

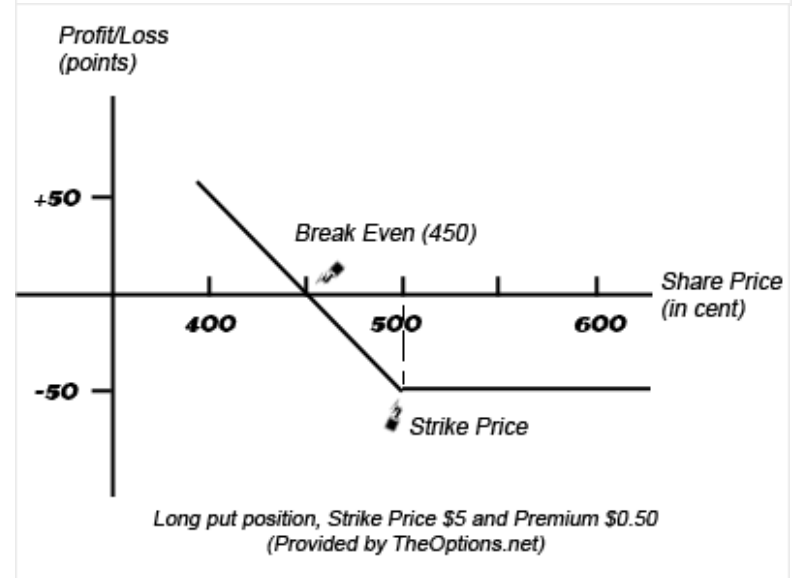
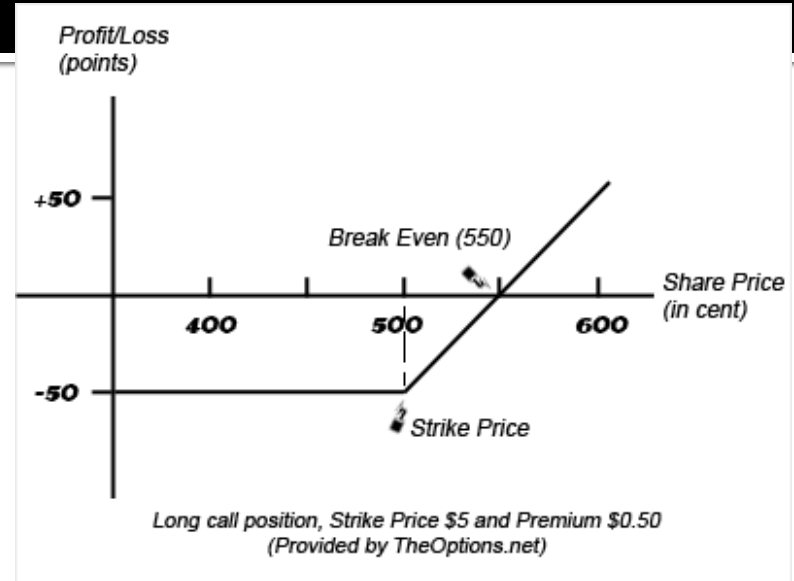
Machine Learning methods for Option Pricing

EEL 709
Pattern Recognition

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An Introduction to Options

- What are options?
 - Derivatives
 - Puts and Calls
 - Payoff profile
- Types of options
 - European
 - American
 - Exotics



Black Scholes Pricing

$$C(S, t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

where:
$$d_1 = \frac{\ln(S / K) + (r + \frac{1}{2} \sigma^2)(T - t)}{\sigma \sqrt{T - t}}$$

$$d_2 = d_1 - \sigma \sqrt{T - t}$$

$N()$ is the cumulative distribution function of the standard normal distribution

$(T-t)$ is the time to maturity

S is the spot price of the underlying asset

K is the strike price

r is the risk free rate (annual rate, expressed in terms of continuous compounding)

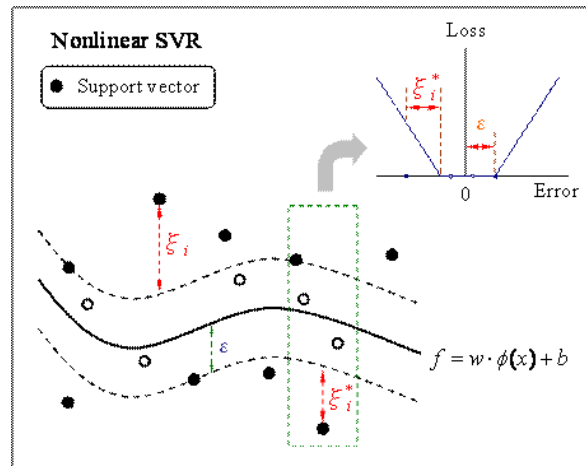
σ is the volatility of returns of the underlying asset

Why Machine Learning?

- The Black Scholes model is only valid for European Options and under certain specific conditions.
- It is possible to borrow and lend cash at a known constant risk free interest rate.
- It is possible to buy and sell any amount, even fractional, of stock.
- There are no transaction costs involved.
- The stock price follows a Geometric Brownian Motion with constant drift and volatility
- The underlying asset does not pay a dividend.
- Black Swans

Approach Used: Support Vector Regression

- SVR: epsilon-intensive function minimization



- Kernel Used:
 - RBFkernel : $K(x_i; x_j) = \exp(-\gamma \|x_i - x_j\|^2); \gamma > 0$

Data

S&P CNX Nifty CE (83518 data points)

- Feature Set:
 - Strike Price
 - Time to Maturity
 - Implied Volatility
 - Current Stock Price
 - Last Day Price
 - Black Scholes Price

Date	Expiry	Strike Price	Open	High	Low	Close	LTP	Settle Price	No. of contracts	Turnover in Lacs	Open Int	Change in OI	Underlying Value
40441.00	40479.00	4800.00	1122.00	1190.00	1120.00	1186.70	1190.00	1186.70	82.00	244.61	45950.00	3600.00	5980.45
40442.00	40479.00	4800.00	1214.10	1240.00	1191.00	1208.05	1208.05	1208.05	53.00	159.59	48300.00	2450.00	6009.05
40443.00	40479.00	4800.00	1206.00	1222.00	1175.00	1213.05	1209.75	1213.05	200.00	600.98	41800.00	-6500.00	5991.00
40444.00	40479.00	4800.00	1190.00	1207.00	1159.00	1181.90	1179.00	1181.90	97.00	290.20	42350.00	550.00	5959.55
40445.00	40479.00	4800.00	1175.05	1250.00	1175.05	1243.35	1235.00	1243.35	85.00	255.68	46250.00	3900.00	6018.30
40448.00	40479.00	4800.00	1271.00	1294.00	1250.45	1258.30	1260.00	1258.30	352.00	1068.86	62450.00	16200.00	6035.65
40449.00	40479.00	4800.00	1254.50	1271.00	1210.00	1247.15	1266.00	1247.15	503.00	1520.22	87150.00	24700.00	6029.50
40450.00	40479.00	4800.00	1262.00	1265.00	1202.00	1218.10	1212.50	1218.10	816.00	2460.24	125800.00	38650.00	5991.30
40451.00	40479.00	4800.00	1200.00	1230.00	1172.00	1215.90	1218.45	1215.90	4185.00	12532.75	327150.00	201350.00	6029.95
40452.00	40479.00	4800.00	1269.00	1380.00	1269.00	1365.55	1365.50	1365.55	599.00	1841.82	314150.00	-13000.00	6143.40
40455.00	40479.00	4800.00	1411.00	1442.60	1372.20	1374.10	1374.00	1374.10	114.00	353.44	311250.00	-2900.00	6159.45
40456.00	40479.00	4800.00	1377.75	1412.15	1371.70	1374.85	1375.00	1374.85	73.00	225.97	309000.00	-2250.00	6145.80
40457.00	40479.00	4800.00	1413.05	1418.50	1385.10	1408.50	1405.00	1408.50	78.00	242.00	306200.00	-2800.00	6186.45
40458.00	40479.00	4800.00	1416.00	1426.35	1338.00	1345.15	1340.00	1345.15	77.00	237.72	304550.00	-1650.00	6120.30
40459.00	40479.00	4800.00	1346.30	1354.15	1294.50	1326.55	1338.30	1326.55	55.00	168.40	303900.00	-650.00	6103.45
40462.00	40479.00	4800.00	1375.00	1444.40	1334.35	1355.15	1350.45	1355.15	57.00	175.67	302100.00	-1800.00	6135.85
40463.00	40479.00	4800.00	1319.55	1319.55	1281.20	1311.40	1307.00	1311.40	180.00	548.71	302450.00	350.00	6090.90
40464.00	40479.00	4800.00	1324.45	1474.35	1322.00	1469.10	1465.00	1469.10	118.00	366.68	300550.00	-1900.00	6233.90
40465.00	40479.00	4800.00	1500.00	1503.00	1401.00	1401.00	1401.00	1401.00	275.00	860.93	307700.00	7150.00	6177.35
40466.00	40479.00	4800.00	1377.30	1377.30	1271.50	1279.85	1271.50	1279.85	131.00	402.68	301350.00	-6350.00	6062.65
40469.00	40479.00	4800.00	1250.00	1330.00	1203.00	1321.50	1330.00	1321.50	301.00	915.06	290600.00	-10750.00	6075.95
40470.00	40479.00	4800.00	1289.30	1350.00	1224.90	1230.40	1230.00	1230.40	124.00	377.06	285950.00	-4650.00	6027.30
40471.00	40479.00	4800.00	1249.90	1275.00	1205.00	1219.05	1211.00	1219.05	543.00	1642.88	265600.00	-20350.00	5982.10
40472.00	40479.00	4800.00	1253.00	1348.00	1244.95	1344.15	1326.00	1344.15	82.00	250.41	262950.00	-2650.00	6101.50
40473.00	40479.00	4800.00	1339.70	1343.50	1264.00	1289.85	1282.50	1289.85	232.00	708.71	254000.00	-8950.00	6066.05
40476.00	40479.00	4800.00	1327.25	1377.00	1327.25	1334.90	1340.00	1334.90	476.00	1464.67	231650.00	-22350.00	6105.80
40477.00	40479.00	4800.00	1324.85	1332.00	1300.00	1305.45	1314.00	1305.45	239.00	731.01	220350.00	-11300.00	6082.00
40478.00	40479.00	4800.00	1289.40	1300.45	1208.60	1240.40	1233.50	1240.40	552.00	1674.32	201450.00	-18900.00	6012.65

Choosing from Data

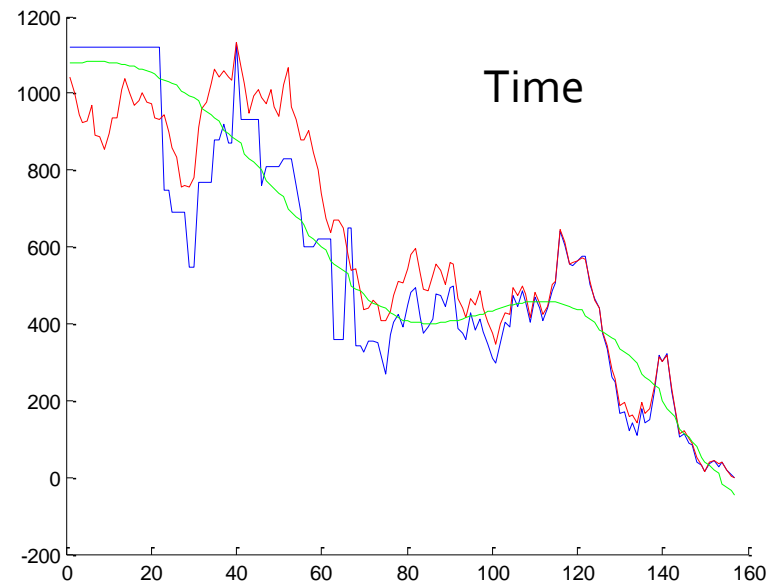
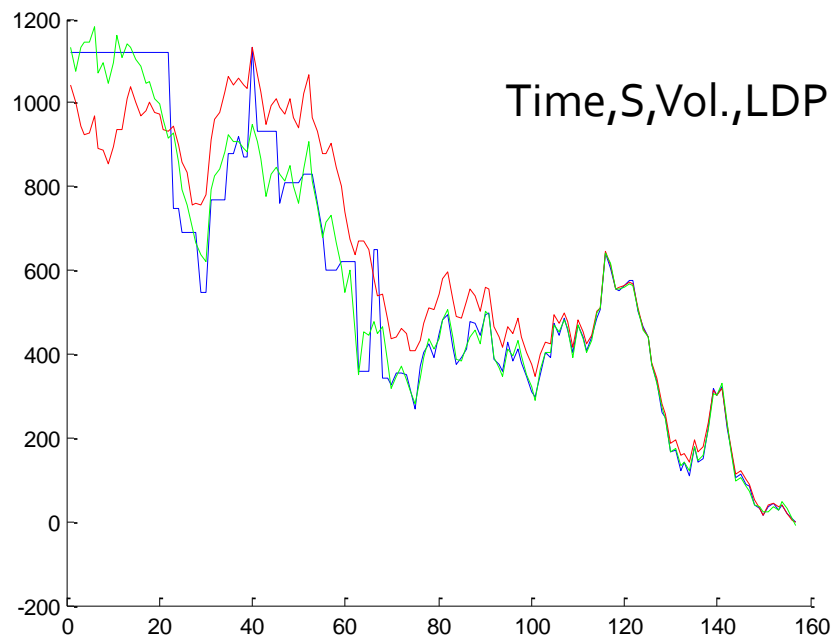
- After observing the data 2 types of options
 - Frequently traded
 - Dormant
- Without this classification the results were very unsatisfactory.

New Approach

- Train on in the regularly traded options
- Add new features like last day price.
- Black Scholes RMSE = 124.62

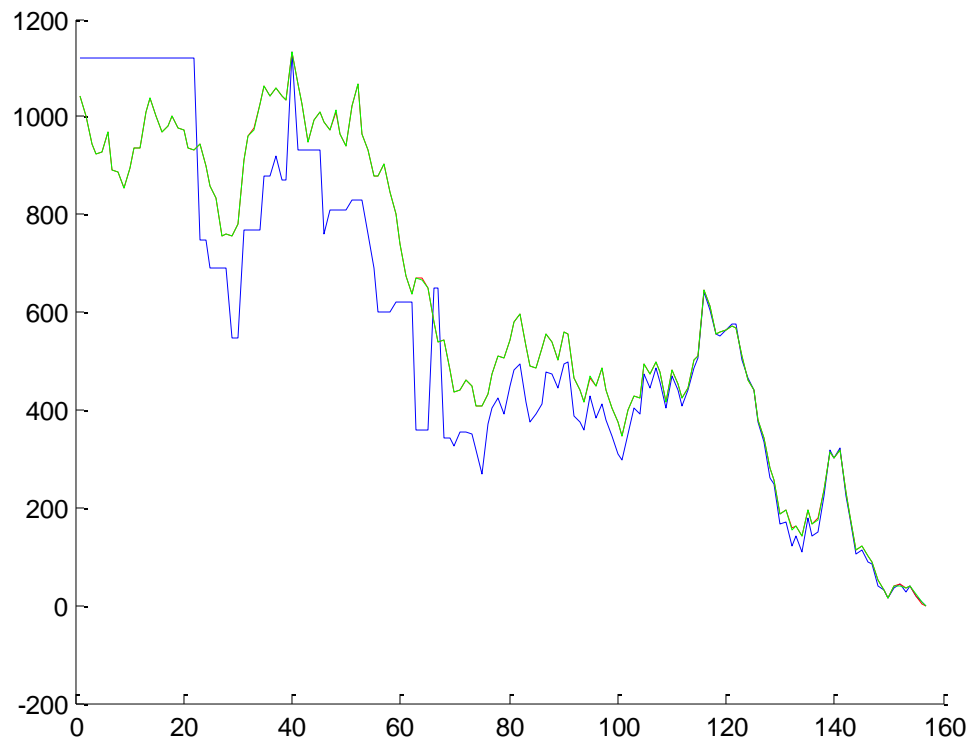
Root mean square error	Features
120.84	Underlying Value, Volatility
90.82	Time till Expiry, Volatility
60.49	Time till Expiry, Volatility, Last Day Price
50.95	Time till Expiry, Volatility, Last Day Price, underlying value

- Standard Deviation in Error = 50.52
- Standard Deviation in Error Black Scholes = 116.2



Some Experiments

- I tried to train it for Black Scholes prices and the curves came out to be exactly similar.



Conclusions

- Significant Improvement from Black Scholes
- Last Day price turned out to be an important feature, along with other features
- SVR adjusts more rapidly to changing market behaviour and is able to capture the pattern more effectively compared to the Black Scholes model.

Thank You!!