Tutorial #2
ELL-225: Control Engineering

Session: Semester-II (2022-23)

1. Figure (1) shows a crane hoisting a load. Although the actual system's model is highly non-linear, if the rope is considered to be stiff with a fixed length L, the system can be modeled using the following equations :

$$
\begin{aligned}
m_L \ddot{x}_{La} &= m_L g \phi \\
m_T \ddot{x}_T &= f_T - m_L g \phi \\
x_{La} &= x_T - x_L \\
x_L &= L\phi
\end{aligned}
$$

where m_L is the mass of the load, m_T is the mass of the cart, x_T and x_L are displacements as defined in the figure, ϕ is the rope angle with respect to vertical, and f_T is the force applied to the cart.
a. Obtain the transfer function from cart velocity to rope angle $\Phi(s)/V_T(s)$
b. Find the transfer function from the applied force to the cart's position, $X_T(s)/F_T(s)$.

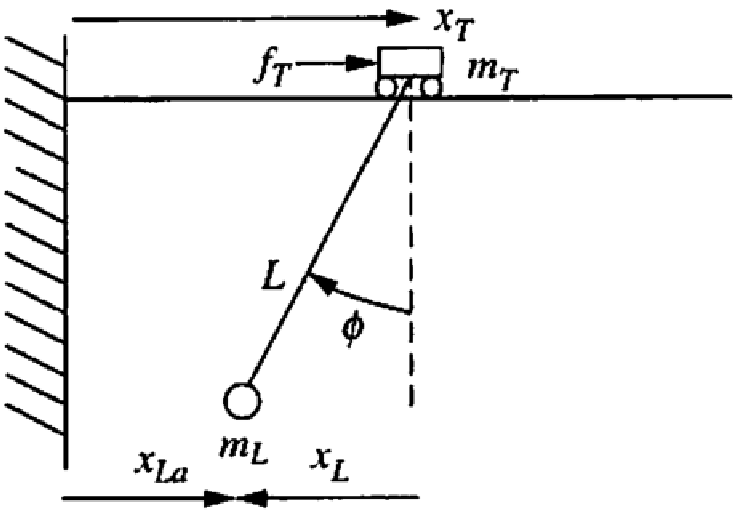

Figure 1: Crane hoisting a load.

2. Consider the restaurant plate dispenser shown in the following Fig. (2), which consists of a vertical stack of dishes supported by a compressed spring. As each plate is removed, the reduced weight on the dispenser causes the remaining plates to rise. Assume that the mass of the system minus the top plate is M, the viscous friction between the piston and the sides of the cylinder is f_v, the spring constant is K, and the weight of a single plate is W_D. Find the transfer function, $Y(s)/F(s)$, where $F(s)$ is the step reduction in force felt when the top plate is removed, and $Y(s)$ is the vertical displacement of the dispenser in an upward direction.

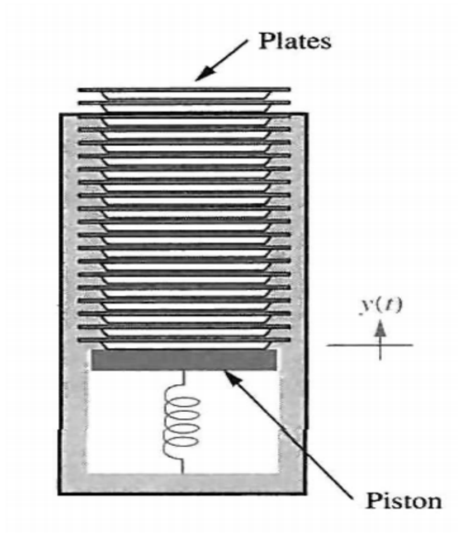

Figure 2: Plate dispenser

3. (a) Determine the transfer function $V_2(s)/V_1(s)$ for the operational amplifier shown in the Fig. (3). Assume an ideal operational amplifier.

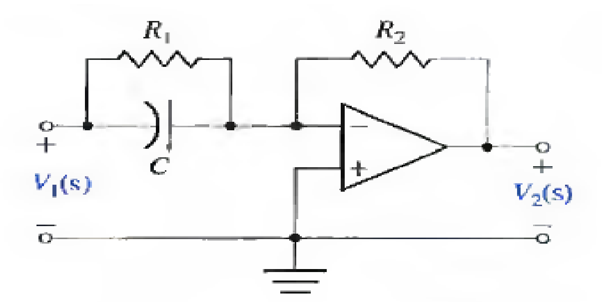

Figure 3: Op-amp circuit

(b) Find the transfer function $G(s) = V_0(s)/V_i(s)$, for operational amplifier circuit shown in Fig. (4).

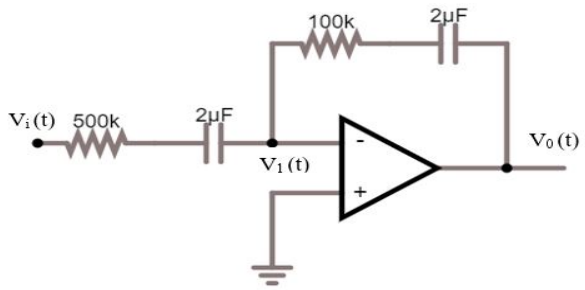

Figure 4: Op-amp circuit

(c) Find the transfer function $G(s) = V_0(s)/V_i(s)$, for operational amplifier circuit shown as in Fig. (5):

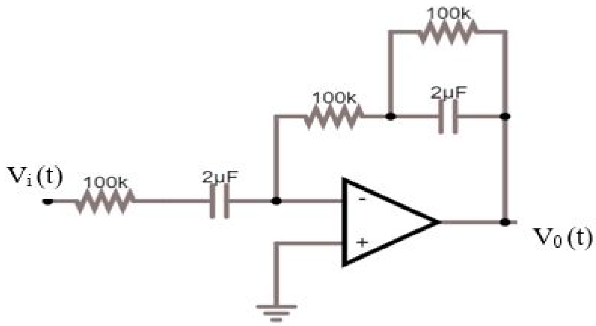

Figure 5: Op-amp circuit

4. The pantograph-catenary mechanism for high-speed rail systems can be modelled as given below using springs and viscous dampers. Here, the catenary is represented by a spring. A vertical force f_{up} is applied to the pantograph resulting in an outward force f_{out} applied to the catenary at the top. The head of the pantograph and the catenary is connected by a spring. The output force is proportional to the displacement of this spring, i.e. the vertical displacement between the catenary and the pantograph head. Find the resulting transfer function for f_{out}/f_{up}.

   Hint: f_{out} is directly proportional to spring displacement

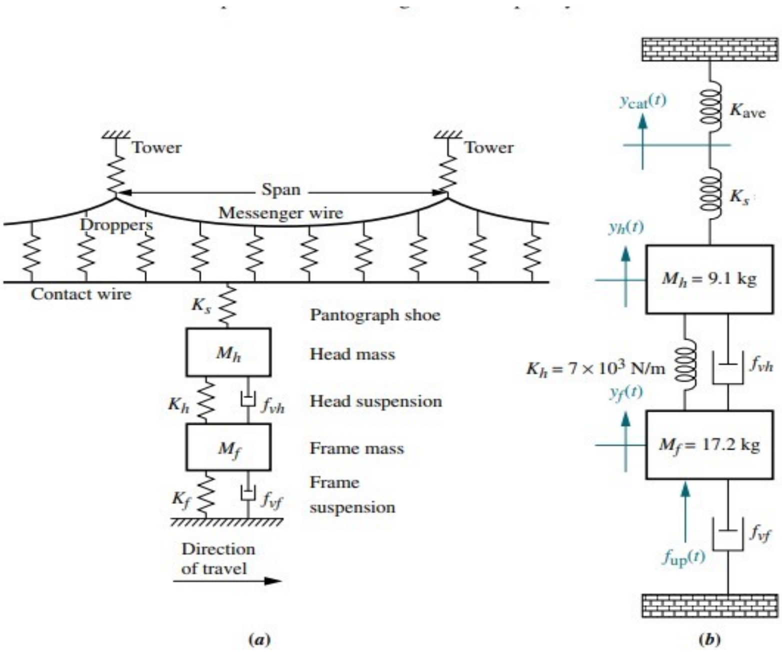

Figure 6: Pantograph-Catenary Model