ELL805

Lecture - 21

Assume that there are n - no. of agents in the n/w. The agent dynamics:

$$\dot{x}_i = u_i \qquad x_i \in \mathbb{R}, \; u_i \in \mathbb{R}$$

→ Assume a *feasible* distance specification D is given $D := \{ \ldots d_{ij} \ldots \}$

$$|x_i^* - x_j^*| = d_{ij}$$

→ For a feasible distance specification one can solve the set of distance equations & compute a set of points $x_1^*, x_2^*, \ldots x_n^*$ s.t.

$$|x_i^* - x_j^*| = d_{ij}$$

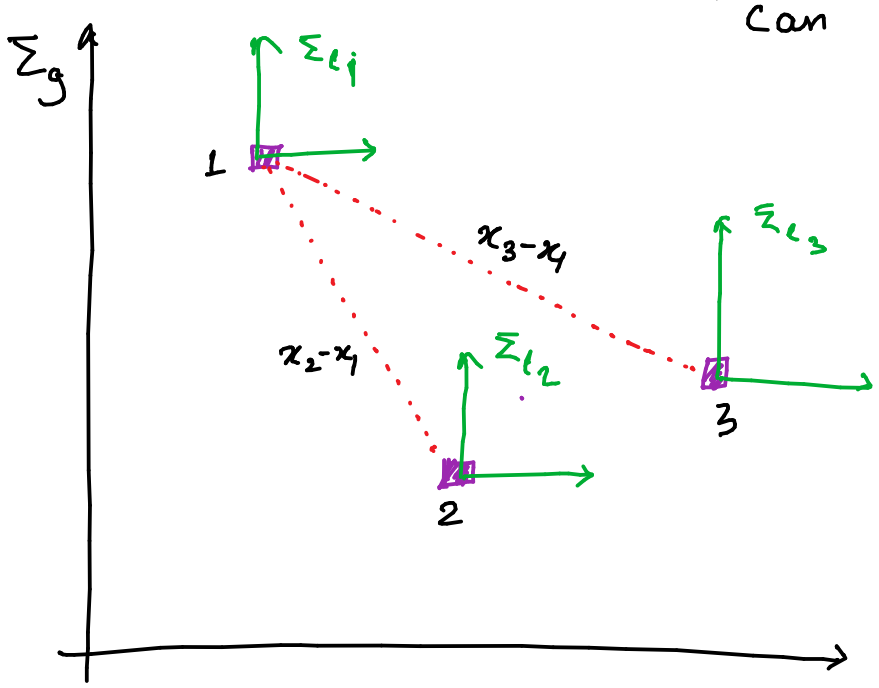
Let the starting points $x_i(0) = x_{i0}$.

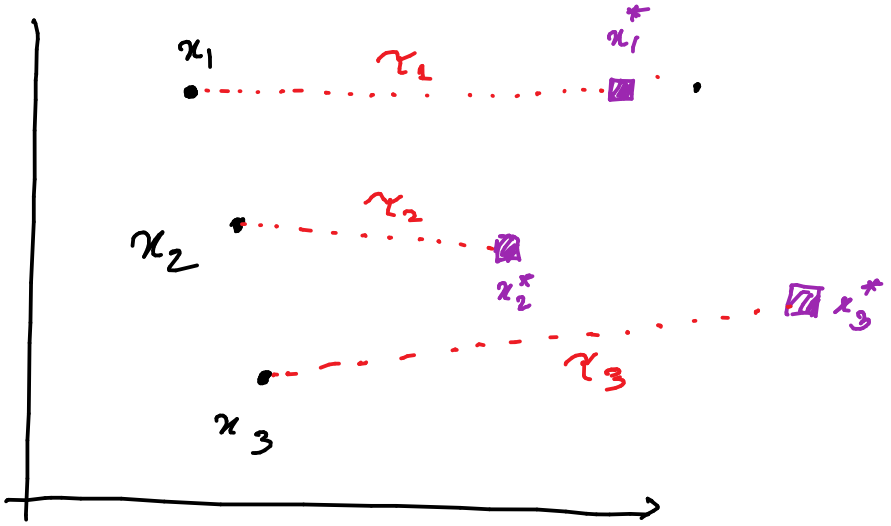
Let the global co-ordinate system be = Σ_g

Let the individual agents coordinate system (local-coordinate system) be = Σ_{l_i}

- Assume that the agents coordinate system Σ_{l_i} is aligned with global coordinate system Σ_g

Can be done using magnetic sensors.





Define a new variable

$$\gamma_i = x_i - x_i^*$$

$$\Rightarrow x_i = \gamma_i + x_i^*$$

$$\downarrow$$

Assume that at steady state

$$\gamma_1 = \gamma_2 = \gamma_3 = \cdots = \gamma_n$$

At steady state

$$|x_i - x_j| = |\gamma_i + x_i^* - \gamma_j - x_j^*|$$

$$= |x_i^* - x_j^*|$$

$$= d_{ij}$$

→ If γ_i's reach to a consensus, then desired formation can be realized.

$$x_i = \gamma_i + x_i^*$$

$$\dot{x}_i = \dot{\gamma}_i = u_i$$

Let the control signal be produced as follows

$$u_i = \gamma \left[\sum_{j \in \mathcal{N}_i} (\gamma_j - \gamma_i) \right]$$

γ is a +ve scalem

$\mathcal{N}_i$: ith agent's neighborhood set

In terms of original variables

$$u_i = \gamma \left[\sum_{j \in \mathcal{N}_i} (x_j - x_j^* - x_i + x_i^*) \right]$$

$$= \gamma \left[\sum_{j \in \mathcal{N}_i} (x_j - x_i) + (x_i^* - x_j^*) \right]$$

$(x_j - x_i)$: measurable points.

$(x_i^* - x_j^*)$: given points, which can be considered as ref. pt.

$$\dot{r}_i = u_i$$

$$= \gamma \left[\sum_j (r_j - r_i) \right]$$

$$\Rightarrow \dot{r} = -\gamma L r \quad \cdots (*) \qquad r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$$

L: graph Laplacian matrix.

→ Under the assumption that the n/w graph is undirected & connected, the protocol (*) reach to a consensus.

At steady state ($t \to \infty$)

$$r_1 = r_2 = \cdots = r_n$$

$$\Rightarrow |x_i - x_j| = d_{ij}$$

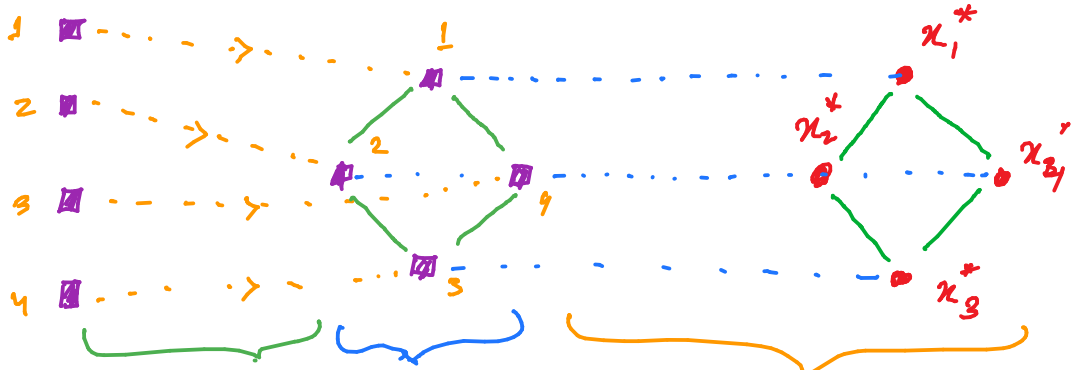
$$\gamma_i = x_i - x_i^*$$

$$\Rightarrow \quad x_i = \gamma_i + x_i^*$$

At steady state $\gamma_1 = \gamma_2 = \cdots = \gamma_n = \gamma$

$$\Rightarrow \quad \begin{aligned} x_1 &= \gamma + x_1^* \\ x_2 &= \gamma + x_2^* \\ &\vdots \\ x_n &= \gamma + x_n^* \end{aligned}$$

Translation invariant formation.



How quickly one can reach to the desired formation, is determined by γ.

Desired formation.

Translation of the formation in the direction γ to reach x_i^*.

→ Allowing the inter-agent communication, we could move the formation in a certain direction, which is determined by γ.

↑

Translation of a formation

- The proposed control can not ensure collision avoidance among the agents, before reaching to a formation.

→ For instance in a multi-robot system, if the robots are not placed appropriately, then there is a possibility of collision among them, before reaching to the desired formation.