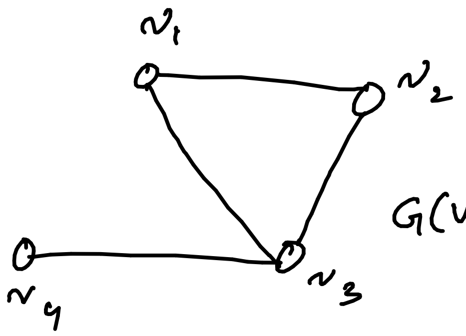
ELL 805

Lecture - 2

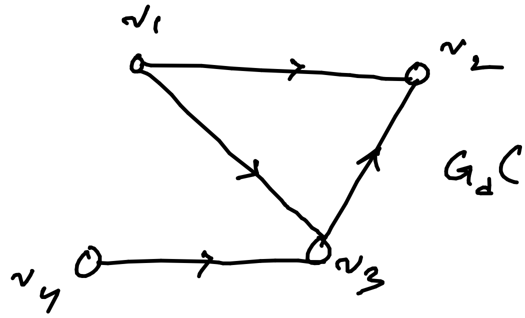
Multiagent System

Represented by graphs
directed / undirected
(depends on nature of communication)



$G(V,E)$

undirected Graph
(bidirectional communication between the agents)



$G_d(V,E)$

Directed graph
(unidirectional communication)

Graph : vertex set $V := \{v_1, v_2 \cdots v_n\}$

$G(V,E)$ Edge set $E := \{(v_1, v_2), (v_2, v_3) \cdots\}$

(v_1, v_2)

Initial vertex of the edge

Final vertex of the edge

can also be referred to as "**End vertices**".

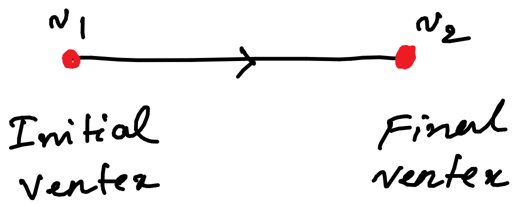
For an undirected graph, initial & final vertices can interchangibly be used.

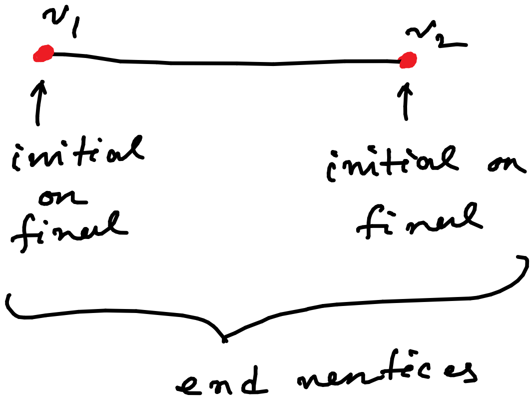
- <u>Adjacent Vertices</u> : If there exists an edge between the vertices v_i & v_j, then v_i & v_j are called adjacent vertices

Ex : v_1 & v_3 are adjacent
v_1 & v_4 are not adjacent } in previous figure.

- <u>Incident vertex</u> : A vertex v_i is incident with an edge (v_i, v_j) if it is either the initial or final vertex of that edge (v_i, v_j)

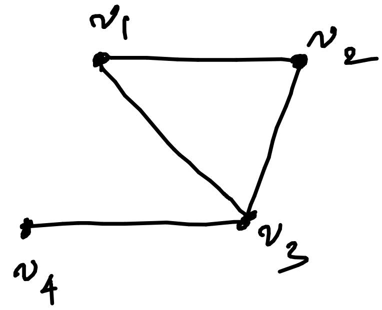
- <u>Directed edge</u> : If there is a direction associated with an edge in the directed graph $G_d(V, E)$ then it is called directed edge.





- Neighborhood set of a vertex v_i

$$\mathcal{N}(i) := \{ v_j \in V \mid v_i \; \& \; v_j \text{ are adjacent} \}$$



Neighborhood set of

$v_2 \rightarrow \mathcal{N}(2) := \{ v_1, v_3 \}$

$v_4 \rightarrow \mathcal{N}(4) := \{ v_3 \}$

- Path : A path between the vertices v_i & v_j is a sequence of vertices where all of the following conditions hold:

(i) no edge enters at v_i

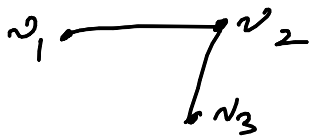
initial vertex of the path

(ii) no edge leaves from v_j

final vertex of the path

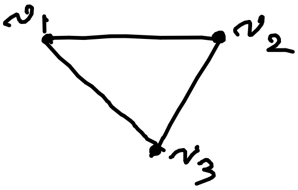
(iii) only one edge enters & one edge leaves from the intermediate vertices.

path $\{ v_1 \; v_2 \; v_3 \} \rightarrow$ the associated edges are : $\{ (v_1, v_2), (v_2, v_3) \}$



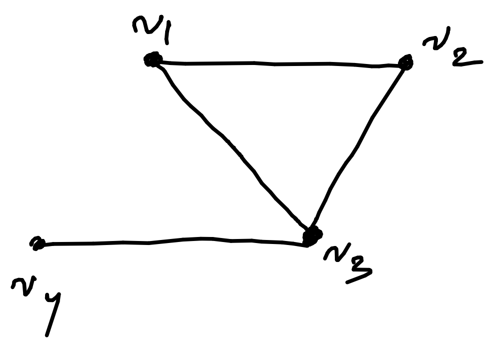
- For a directed graph (digraph), same definition can be used to define a <u>directed path</u>.

- Cycle: It is a closed path in $G(V,E)$ or $G_d(V,E)$ whose initial & final vertices are same.

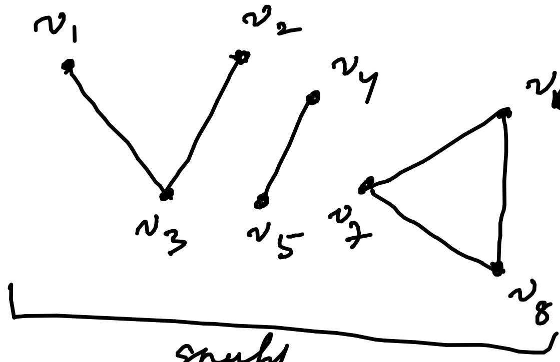


- <u>Tree</u>: A graph on 'n' vertices has $n-1$ edges, the the graph is a tree.

- <u>Connected graph</u>: A graph $G(V,E)$ is connected if for <u>every</u> pair of vertices v_i & v_j in $G(V,E)$, there is a path between v_i & v_j, where v_i & v_j are end vertices of that path.

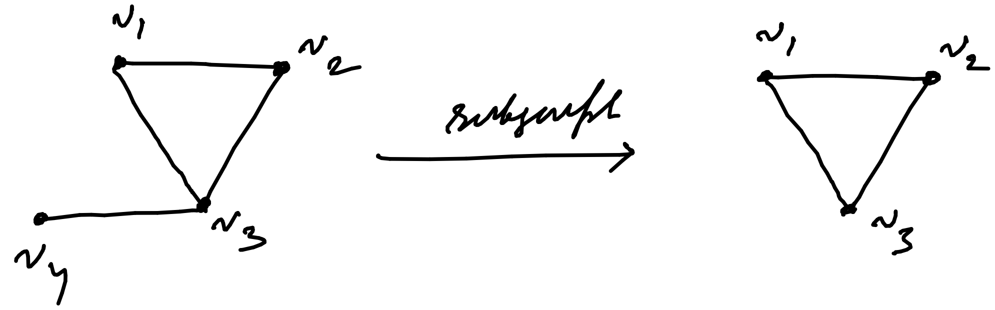


Connected graph



Dis connected graph.

- <u>Subgraph</u> : A graph $G(V', E')$ is a subgraph of graph $G(V, E)$ if
$$V' \subseteq V \text{ and } E' \subseteq E.$$



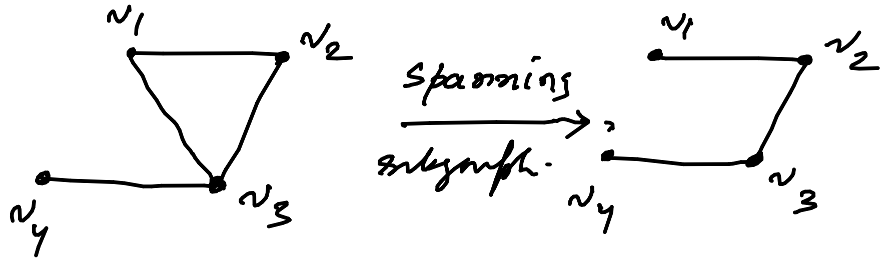
$V = \{v_1, v_2, v_3, v_4\}$

$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_1), (v_3, v_4)\}$

$V' = \{v_1, v_2, v_3\}$

$E' = \{(v_1, v_2), (v_2, v_3), (v_3, v_1)\}$

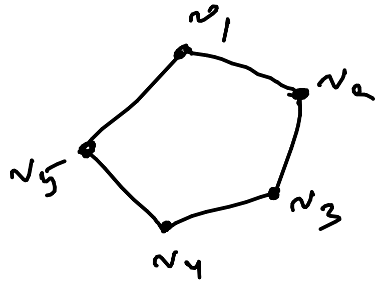
- <u>Spanning Subgraph</u> : A subgraph $G(V', E')$ is a spanning subgraph of $G(V, E)$ if $V' = V$ and $E' \subseteq E$.



- <u>Connected Components</u> : In a disconnected graph one can find multiple connected subgraphs. The connected subgraphs are called connected components of the original disconnected graph.

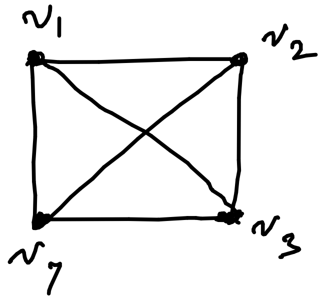
- In a connected graph, there is only one connected component.

- Cyclic graph : It is a connected graph where each vertices has exactly two neighbors.



Cyclic graph on 'n' vertices, denoted as C_n.

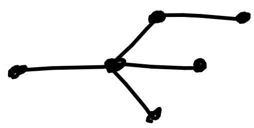
- Complete graph : A graph is complete if there exists an edge between every two vertices.



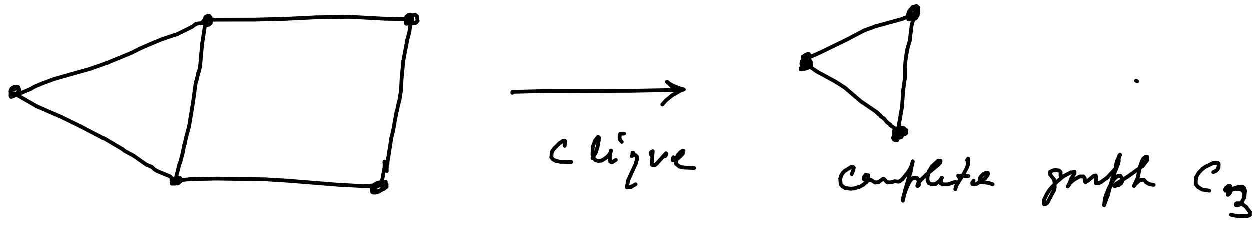
Complete graphs on 'n' vertices are denoted as K_n.

- Acyclic graph : A graph $G(V, E)$ is acyclic if it has no cyclic subgraphs.

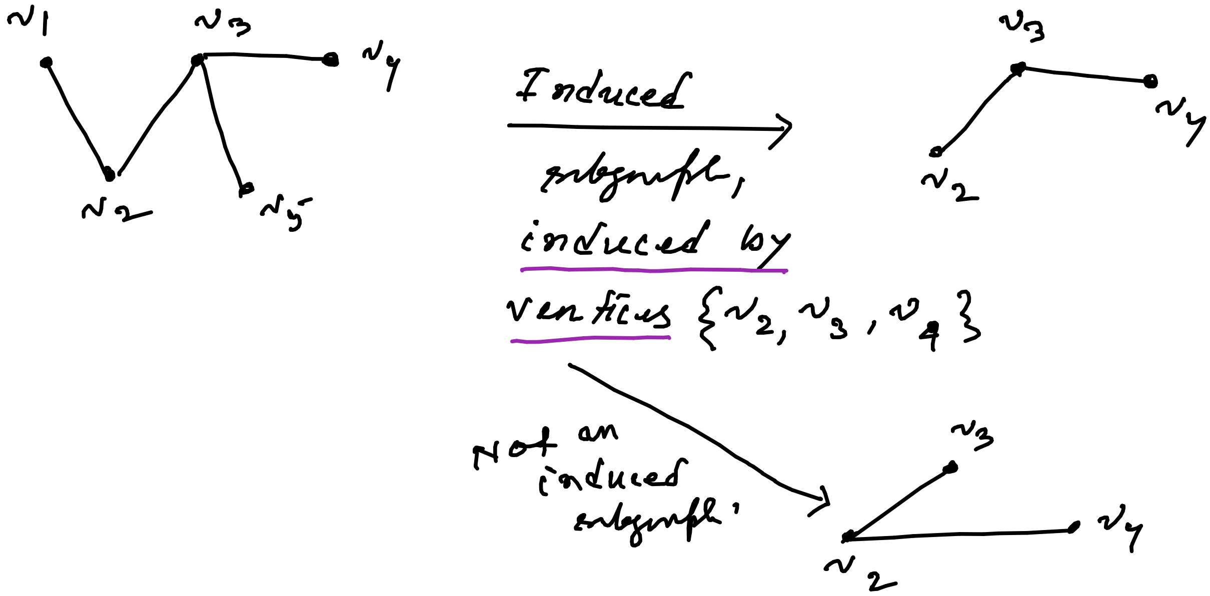
(Ex : tree)



- <u>Clique</u>: A clique is a subgraph of $G(V,E)$ which is a complete graph.



- <u>Induced Subgraph</u>: A subgraph $G(V',E')$ of a graph $G(V,E)$ is said to be an induced subgraph of $G(V,E)$ if following condition hold:
    - two vertices of $G(V',E')$ are adjacent in $G(V',E')$ if they are also adjacent in $G(V,E)$.



- Any induced subgraph of $G(V,E)$ can be obtained by deleting some of the vertices of $G(V,E)$ along with the edges that are incident to the deleted vertices.