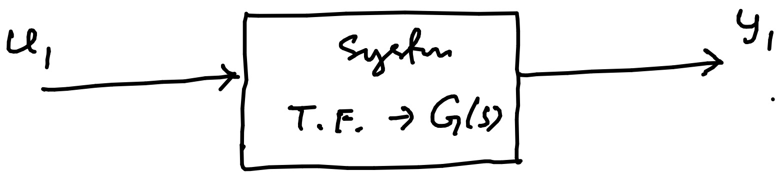
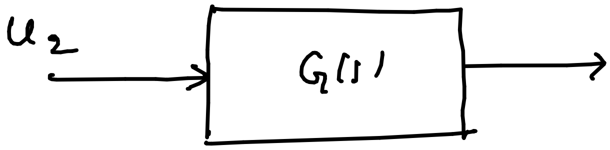
ELL225

Lecture - 5

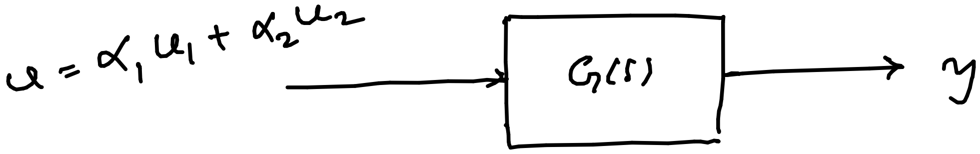
A dynamical system represented by a set of linear constant coefficient differential equations has also a transfer function representation.



$Y_1(s) = G(s)\,U_1(s)$



$y_2 = G(s)\,U_2(s)$



where α_1, α_2 are some scalars

$$Y(s) = G(s)\,U(s)$$

$$= G(s)\left[\alpha_1 U_1(s) + \alpha_2 U_2(s)\right]$$

$$= \alpha_1 G(s) U_1(s) + \alpha_2 G(s) U_2(s)$$

$$= \alpha_1 Y_1(s) + \alpha_2 Y_2(s)$$

$\Downarrow$

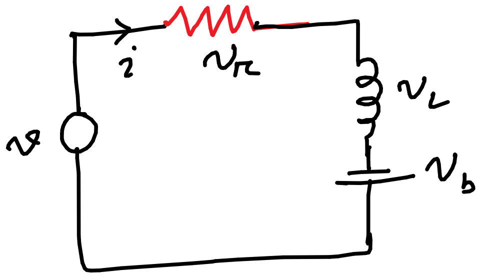
The system satisfies the principle of "superposition".

$\Downarrow$

The system linear system.

Linear System : A system whose input-output relation satisfies the principle of "superposition".

Non-linear System : A system whose input-output relation does not satisfy principle of superposition.



$$v_R = iR$$

$$v_R = 10 \ln\left(\frac{i_R}{2}\right) \qquad i = i_R$$

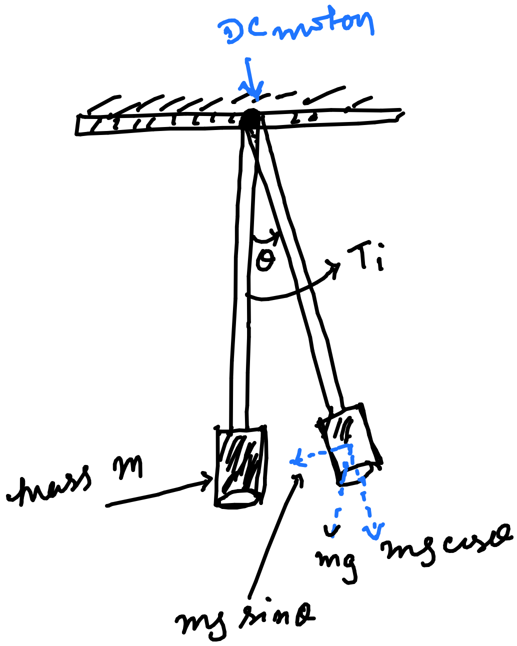
$$v = v_R + v_L - v_b \qquad \text{Let } v_b = 20$$

$$v = 10 \ln\left(\tfrac{1}{2} i\right) + L \frac{di}{dt} - 20$$

$$\boxed{\Rightarrow \frac{di}{dt} = -\frac{1}{L} 10 \ln\left(\frac{i}{2}\right) + \frac{1}{L} 20 + \frac{1}{L} v}$$

T.F

$$\frac{I(s)}{V(s)}$$



Assume that the rod is mass-less

$$J = ml^2$$

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + mgl \sin\theta = T_i$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{D}{ml^2} \frac{d\theta}{dt} + \frac{g}{l} \sin\theta = \frac{1}{ml^2} T_i}$$

Obtain transfer fn? $\dfrac{\Theta(s)}{T_i(s)} = ?$

Since the differential equations are non-linear the transfer function can not be computed.

Non-linear differential eq^n

$$\boxed{\dot{x} = f(x, u)} \leftarrow \text{Non-linear system}$$

$$\frac{d^2\theta}{dt^2} + \frac{D}{ml^2}\frac{d\theta}{dt} + \frac{g}{l}\sin\theta = \frac{1}{ml^2}T_i$$

$$x_1 = \theta \qquad x_2 = \dot{\theta}$$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{D}{ml^2}x_2 - \frac{g}{l}\sin x_1 + \frac{1}{ml^2}T_i \end{aligned}}$$

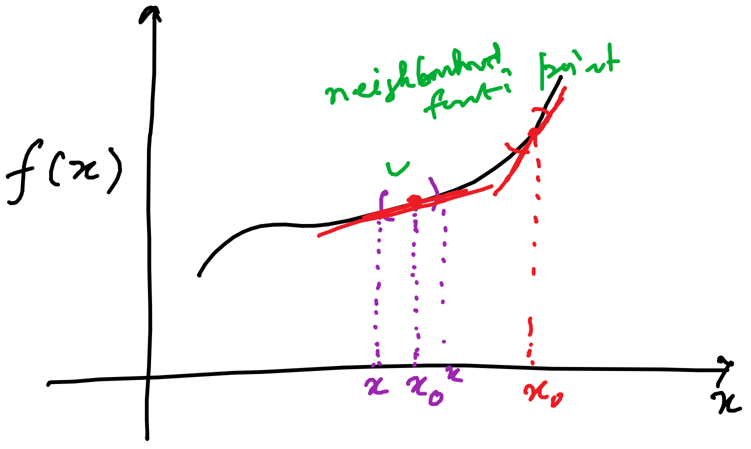
→ Assume that there are <u>no inputs to the system</u>

↑ Autonomous system

$$\dot{x} = f(x)$$

↓

<u>Linearization</u>



The function in the neighborhood of x_0
↓
Taylor series expansion
↓

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x-x_0) + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x_0} \frac{(x-x_0)^2}{2!} + \cdots$$

Let $\Delta x = x - x_0$

- For small values of Δx, the quantities Δx^2, $\Delta x^3 \ldots$ can be neglected in comparison to Δx.

↓

$$\boxed{f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} \Delta x}$$

$$\Delta x = x - x_0$$

$$\Delta \dot{x} = \dot{x}$$

const. parameter

$$\Delta \dot{x} = \dot{x} = f(x)$$

$$\boxed{\Delta \dot{x} = \underline{f(x_0)} + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} \Delta x}$$

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0}$$

↑
const.