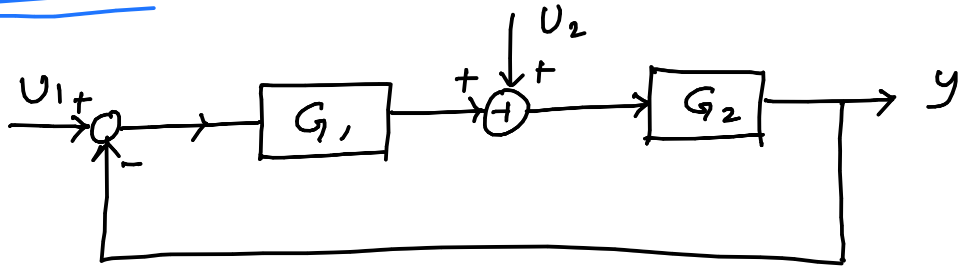
ELL225

Lecture-8

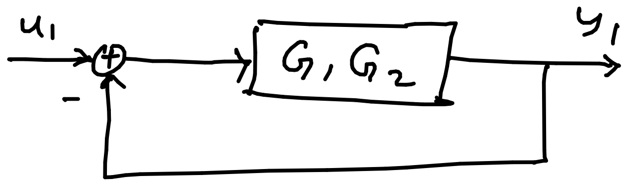


Multiple inputs to the system.

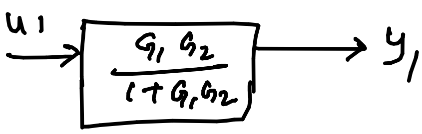
- Since we are considering LTI system. superposition rule will hold.

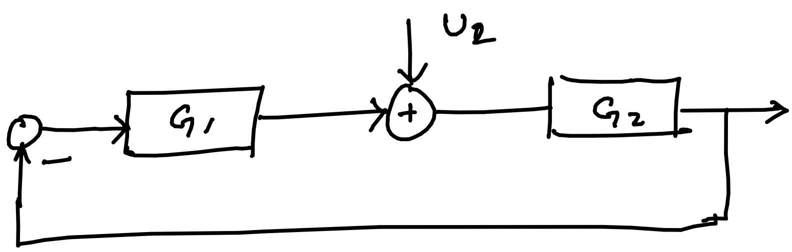
- Set all the inputs to the system to '0', except one.

Let $U_2 = 0$

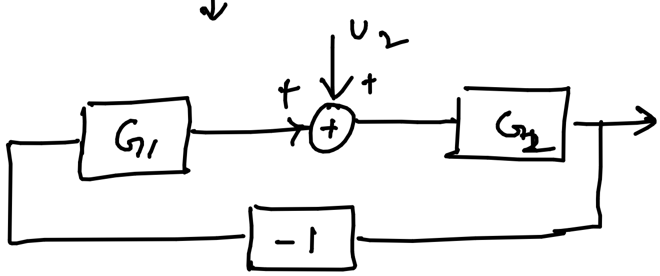


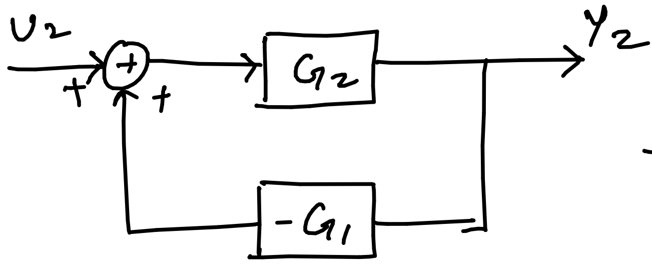
$$Y_1 = \frac{G_1 G_2}{1 + G_1 G_2} U_1$$





↓





$$\rightarrow \; Y_2 = \frac{G_2}{1 + G_1 G_2} u_2$$

$$Y = Y_1 + Y_2$$

$$= \frac{G_2}{1 + G_1 G_2} \left[G_1 U_1 + u_2 \right]$$

- Large order interconnected systems (T.F. computation)

Block Reduction | Using Signal flow graph

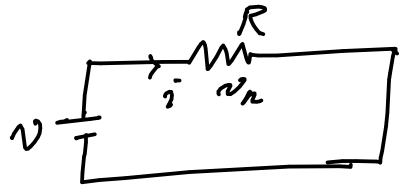
→ <u>Signal-flow graph</u>

- Two signals u & y
- A mathematical operator G, which maps u to y.

$$y = Gu$$

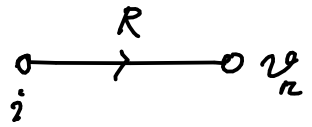
$$u \xrightarrow{G} y$$

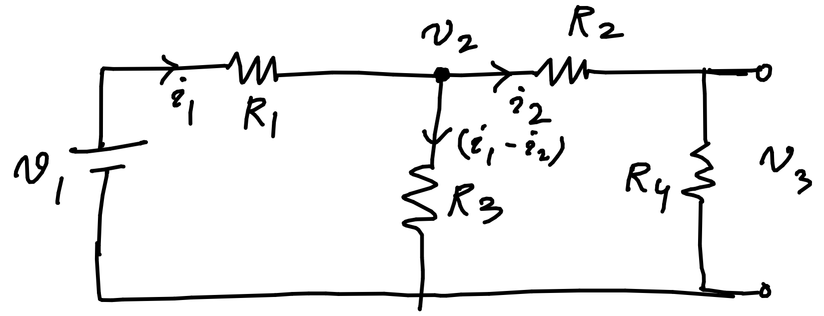
- The signals are represented by nodes
- The operator is represented by branch



$\underline{v_r} = iR$

$\downarrow$



- 

- The associated signals:

  $v_1, \; i_1, \; v_2, \; i_2, \; v_3$

- $$\frac{v_1 - v_2}{R_1} = i_1$$

  $$\Rightarrow i_1 = \frac{1}{R_1} v_1 + \left(-\frac{1}{R_1}\right) v_2$$

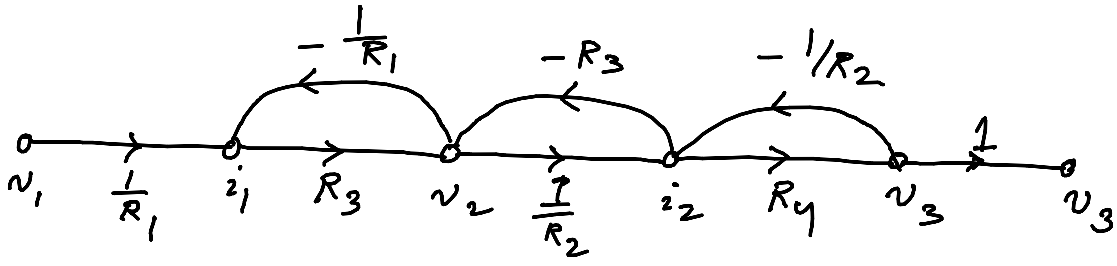
- $$\frac{v_2}{R_3} = i_1 - i_2$$

  $$\Rightarrow v_2 = R_3 i_1 + (-R_3) i_2$$

- $$\frac{v_2 - v_3}{R_2} = i_2$$

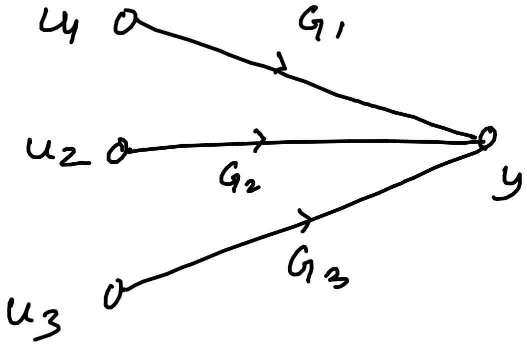
  $$\Rightarrow i_2 = \left(\frac{1}{R_2}\right) v_2 + \left(-\frac{1}{R_2}\right) v_3$$

- $v_3 = R_4 i_2$



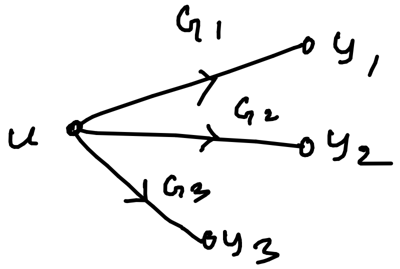
- Signal-flow graph Algebra

(1) Addition Rule



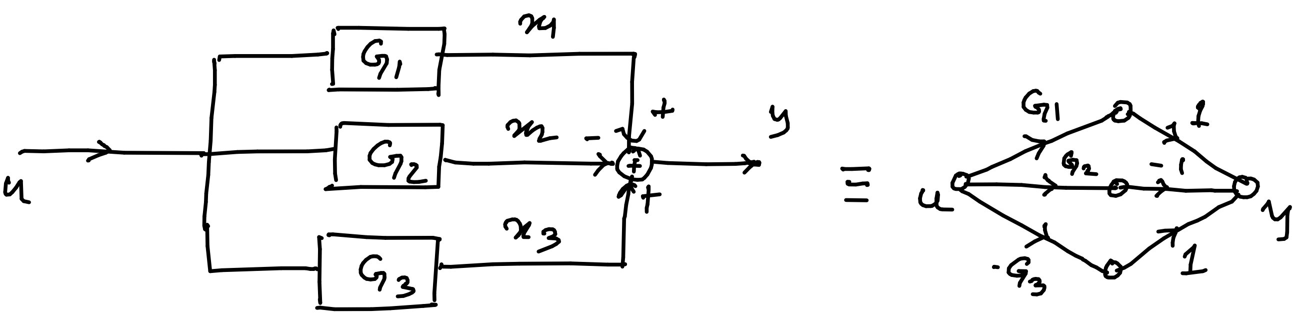
$y = G_1 u_1 + G_2 u_2 + G_3 u_3$

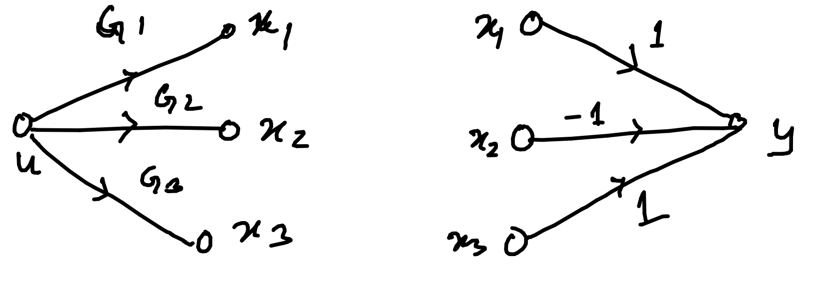
(2) Transmission Rule :

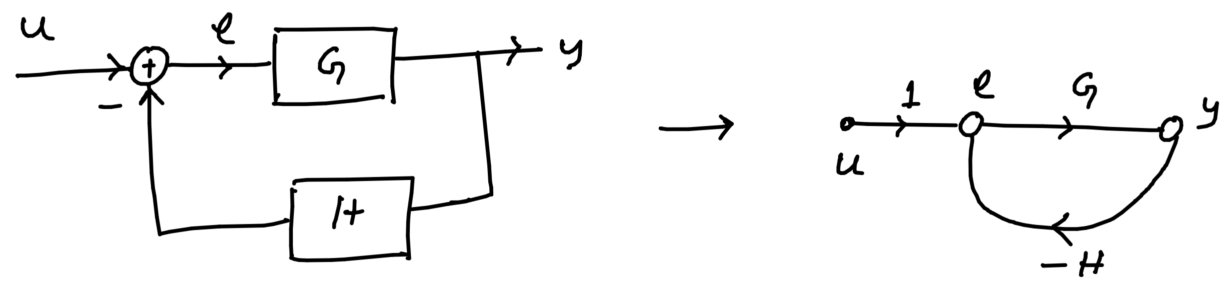


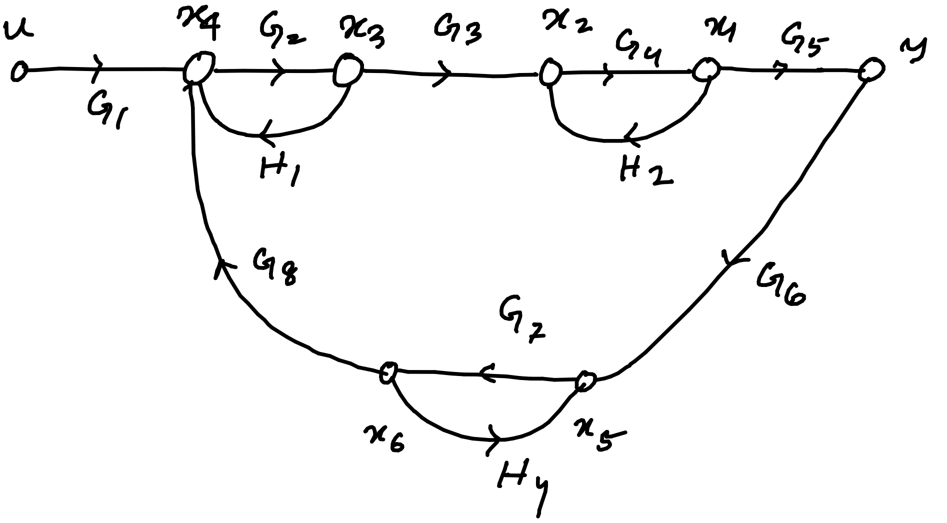
$y_1 = G_1 u$
$y_2 = G_2 u$
$y_3 = G_3 u$

(3) Multiplication Rule :









- Loop gains : The product of branch gains found by traversing a path that starts at a node & ends at the same node, without passing through any node more than once.

① $G_2 H_1$ ② $G_4 H_2$ ③ $G_7 H_4$

④ $G_2 G_3 G_4 G_5 G_6 G_7 G_8$

- Forward path gain : The product of gains found by traversing a path from input node to output node in the direction of signal flow.

  - $G_1 G_2 G_3 G_4 G_5$

- Non-touching loops : Loops that do not have any nodes common

• $G_2 H_1$ • $G_4 H_2$ • $G_7 H_4$

The overall transfer function of the system

$$\boxed{\frac{Y(s)}{U(s)} = G(s) = \frac{\sum T_k \Delta_k}{\Delta}}$$ ← Mason's Gain formula

k : the number of forward paths

T_k : the k^{th} forward path gain

$\Delta := 1 - \sum$ loop gains $+ \sum$ non-touching loop gains taken 2 at a time $- \sum$ non-touching loop gains taken 3 at a time $+ \sum$ non-touching loop gains take 4 at a time - . . . .

$\Delta_k :=$ $\Delta - \sum$ loop gains in Δ that touch the k^{th} forward path (it can be obtained by eliminating from Δ those loop gains that touch the k^{th} forward path.

$$\Delta = 1 - [G_2H_1 + G_4H_2 + G_7H_4 + G_2G_3G_4G_5G_6G_7G_8]$$
$$+ [G_2H_1G_4H_2 + G_2H_1G_7H_4 + G_4H_2G_7H_4]$$
$$- [G_2H_1G_4H_2G_7H_4$$

$$\Delta_1 = 1 - G_7 H_4$$

$$G(s) = \frac{T_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 G_4 G_5 (1 - G_7 H_4)}{\Delta}$$