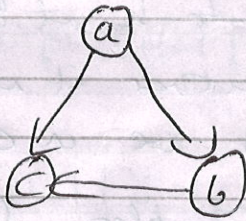
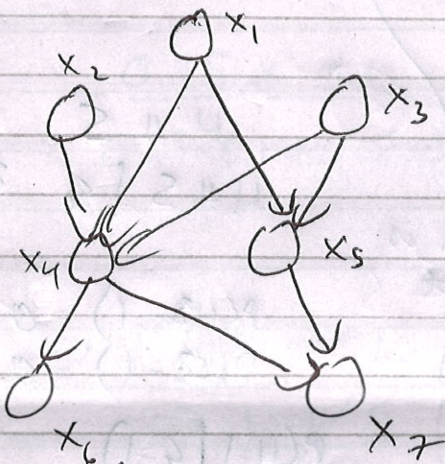
Bayesian Nets



(Intuition)

$$p(a,b,c) = p(a) \cdot p(b|a) \cdot p(c|a,b)$$



Directed Acyclic Graph (DAG)

$$p(x_1)\, p(x_2)\, p(x_3)\, p(x_4 | x_1, x_2, x_3)\, p(x_5 | x_1, x_3)\, p(x_6 | x_4)\, p(x_7 | x_4, x_5)$$

$$p(\underline{x}) = \prod_{k=1}^{K} p(x_k | pa_k)$$

pa_k: parents of k

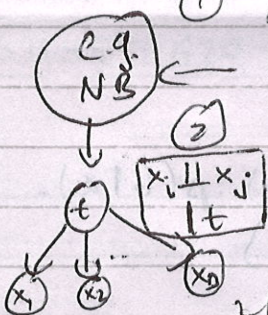
D- seperation

$A \perp\!\!\!\perp B \mid C$?

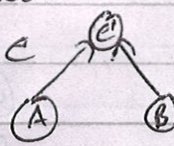
yes: If all paths from A to B are **blocked** by C   tail(t) $\longrightarrow$ head(h)

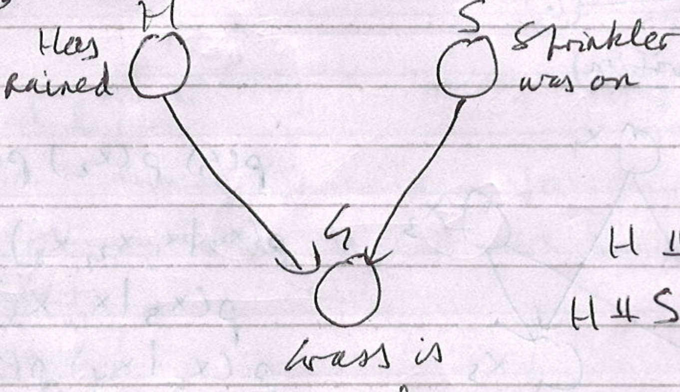
↓ Defn.

(1) Arrows meet h-to-t or t-to-t at a node in C



(2) Arrows meet h-to-h at a node, s.t. neither it nor its descendants are in C





$H \perp\!\!\!\perp S$

$H \perp\!\!\!\perp S \mid G$? ×

E.g.

H	S	$P(G=1 \mid H,S)$
0	0	0.1
0	1	0.8
1	0	0.9
1	1	0.95

$P(H=1) = 0.1$

$P(S=1) = 0.2$

$P(H=1 \mid G=1)$

$= \dfrac{P(G=1 \mid H=1) \cdot P(H=1)}{P(G=1)}$

$$p(G=1) = \sum_{H} \sum_{S} p(G=1 \mid H,S)\, p(H)\, p(S)$$

$$= 0.1 \times 0.9 \times 0.8 + 0.8 \times 0.9 \times 0.2 + 0.9 \times 0.1 \times 0.8 + 0.95 \times 0.1 \times 0.2$$

$$= 0.307$$

$$p(G=1 \mid H=1) = \sum_{S} p(G=1 \mid H=1, S)\, p(S)$$

$$= 0.9 \times 0.8 + 0.95 \times 0.2$$

$$= 0.91$$

$$\Rightarrow p(H=1 \mid G=1) = \frac{0.91 \times 0.1}{0.307}$$

$$= \underline{0.296}$$

$$p(H=1 \mid G=1, S=1)$$

$$= \frac{p(G=1 \mid S=1, H=1) \cdot p(H=1)}{p(G=1 \mid S=1)}$$

$$= \frac{0.95 \times 0.1}{0.95 \times 0.1 + 0.8 \times 0.9} = \frac{0.095}{0.815}$$

$$= \underline{0.117}$$

→ Why is this lower?

$$\Rightarrow H \not\perp\!\!\!\perp S \mid G$$

Inference (writing down likelihood of unknowns in terms of knowns)

specified forms

$$p(x,y) = p(x)\,p(y|x)$$

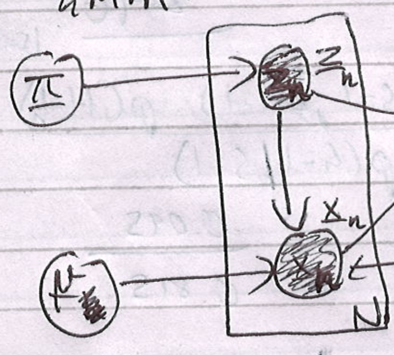
y observed:

$$p(x|y) = \frac{p(y|x)\cdot p(x)}{p(y)}$$

$$L(x) = \frac{p(y|x)\cdot p(x)}{\sum_{x'} p(y|x')\,p(x')}$$

(posterior)

GMM:



filled in ⇒ observed

$$P(X, Z \mid \underline{\pi}, \underline{\mu}, \Sigma\} \longrightarrow L(\underline{x}, \underline{\mu}, \Sigma)$$