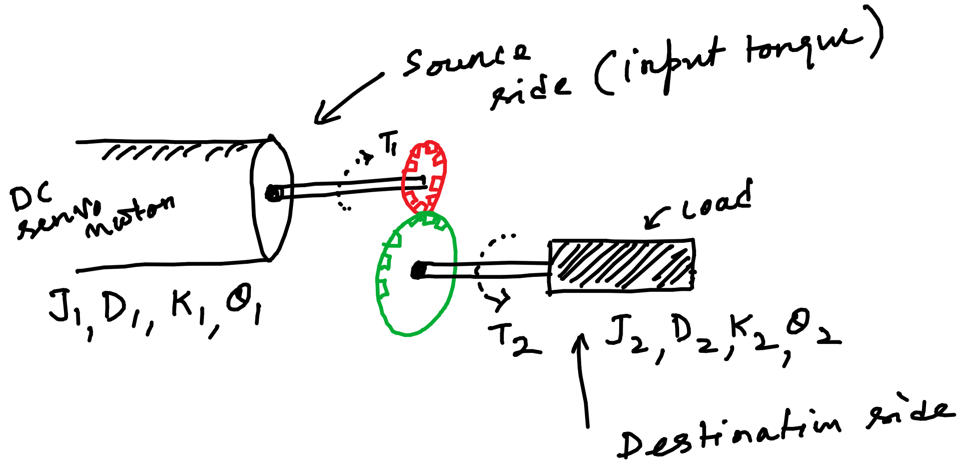
Gear mechanism to change the speed of load.



- $r_1\theta_1 = r_2\theta_2$
- $T_1\theta_1 = T_2\theta_2$
- $\dfrac{r_1}{r_2} = \dfrac{N_1}{N_2}$

$$\boxed{\begin{aligned} \theta_2 &= \frac{N_1}{N_2}\theta_1 \\ \theta_1 &= \frac{N_2}{N_1}\theta_2 \end{aligned}}$$

$$\boxed{\begin{aligned} T_2 &= \frac{N_2}{N_1}T_1 \\ T_1 &= \frac{N_1}{N_2}T_2 \end{aligned}}$$



$$s^2 J_2\theta_2(s) + sK_2\theta_2(s) + K_s\theta_2 = T_2$$

$$J_1 \frac{d^2\theta_1}{dt^2} + D_1 \frac{d\theta_1}{dt} + K_{s_1}\theta_1 = T_1$$

$$\mathcal{L} \Rightarrow s^2 J \theta_1(s) + s D_1 \theta_1(s) + K_{s_1}\theta_1(s) = T_1(s)$$

$$s^2 J \left(\frac{N_2}{N_1}\right)\theta_2(s) + s D_1\left(\frac{N_2}{N_1}\right)\theta_2(s) + K_{s_1}\left(\frac{N_2}{N_1}\right)\theta_2(s) = \frac{N_1}{N_2} T_2(s)$$

$$\Rightarrow \underbrace{s^2 J \left(\frac{N_2}{N_1}\right)^2}_{J_1^D}\theta_2(s) + s\,\underbrace{D_1\left(\frac{N_2}{N_1}\right)^2}_{D_1^D}\theta_2(s) + \underbrace{K_{s_1}\left(\frac{N_2}{N_1}\right)^2}_{K_{s_1}^D}\theta_2(s) = T_2(s)$$

J_1^D : The inertia of source side when it is referred to destination side

D_1^D : The damping coefficient of source side when it is reffered to destination side.

$K_{s_1}^D$: "

$$J_2 \frac{d^2\theta_2}{dt^2} + D_2 \frac{d\theta_2}{dt} + K_{s_2}\theta_2 = T_2$$

$$\boxed{\begin{array}{lll} J_1^D = J_1\left(\frac{N_2}{N_1}\right)^2 & D_1^D = D_1\left(\frac{N_2}{N_1}\right)^2 & K_{s_1}^D = K_{s_1}\left(\frac{N_2}{N_1}\right)^2 \\ J_2^S = J_2\left(\frac{N_1}{N_2}\right)^2 & D_2^S = D_2\left(\frac{N_1}{N_2}\right)^2 & K_{s_2}^S = K_{s_2}\left(\frac{N_1}{N_2}\right)^2 \end{array}}$$

J_2^S : The inertia of destination side when it is reffered to source side.

D_2^S $K_{s_2}^S$ ..

- Transfer all quantities to one side only while computing transfer function.

- The equivalent inertia at armature side (motor side) :

$$J_e = J_1 + J_2^s$$
$$= J_1 + J_2\left(\frac{N_1}{N_2}\right)^2$$

$$D_e = D_1 + D_2^s$$
$$= D_1 + D_2\left(\frac{N_1}{N_2}\right)^2$$