# ELL805

## Lecture - 22

→ Formation specifications

↳ Exact position / state information based specificat.

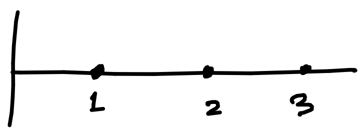
↳ Feasible distance / displacement based specificati.

↳ Relative state based specificat.

→ <u>Relative State Specification (R.S.S)</u>

Agent dynamics $\dot{x}_i = u_i$ $\quad x_i \in R$ is the state of $i^{th}$ agent.

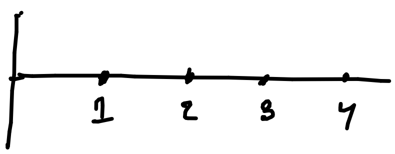
3 - agent system



$$\text{R.S.S.} \quad z^* = \begin{bmatrix} x_2^* - x_1^* \\ x_3^* - x_2^* \end{bmatrix}$$

the 3rd component $x_3^* - x_1^*$ is implicitely specified by the above two.

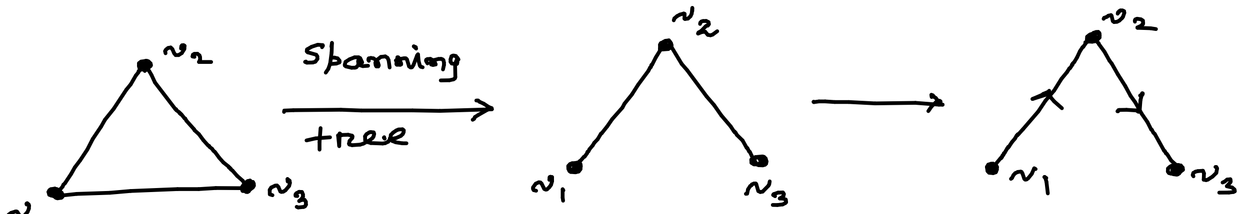
- 4 - agent system



$$z^* = \begin{bmatrix} x_2^* - x_1^* \\ x_3^* - x_2^* \\ x_4^* - x_3^* \end{bmatrix}$$

→ This specification does not specify the exact geometrical pattern (<u>shape</u>) of the formation.

- Representation of R.S.S.




original graph of communication n/w

Directed spanning tree of the original graph

Construct the incidence matrix of directed spanning tree

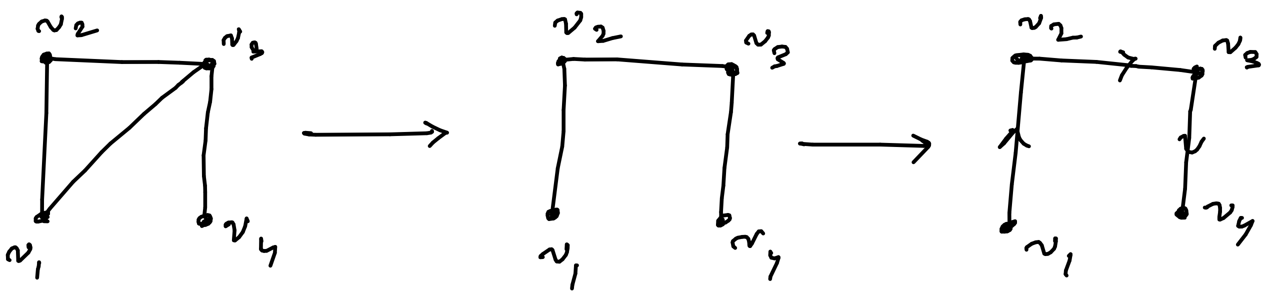
$$D = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

The R.S.S.

$$\boxed{z^* := D^T x^*} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix}$$

$$= \begin{bmatrix} x_1^* - x_2^* \\ x_3^* - x_1^* \end{bmatrix}$$

→ 4-agent system with following communication topology




Incidence matrix

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} x_2^* - x_1^* \\ x_3^* - x_2^* \\ x_4^* - x_3^* \end{bmatrix}$$

Given a R.S.S. $(z^*)$, we can represent it as follows

$$\boxed{z^* = D^T x^*}$$

→ Given the agent dynamics

$$\dot{x}_i = u_i$$

Objective : Design $u_i$ s.t.

$$\underset{z}{(x_i - x_j)} \xrightarrow[t\to\infty]{} \underset{z^*}{(x_i^* - x_j^*)}$$

Define a vector $z := D^T x$

We need $z \longrightarrow z^*$

Define an error vector

$$\delta = z - z^*$$

$$\dot{\delta} = \dot{z} = D^T \dot{x} = D^T u$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Let the control law is as follows.

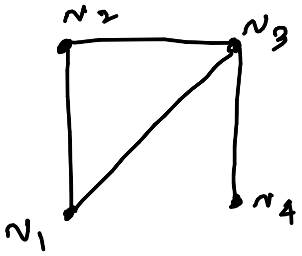
$$\boxed{u = -\gamma D \delta} \leftarrow \text{Distributed in nature}$$

$$u = -\gamma D(D^T x - D^T x^*)$$

$$= -\gamma D D^T (x - x^*)$$

$$= -\gamma L (x - x^*)$$

$L \rightarrow$ Laplacian matrix




$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$u = -\gamma L (x - x^*)$$

$$u_1 = -\gamma \left[ 2(x_1 - x_1^*) - (x_2 - x_2^*) - (x_3 - x_3^*) \right]$$

$$= \gamma \left[ \left( (x_2 - x_2^*) - (x_1 - x_1^*) \right) + \left( (x_3 - x_3^*) - (x_1 - x_1^*) \right) \right]$$

$$= \gamma \left[ \left( (x_2 - x_1) - (x_2^* - x_1^*) \right) + \left( (x_3 - x_1) - (x_3^* - x_1^*) \right) \right]$$

specified.

measurable neighborhood information.

$$u_2 = \gamma \left[ \left( (x_1 - x_2) - (x_1^* - x_2^*) \right) + \left( (x_3 - x_2) - (x_3^* - x_2^*) \right) \right]$$

$u_3$ :=

$\vdots$

$u_4$ :=

Then error dynamics $\dot{\delta} = D^T u$

Edge Agreement Protocol

$$\dot{\delta} = -\gamma D^T D \delta$$

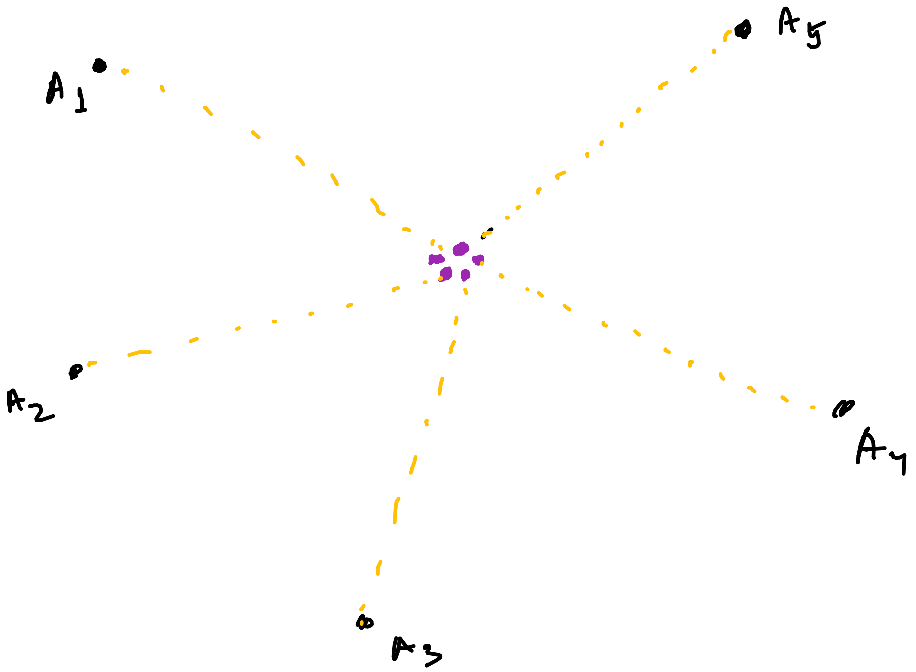
$$= -\gamma L_e \delta \qquad L_e : \text{Edge Laplacian matrix}$$

We have already studied that

$$\delta(t) \longrightarrow 0 \quad \text{as } t \to \infty$$

$$\Rightarrow \quad x - x^* \longrightarrow 0 \quad \Rightarrow \quad (x_i - x_j) \xrightarrow[t \to \infty]{} (x_i^* - x_j^*)$$

→ <u>Rendezvous Problem</u>




$A_i$ : $i$th agent.

Objective is to reach to a common point.

The RSS for this problem $x^* = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

$$x_i^* - x_j^* = 0$$

$$\dot{x}_i = u_i$$

$$\delta = z - z^*$$

$$\Rightarrow \delta = z$$

$$\dot{\delta} = D^T \dot{x} \Rightarrow \dot{z} = D^T \dot{x}$$

$$= -r L_e z$$

We need $z \longrightarrow 0 \Rightarrow x_i - x_j \longrightarrow 0$ as $t \to \infty$

$$\delta = z = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ \vdots \end{bmatrix}$$

Since it is an Edge-agreement protocol,

We can achieve $x_i - x_j = 0$ as $t \to \infty$

This is also a consensus problem.