

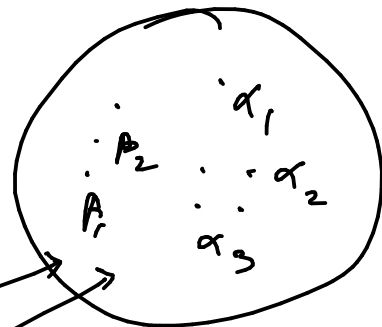
L-1

ELL7100

$\{ \underset{\uparrow}{*} \quad * \quad * \quad * \quad * \}$ Set
elements of a set

$\rightarrow \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$

$$\alpha_1 + \alpha_2 \in S$$



$\rightarrow \alpha_1, \alpha_2$

$\rightarrow \alpha_1 + \alpha_2 = \alpha_2 + \alpha_1$ Commutative

$\rightarrow \alpha_1 + (\alpha_2 + \alpha_3) = (\alpha_1 + \alpha_2) + \alpha_3$ Associative

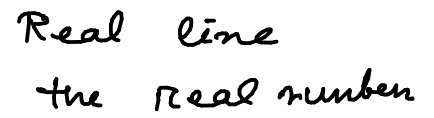
$\rightarrow \alpha_1 (\beta_1 + \beta_2) = \alpha_1 \beta_1 + \alpha_1 \beta_2$ Distributive

$\rightarrow$ For every α , $\exists (-\alpha)$ (Additive inverse)
s.t. $\alpha + (-\alpha) = 0$

$\rightarrow$ For every α , $\exists \frac{1}{\alpha}$ s.t. $\alpha \cdot \frac{1}{\alpha} = 1$
(multiplicative inverse)

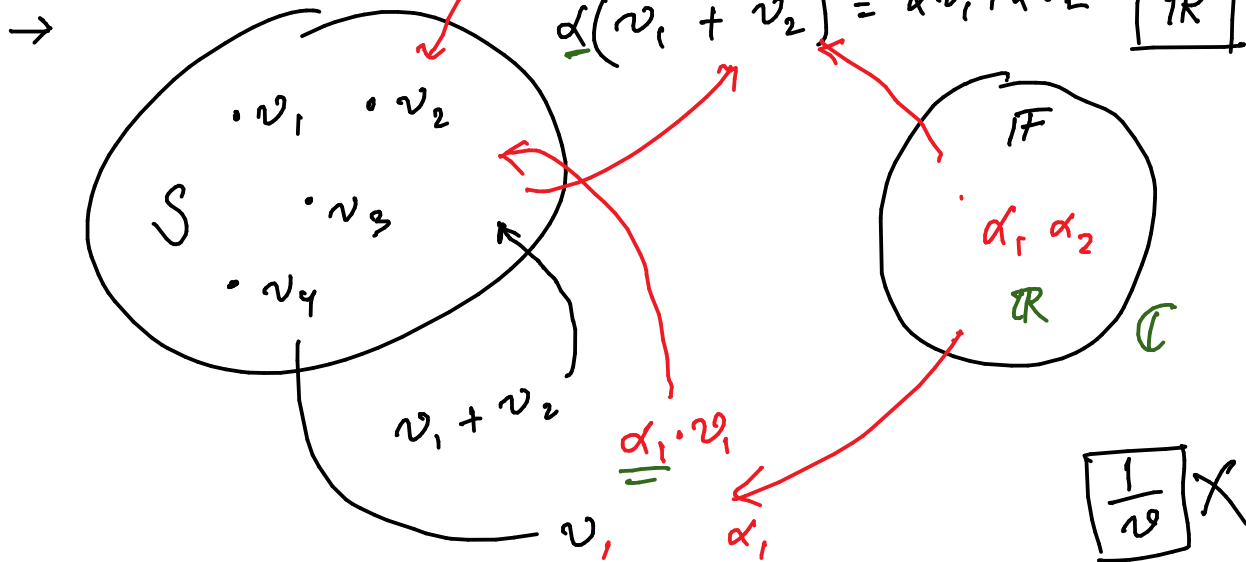
$\rightarrow \alpha + 0 = \alpha \quad \alpha \cdot 1 = \alpha$

The set 'S' satisfies above rules
"Field"



↓
Field of Real
members

$$\alpha(v_1 + v_2) = \alpha v_1 + \alpha v_2 \quad \boxed{\text{TR}}$$



$$v_1 + (-v_1) = 0$$

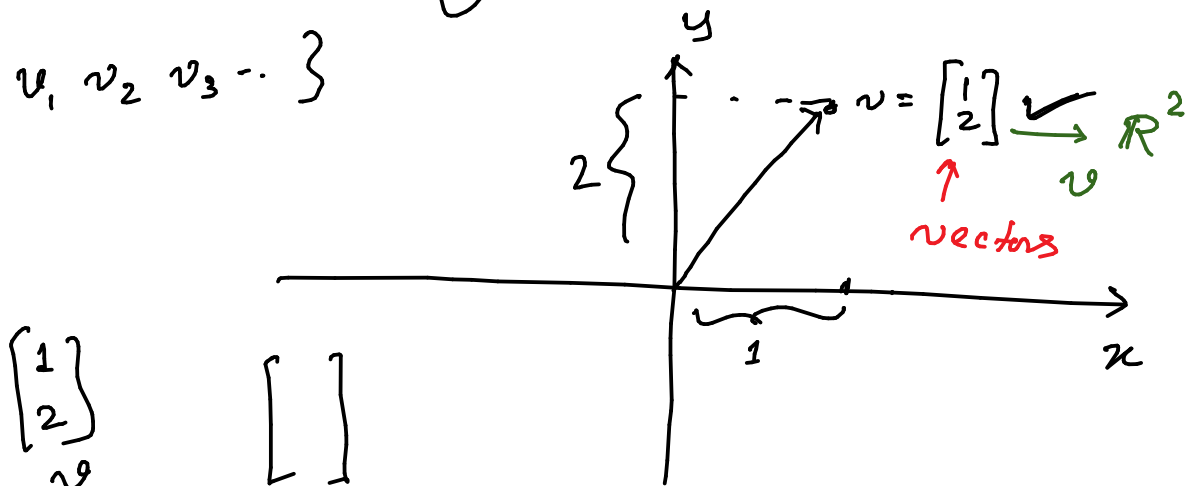
$$v_1 + v_2 = v_2 + v_1$$

$$v_1 + (v_2 + v_3) = (v_1 + v_2) + v_3$$

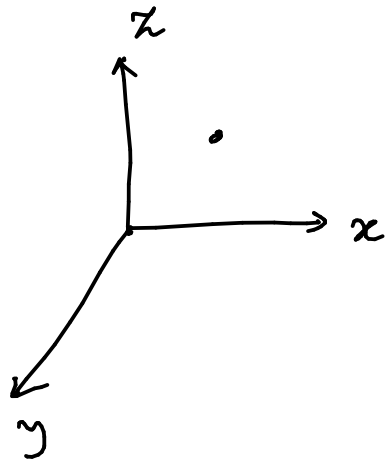
"Vector space" over field F



$$S = \{v_1, v_2, v_3, \dots\}$$



- The set of points lying on a plane is a vector space.



$$v = \begin{bmatrix} * \\ * \\ * \\ * \\ * \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

↑ vector space

$$\begin{bmatrix} * \\ * \\ * \\ \vdots \\ * \end{bmatrix}$$

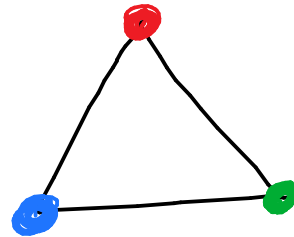
$$\rightarrow \mathbb{R}^n$$

$$n = 1, 2, 3 \dots$$

*

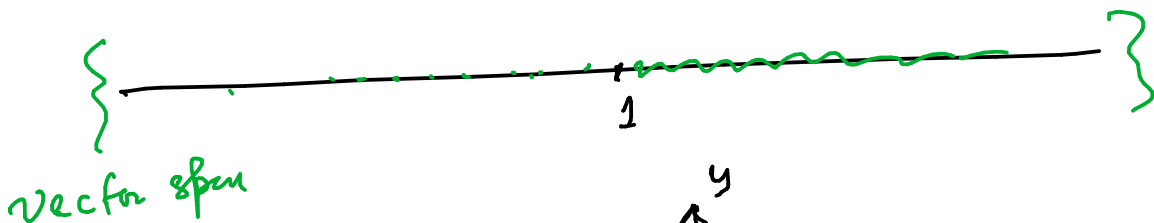
Color

Primary colors

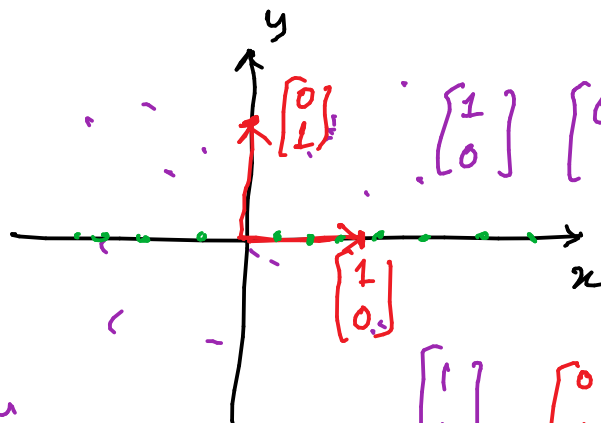


$$()R + ()G + ()B$$

"1"



vector space



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$v \rightarrow$ all elements on a plane

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \leftarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

v

$$\begin{bmatrix} 5 \\ 4 \end{bmatrix} \leftarrow \underline{5} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \underline{4} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$v_1 \qquad v_2$

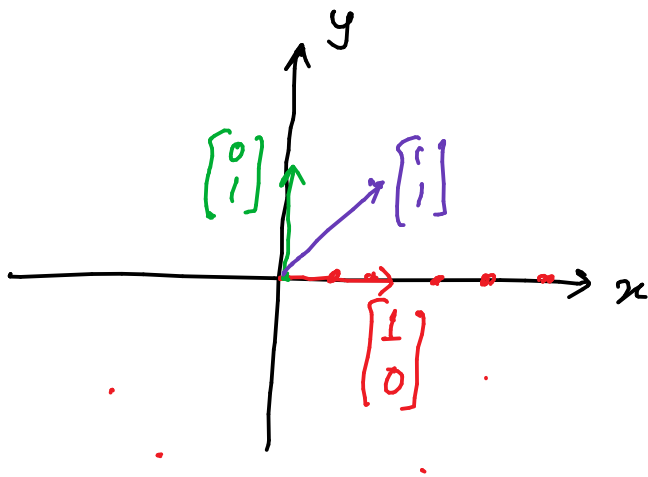
L-2

$$\mathbb{R} := \{ \sim \underline{1} \sim \}$$

$$\mathbb{R}^2 = \left\{ \begin{bmatrix} * \\ * \end{bmatrix} \begin{bmatrix} * \\ * \end{bmatrix} \dots \right\}$$

↑
vectors

$$\left\{ \sim \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sim \right\} \rightarrow \mathbb{R}^2$$



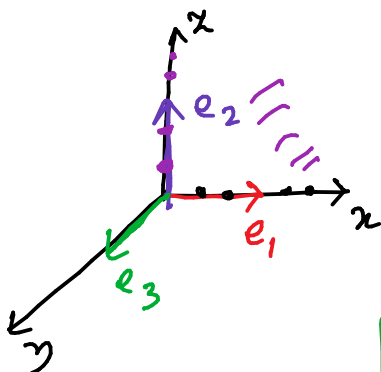
$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

v_1 v_2 v_3

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = r_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + r_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

v_3 can be obtained using
 v_1 & v_2

$$v_3 = () v_1 + () v_2$$



$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \{ e_1, e_2, e_3 \}$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} \rightarrow (r_1) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + (r_2) \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + r_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + r_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{"Basis"} = [v_1, v_2, v_3, v_4 \dots]$$

$$\mathbb{R} \rightarrow \text{Basis} \rightarrow [1], \mathbb{R}^2 \rightarrow \text{Basis} \rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$\mathbb{R}^3 \rightarrow \text{Basis} \rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

→ The redundant vectors can be obtained by taking appropriate combination of basis vectors.

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$



Redundant since it can be generated/obtained by linear combination of other two vectors

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow$ linearly dependent on $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \mathbb{R}^2 = \left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{v_2} \right\}$$

Basis $\nwarrow$ linearly independent

Linearly dependent :

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0 \quad \dots \textcircled{*}$$

$$\alpha_i \in \mathbb{R}$$



$$\begin{aligned} \alpha_1 &= 1 \\ \alpha_2 &= 1 \\ \alpha_3 &= -1 \end{aligned}$$

One can find out a
nontrivial set of α_i s.t. $\textcircled{*}$
holds.
Not all $\alpha_i \neq 0$

$$\alpha_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

→ "Dimension of a Vector Space":

||

the number of elements
in a basis

$$\mathbb{R}^3 \rightarrow \text{No of basis vectors} \rightarrow 3$$



3-Dimensional

$\mathbb{R}^n$

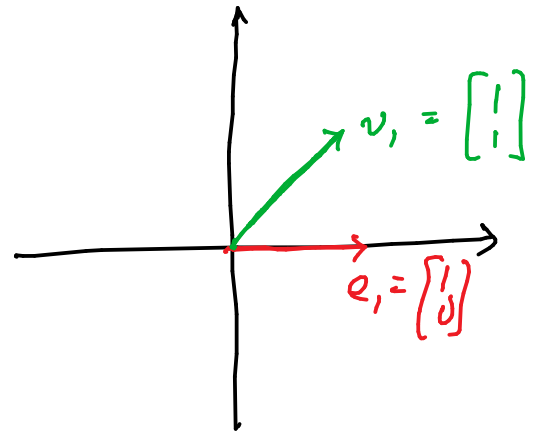
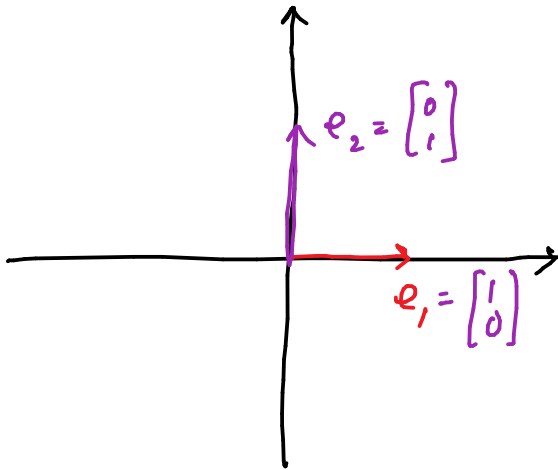
$$\mathbb{R}^2 \rightarrow \text{No of basis} \rightarrow 2$$

Vector
Space.



2-Dimensional vector
space.

→ Choice of Basis :



$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = (-1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (5) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

↑ ↑

$$\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} \text{ Basis for } \mathbb{R}^2$$

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\} \text{ Basis for } \mathbb{R}^2$$

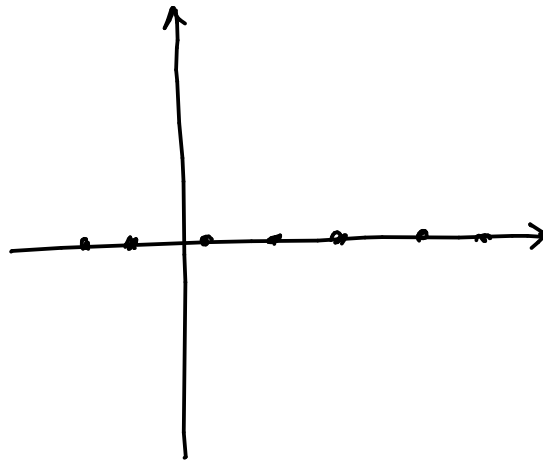
→ The choice of basis for a vector space is not "unique".

↑ One can choose any linearly independent vectors.

→ The number of basis vectors for a particular vector space is "unique".

$$\begin{bmatrix} * \\ * \\ * \end{bmatrix}$$

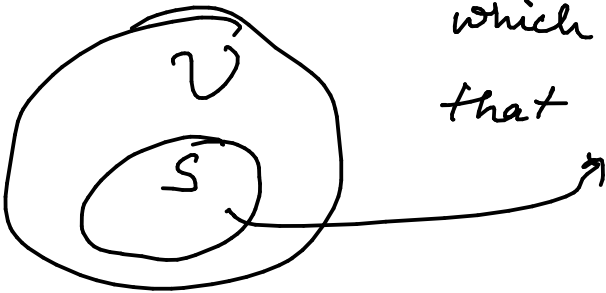
↓
Dimension



$$\begin{bmatrix} * \\ * \end{bmatrix} \in \mathbb{R}^2$$

$$\begin{bmatrix} * \\ 0 \end{bmatrix}$$

→ Subspace : The subset of a vector space which satisfies all the rules that are valid for a vector space.

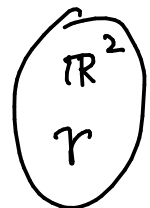
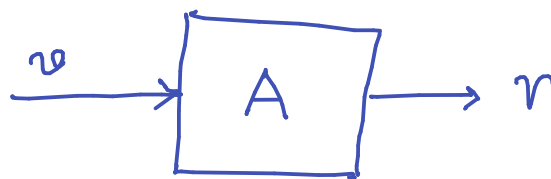
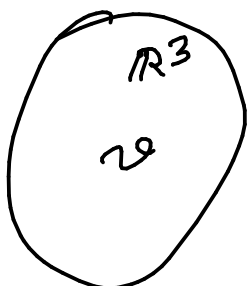


→ Matrix →

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \rightarrow * \in \mathbb{R}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}_{2 \times 3} \rightarrow \in \mathbb{R}^{2 \times 3}$$

$$\begin{matrix} A & v & = & r \\ \downarrow & \downarrow & & \downarrow \\ 2 \times 3 & 3 \times 1 & & 2 \times 1 \end{matrix}$$



$$A : \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$A \in \mathbb{R}^{2 \times 3} \rightarrow A$ is an Operator funf

L-3

Minon - 35

Major - 40

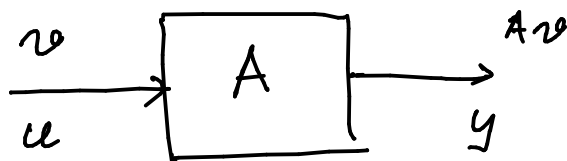
Project - 25

Report + Presentation

$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \leftarrow \text{Basis for } \mathbb{R}^2$
linearly independent

$\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

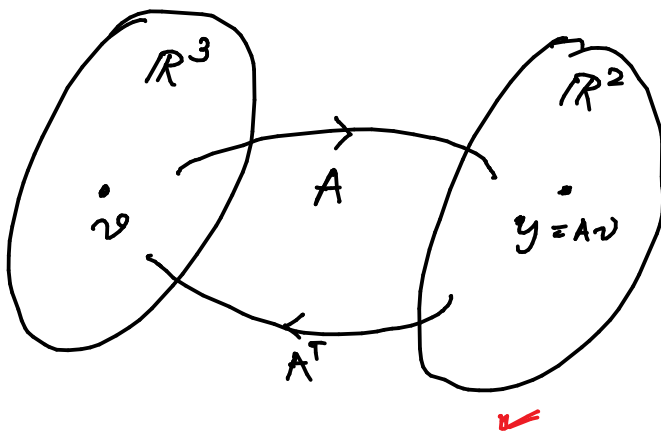
↑ The subspace generated by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



A is a matrix

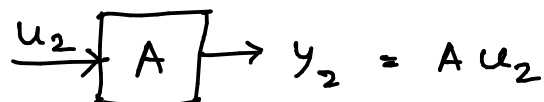
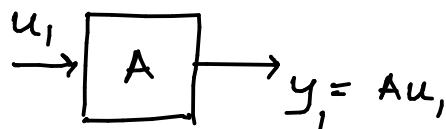
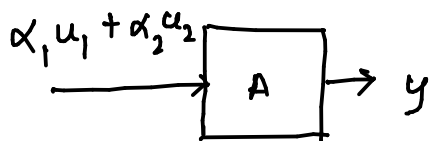
$$A \in \mathbb{R}^{m \times n}$$

↑ A is an operator



$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^T \rightarrow 3 \times 2$$



$$\begin{aligned} y &= A(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 Au_1 + \alpha_2 Au_2 \\ &= \alpha_1 y_1 + \alpha_2 y_2 \end{aligned}$$

- A is a linear operator

→

$$R(A) := \left\{ y \mid Ax = y \right\}$$

Image space of A

or
Range space
of A

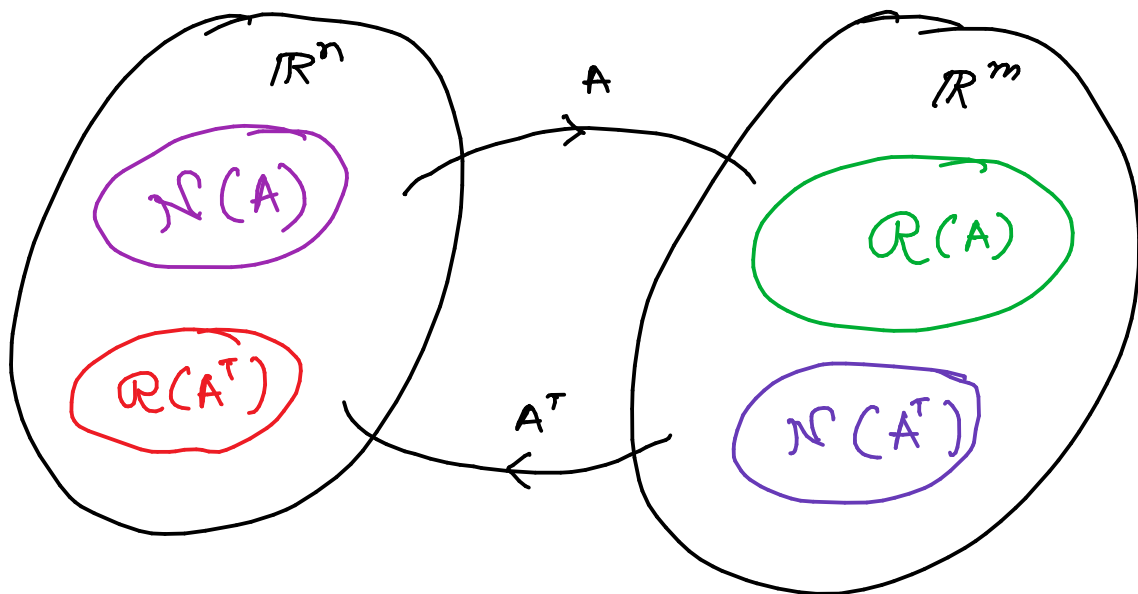
$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 4 \\ 5 \end{bmatrix}}_x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

← Column space of A
The set generated
by taking the
linear combination
of all columns
of A

$$4 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Linear combination of all
columns of A

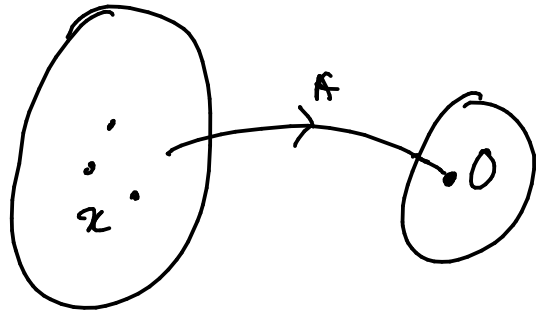
$$A \in \mathbb{R}^{m \times n}$$



$$R(A^T) := \left\{ x \mid A^T x = x \right\}$$

← Range space of A^T
//
Row space of A

$$\mathcal{N}(A) := \{ x \mid Ax = 0 \} \quad \begin{array}{l} \leftarrow \text{Null space of } A \\ \leftarrow \text{Kernel space of } A \end{array}$$



$$\mathcal{N}(A^T) := \{ z \mid A^T z = 0 \} \quad \begin{array}{l} \leftarrow \text{Null space} \\ \leftarrow \text{of } A^T \end{array}$$

$\rightarrow \underbrace{\mathcal{R}(A), \mathcal{R}(A^T), \mathcal{N}(A) \text{ \& } \mathcal{N}(A^T)}_{\text{Fundamental subspaces associated with } A.}$

→ Basis for $R(A) \subseteq \mathbb{R}^4$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ -1 & 1 & -2 & 1 \\ 0 & 3 & 0 & 6 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

$\mathbb{R} \quad \mathbb{R}$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ -1 & 1 & -2 & 1 \\ 0 & 3 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

$$\begin{matrix} 2C_1 - C_3 \\ C_1 - C_4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & -2 \\ 0 & 3 & 0 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$2C_2 + C_4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Linearly independent
↑
Basis for $R(A)$

Linearly independent
↓
Basis for $N(A)$

→ L=4

- Rank of a matrix : The number of linearly independent columns of A .

The no. of linearly independent col in A

↓

Column rank of A

- Row rank of A = the number of linearly independent rows in A

$$\begin{cases} \text{row rank}(A) = \text{column rank}(A^T) \\ \text{column rank}(A) = \text{row rank}(A^T) \end{cases}$$

Facts

(i) $\text{column rank}(A) = \text{row rank of } (A)$

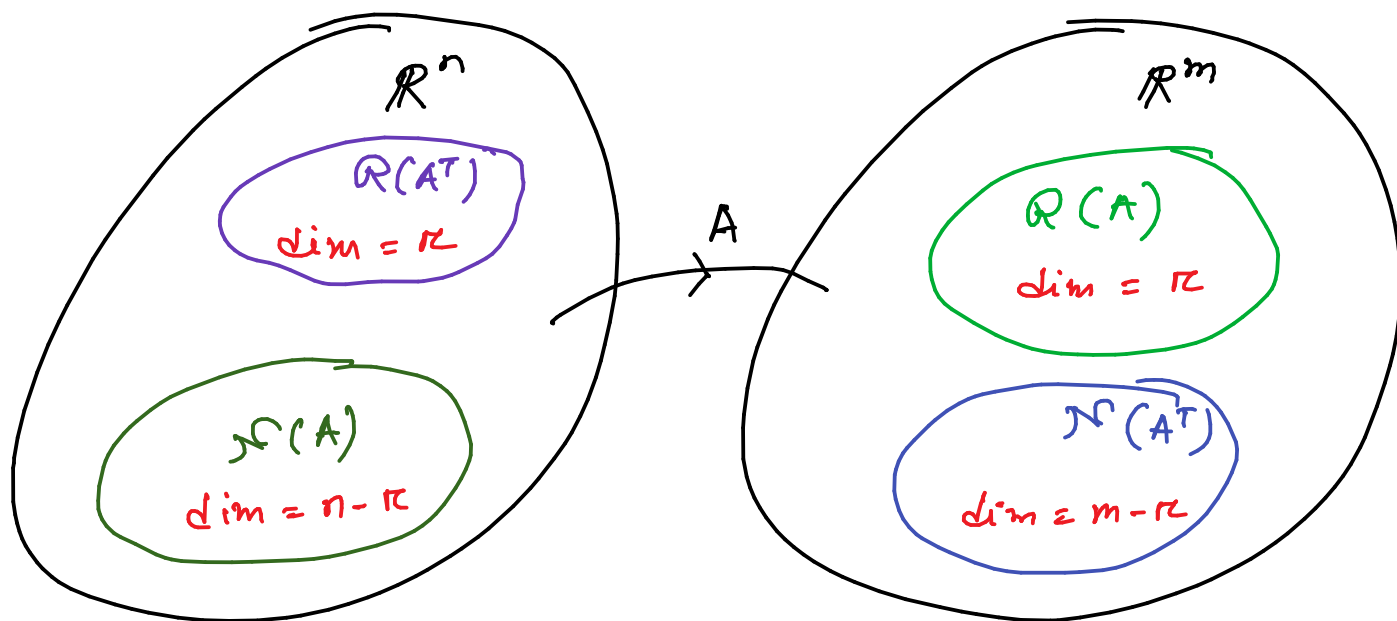
|||

$$\text{rank}(A) = \text{rank}(A^T)$$

(ii) $\text{rank}(A) = \text{rank}(A^T A) = \text{rank}(A A^T)$

(iii) For $A \in \mathbb{R}^{m \times n} \rightarrow \text{rank}(A) \leq \min(m, n)$

$$A \in \mathbb{R}^{m \times n}$$



Assume that $\text{rank}(A) = r$

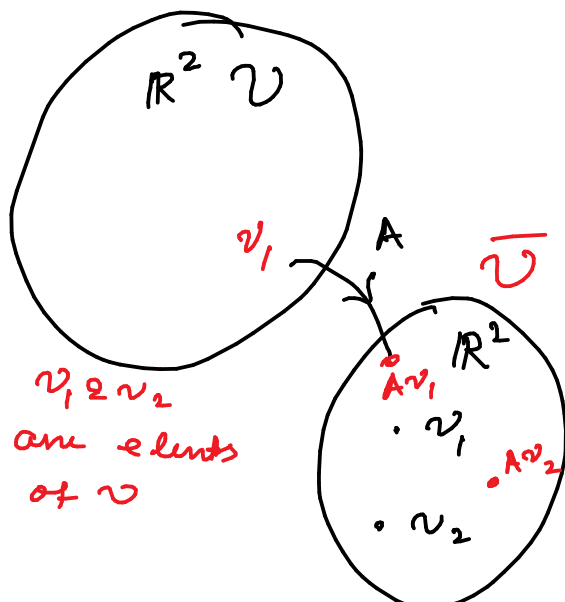
$$\rightarrow \text{rank}(A) + \dim(N(A)) = n$$

|||

$$\text{rank}(A) + \text{Nullity of } A = n$$

$\nwarrow$ rank nullity theorem

$\rightarrow$



$$Av_1 = (\alpha_1)v_1 + (\alpha_2)v_2$$

$$Av_2 = (\gamma_1)v_1 + (\gamma_2)v_2$$

v_1, v_2
are elements
of V

v_1, v_2 are basis for $\bar{V}$.

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{cases} Av_1 = \begin{bmatrix} 2 \\ -2 \end{bmatrix} = (-2)v_1 + (0)v_2 \\ Av_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix} = (0)v_1 + (4)v_2 \end{cases}$$

Basis for $\overline{V}$ space

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Basis $\overline{V}$

$$Av_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = (-2)v_1 + (3)v_2$$

$$Av_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix} = (0)v_1 + (4)v_2$$

$$\begin{aligned}
 Av_1 &= \alpha_1 v_1 + \alpha_2 v_2 & Av_2 &= r_1 v_1 + r_2 v_2 \\
 &= \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} & &= \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}
 \end{aligned}$$

$$\begin{bmatrix} | & | \\ Av_1 & Av_2 \\ | & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ \alpha_1 v_1 + \alpha_2 v_2 & & r_1 v_1 + r_2 v_2 & \\ | & & & | \end{bmatrix}$$

$$= \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} \begin{bmatrix} \alpha_1 & r_1 \\ \alpha_2 & r_2 \end{bmatrix}$$

$$\Rightarrow A \underbrace{\begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix}}_V = \underbrace{\begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix}}_V \underbrace{\begin{bmatrix} \alpha_1 & r_1 \\ \alpha_2 & r_2 \end{bmatrix}}_{\tilde{A}}$$

V is non-singular

$$\Rightarrow AV = V\tilde{A}$$

$$\Rightarrow V^{-1}AV = \tilde{A} \quad \rightarrow \quad A \text{ \& \; } \tilde{A} \text{ are similar matrices.}$$

$\rightarrow$ The matrix $\tilde{A}$ depends on the choice of basis.

For basis $\left\{ \underset{v_1}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}, \underset{v_2}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} \right\}$

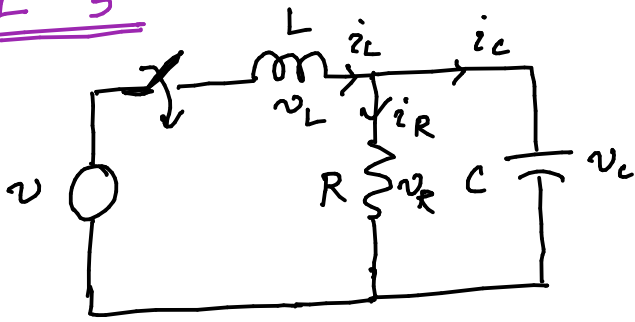
$$V = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$V^{-1}AV = \underset{\tilde{A}}{\begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix}}$$

For basis $\left\{ \underset{v_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \underset{v_2}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} \right\}$

$$\tilde{A} = \begin{bmatrix} -2 & 0 \\ 3 & 4 \end{bmatrix}$$

L-5



choose state variables:

(i) inductor current i_L

(ii) capacitor voltage v_C

$$\frac{dv_C}{dt} = \frac{1}{C} i_L - \frac{1}{RC} v_C$$

$$\frac{di_L}{dt} = -\frac{1}{L} v_C + \frac{1}{L} v$$

$$\underbrace{\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_A \underbrace{\begin{bmatrix} i_L \\ v_C \end{bmatrix}}_x + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_B \underbrace{v}_u \quad \dots \quad (1)$$

→ Another set of state variables:

(i) inductor current i_L

(ii) resistor current i_R

$$\frac{di_L}{dt} = -\frac{R}{L} i_R + \frac{1}{L} v$$

$$\frac{di_R}{dt} = \frac{1}{RC} i_L - \frac{1}{RC} i_R$$

$$\underbrace{\begin{bmatrix} \frac{di_L}{dt} \\ \frac{di_R}{dt} \end{bmatrix}}_{\dot{\tilde{x}}} = \underbrace{\begin{bmatrix} 0 & -\frac{R}{L} \\ \frac{1}{RC} & -\frac{1}{RC} \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} i_L \\ i_R \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{\tilde{B}} v \quad \dots \quad (2)$$

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} 0 & -\frac{R}{L} \\ \frac{1}{RC} & -\frac{1}{RC} \end{bmatrix}$$

↓

$$C = L = 1$$

$$R = 0.5$$

$$A = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} 0 & -0.5 \\ 2 & -2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \dot{z}_L \\ \dot{z}_R \end{bmatrix}}_{\tilde{x}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{R} \end{bmatrix}}_Q \underbrace{\begin{bmatrix} z_L \\ z_R \end{bmatrix}}_x$$

$$\tilde{x} = Qx$$

$$\dot{x} = Ax$$

$$\dot{\tilde{x}} = Q\dot{x}$$

$$Q^{-1}\dot{\tilde{x}} = A Q^{-1}\tilde{x}$$

$$\Rightarrow \dot{\tilde{x}} = \underbrace{Q A Q^{-1}}_{\tilde{A}} \tilde{x}$$

$$\tilde{A} = Q A Q^{-1}$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}_Q \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}}_{Q^{-1}} = \underbrace{\begin{bmatrix} 0 & -0.5 \\ 2 & -2 \end{bmatrix}}_{\tilde{A}}$$