Tutorial #7
ELL-225: Control Engineering

Session: Semester-II (2022-23)

1. Transportation systems that use magnetic levitation can reach very high speeds since contact friction at the rails is eliminated (see Fig. 1). Electromagnets can produce the force to elevate the vehicle. Figure 2 is a simulation model of a control system that can be used to regulate the magnetic gap.

Figure 1: A magnetic levitation transportation system.

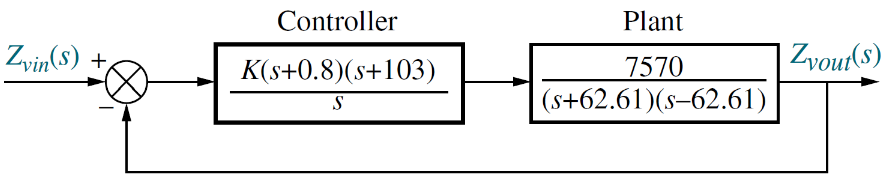

Figure 2: Simplified block diagram.

In the figure, $Z_{vin}(s)$ represents a voltage proportional to the desired amount of levitation, or gap. $Z_{vout}(s)$ represents a voltage proportional to the actual amount of levitation. The plant models the dynamic response of the vehicle to signals from the controller (refer to **(1)**). Use the Routh-Hurwitz criterion to find the range of gain, K, to keep the closed-loop system stable.

2. An interval polynomial is of the form

$$P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \dots$$

with its coefficients belonging to intervals $x_i \le a_i \le y_i$, where x_i, y_i are prescribed constants. *Kharitonov's theorem* says that an interval polynomial has all its roots in the left half-plane if each one of the following four polynomials has its roots in the left half-plane (refer to **(2)**):

$$K_1(s) = x_0 + x_1 s + y_2 s^2 + y_3 s^3 + x_4 s^4 + x_5 s^5 + y_6 s^6 + \dots$$
$$K_2(s) = x_0 + y_1 s + y_2 s^2 + x_3 s^3 + x_4 s^4 + y_5 s^5 + y_6 s^6 + \dots$$
$$K_3(s) = y_0 + x_1 s + x_2 s^2 + y_3 s^3 + y_4 s^4 + x_5 s^5 + x_6 s^6 + \dots$$
$$K_4(s) = y_0 + y_1 s + x_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + y_6 s^6 + \dots$$

Use Kharitonov's theorem and the Routh-Hurwitz criterion to find if the following polynomial has any zeros in the right-half-plane.

$$P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3$$

$$2 \leq a_0 \leq 4; 1 \leq a_1 \leq 2; 4 \leq a_2 \leq 6; a_3 = 1$$

3. Figure. (3) depicts the schematic diagram of a phase shift oscillator.

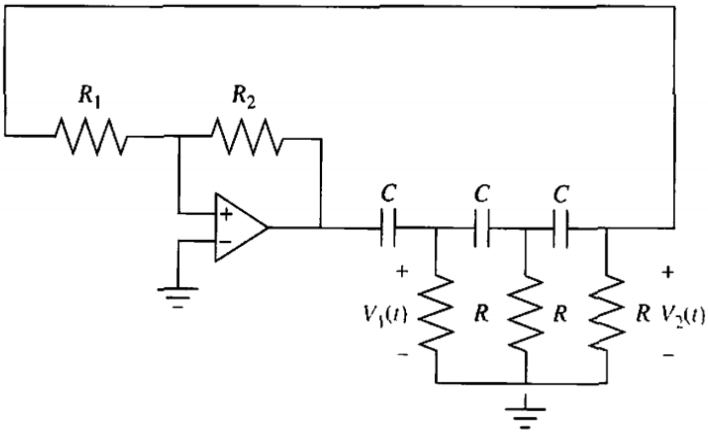

Figure 3: Phase shift oscillator.

The circuit will oscillate if it is designed to have poles on the $j\omega$-axis.

(a) Find the transfer function for the passive network $\left(\frac{V_2(s)}{V_1(s)}\right)$ in the circuit.

(b) Use the Routh-Hurwitz criterion to obtain the oscillation condition and the oscillation frequency.

4. The pantograph-catenary mechanism for high-speed rail systems can be modelled as given below in Figure 4 using springs and viscous dampers.

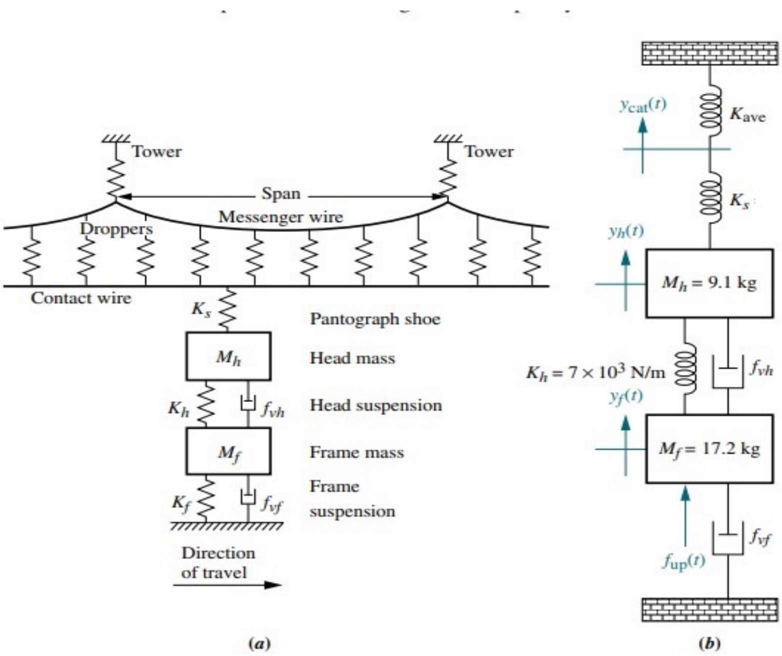

Figure 4: Pantograph-Catenary Model

Here, the catenary is represented by a spring. A vertical force f_{up} is applied to the pantograph resulting in an outward force f_{out} applied to the catenary at the top. The head of the pantograph and the catenary is connected by a spring. The output force is proportional to the displacement of this spring, i.e. the vertical displacement between the catenary and the pantograph head (refer to **(3)**). You have calculated the resulting transfer function for $G(s) = f_{out}/f_{up} = \frac{Y_h(s)-Y_{cat}(s)}{F_{up}(s)}$ in Tutorial-2 (question 4).

Now create a pantograph active-control loop by adding the following components and following the functional block diagram you found in Tutorial-1 (question 4). Input transducer ($G_i(s) = 1/100$), controller ($G_c(s) = K$), actuator ($G_a(s) = 1/1000$), pantograph spring ($K_s = 82.3 \times 10^3 N/m$), and sensor ($H_0(s) = 1/100$).

(a) Using the functional block diagram from your solution of Tutorial 1 (question 4) and the pantograph dynamics, G(s) assemble a block diagram of the active pantograph control system.

(b) Using the Routh-Hurwitz criterion, find the range of controller gain, K that will keep the system stable.

Hint: f_{out} is directly proportional to spring displacement and values of $K_{ave} = 1.535 \times 10^6 N/m$, $K_s = 82.3 \times 10^3 N/m$, $f_{vh} = 130N - s/m$, $f_{vf} = 30N - s/m$.

References

[1] A. Bittar and R. Moura Sales, "H_2 and H_∞ control for MagLev vehicles," *IEEE Control Systems Magazine*, vol. 18, no. 4, pp. 18–25, 1998.

[2] R. Minnichelli, J. Anagnost, and C. Desoer, "An elementary proof of kharitonov's stability theorem with extensions," *IEEE Transactions on Automatic Control*, vol. 34, no. 9, pp. 995–998, 1989.

[3] D. N. O'Connor, S. D. Eppinger, W. P. Seering, and D. N. Wormley, "Active Control of a High-Speed Pantograph," *Journal of Dynamic Systems, Measurement, and Control*, vol. 119, no. 1, pp. 1–4, 03 1997. [Online]. Available: https://doi.org/10.1115/1.2801209