ELL 225

Lecture - 7

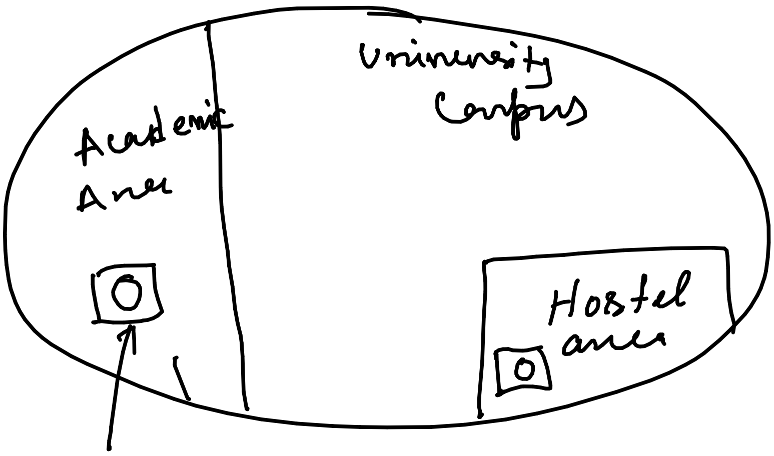
$$\dot{x} = f(x, u)$$

$\downarrow$ linearization about some operating point $(\bar{x}, \bar{u})$

$$\Delta\dot{x} = A\,\Delta x + B\,\Delta u$$

$$A = \left[\frac{\partial f}{\partial x}\right]_{(\bar{x},\bar{u})} \qquad B = \left[\frac{\partial f}{\partial u}\right]_{(\bar{x},\bar{u})}$$

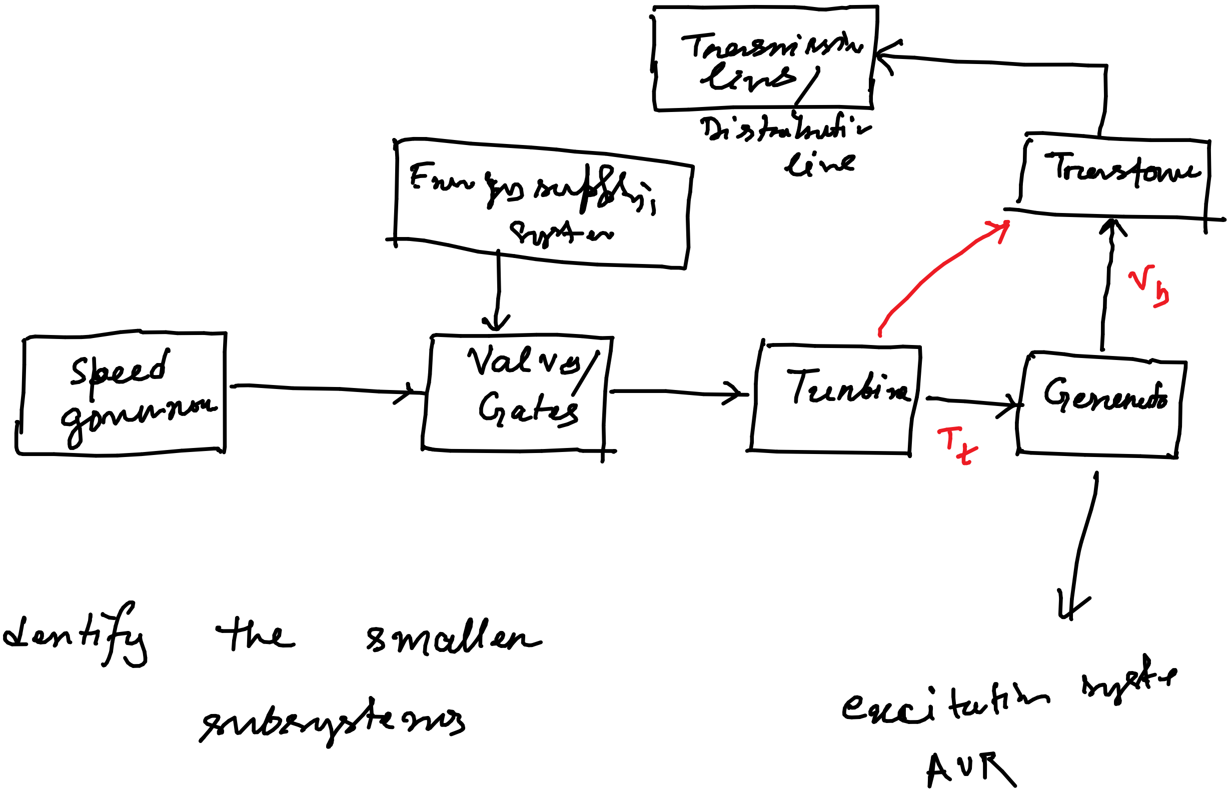
A & B are real matrices.



Camera which will give informat^n^ about the local region

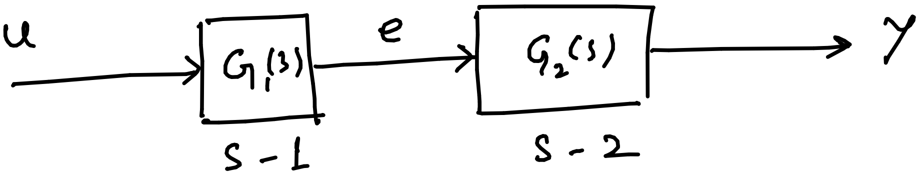
$\Delta x \leftarrow$ the neighborhood states of $\bar{x}$

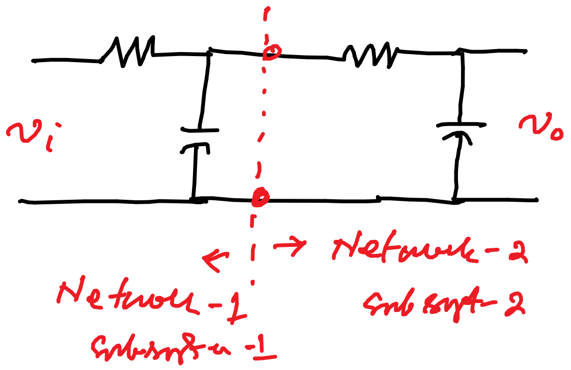
Linearization. Can be considered as putting camera & observing the dynamic behavior of the non-linear system locally.



- Identify the smaller subsystems
- Identify the input/output signals of each subsystem
- Develop mathematical model for each subsystem. (T. F.)
- Perform Block-reduction to obtain the overall t.f. of the system.

→ Block reduction: Obtain a single t.f. of the system by reducing the number of blocks/ subsystems.





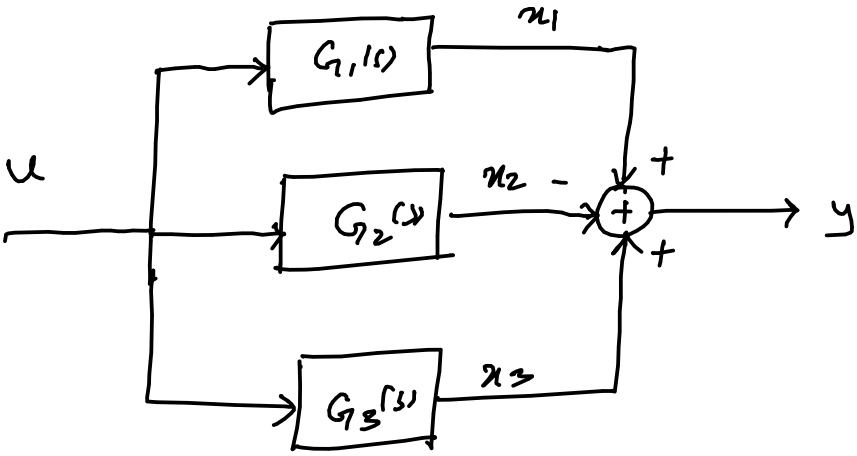
$$y = G_2(s) \cdot e$$

$$= G_2(s) \cdot G_1(s)\, u$$

$$y = G_2(s)\, G_1(s)\, u$$

$$y(s) = G(s)\, U(s)$$

where $\boxed{G(s) = G_2 G_1}$ which is overall t.f. of the .



$$x_1 = G_1 u$$

$$x_2 = G_2 u$$

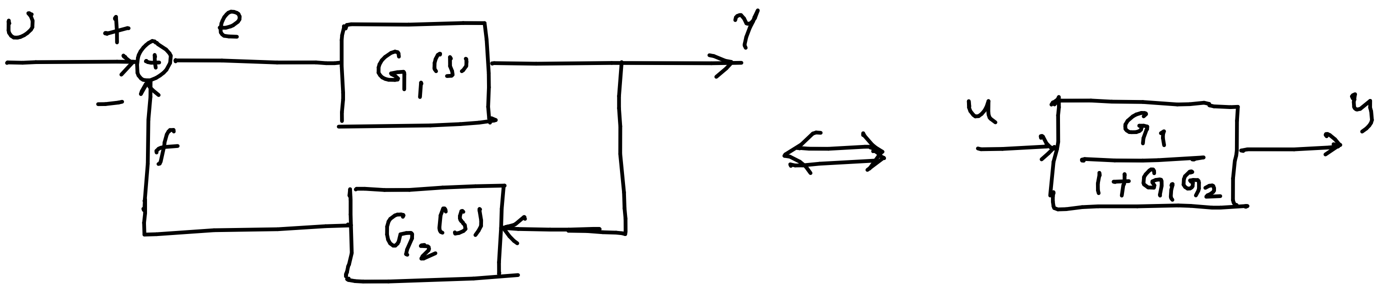
$$x_3 = G_3 u$$

$$y = x_1 - x_2 + x_3$$

$$= G_1 u - G_2 u + G_3 u$$

$$= (G_1 - G_2 + G_3)\, u$$

$Y(s) = G(s)\, U(s)$ when $\boxed{G(s) = G_1 - G_2 + G_3}$

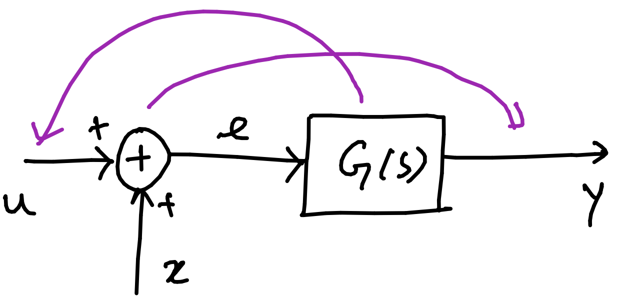


$$
\begin{aligned}
y &= G_1 e \\
&= G_1 (u - f) \\
&= G_1 (u - G_2 y)
\end{aligned}
$$

$$\Rightarrow (1 + G_1 G_2)\, y = G_1 u$$

$$\Rightarrow \quad y = \frac{G_1}{1 + G_1 G_2}\, u$$

$Y(s) = G(s)\, U(s)$ wh $\boxed{G(s) = \dfrac{G_1}{1 + G_1 G_2}}$



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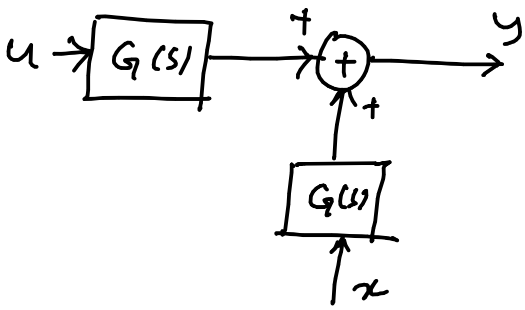
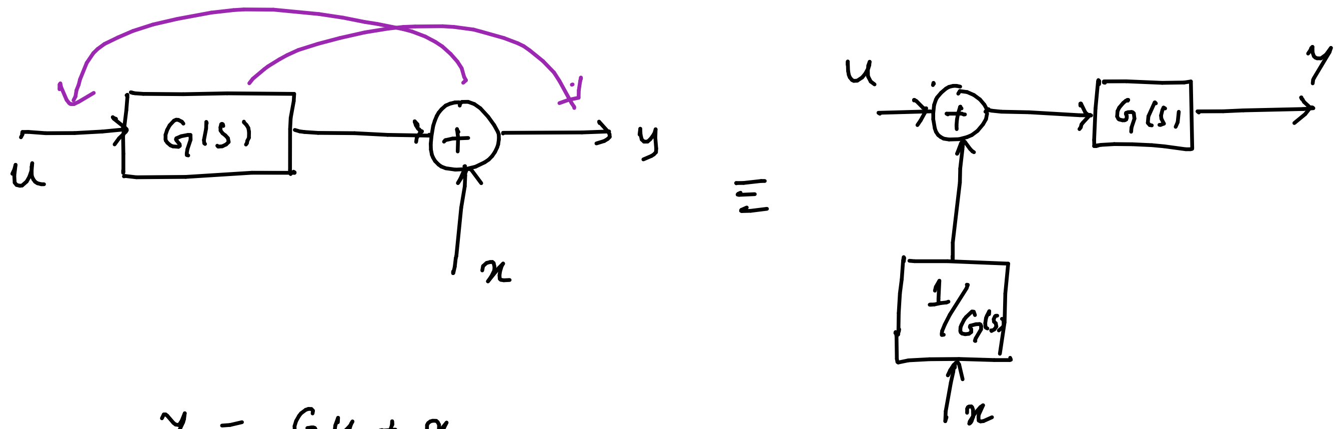


$$
\begin{aligned}
y &= Ge \\
&= G(u + x) \\
&= Gu + Gx
\end{aligned}
$$

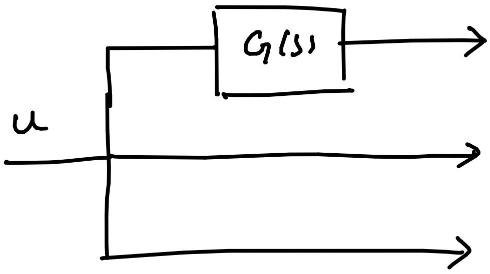
$$y = Gu + Gx$$



$$y = Gu + x$$

$$y = G\left(u + \frac{1}{G}x\right)$$

$$= Gu + x$$



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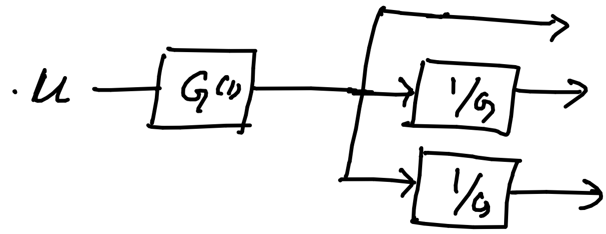
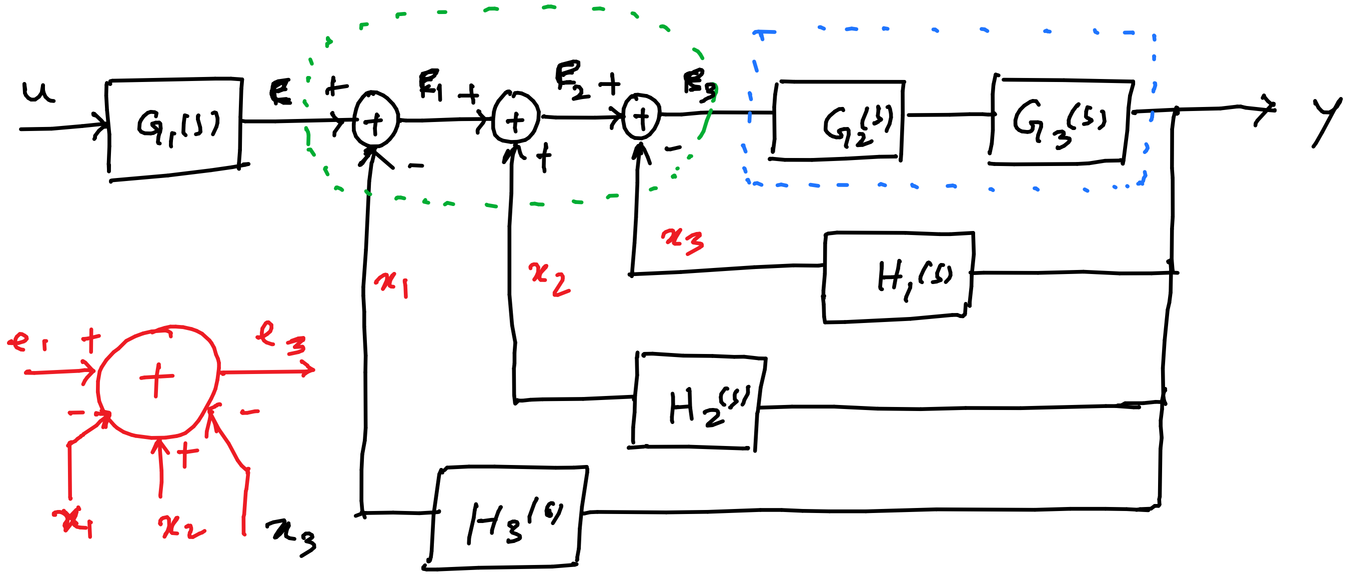
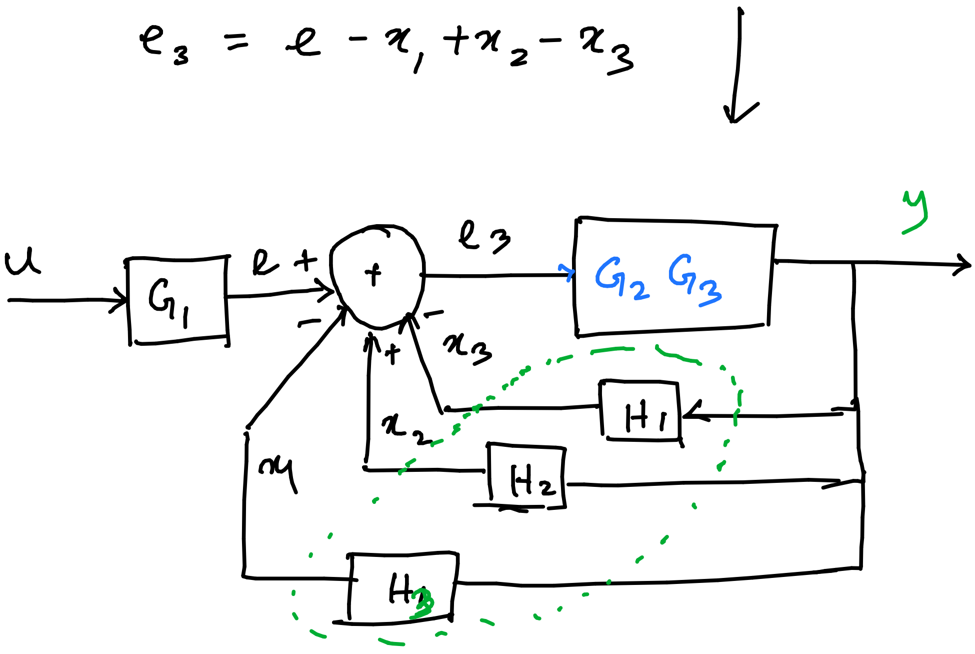
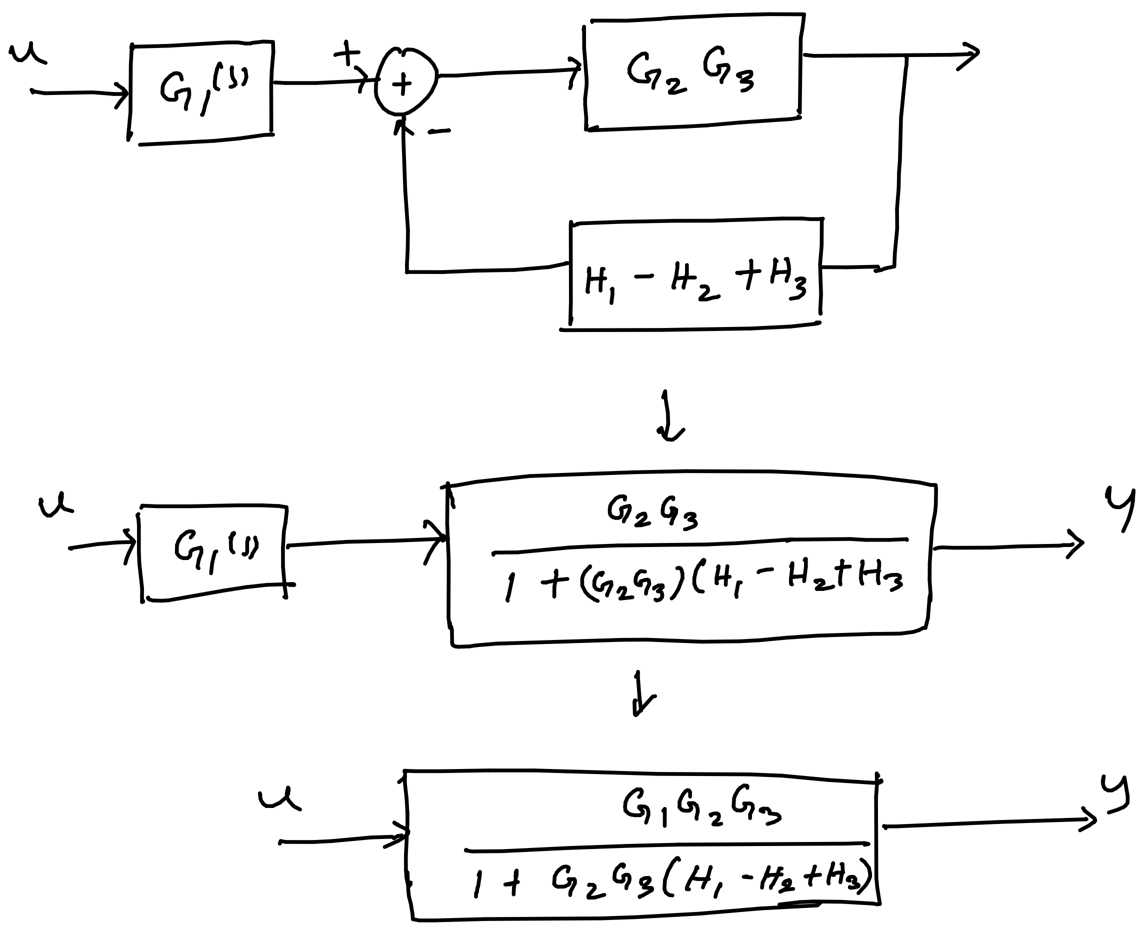




$$e_3 = e - x_1 + x_2 - x_3$$




u
$G_1(s)$
+
+
−
$G_2 G_3$
$H_1 - H_2 + H_3$
u
$G_1(s)$
$\frac{G_2 G_3}{1 + (G_2 G_3)(H_1 - H_2 + H_3}$
y
u
$\frac{G_1 G_2 G_3}{1 + G_2 G_3 (H_1 - H_2 + H_3)}$
y