6. (b) $$\log L = -\frac{N}{2}\log(2\pi) - N\log\sigma - \sum_{n=1}^{N}\frac{(k_n-\lambda)^2}{2\sigma^2}$$

$$\frac{\partial \log L}{\partial \lambda} = +\sum_{n=1}^{N}\frac{\cancel{2}(k_n-\lambda)}{\cancel{2}\sigma^2}$$

(c) Set $$\sum_n \frac{(k_n-\lambda)}{\sigma^2} = 0$$

$$\Rightarrow \sum_n k_n - N\lambda = 0$$

$$\Rightarrow \hat{\lambda}_{ML} = \frac{1}{N}\sum_{n=1}^{N} k_n$$

(d) $$\frac{\partial \log L}{\partial \sigma} = \frac{-N}{\sigma} + \sum_{n=1}^{N}\frac{\cancel{2}(k_n-\lambda)^2}{\cancel{2}\sigma^3}$$

(e) Set $$\frac{-N}{\sigma} + \sum_n \frac{(k_n-\lambda)^2}{\sigma^3} = 0$$

$$\Rightarrow \sum_n \frac{(k_n-\lambda)^2}{\sigma^2} = N$$

$$\Rightarrow \hat{\sigma}_{ML} = \sqrt{\frac{1}{N}\sum_n (k_n-\lambda)^2}$$ use $\hat{\lambda}_{ML}$

(f) $$\hat{\lambda}_{ML} = \frac{1}{5}(6+37+5+7+6) = 12.2 \text{ min.}$$

(g) $$\hat{\sigma}_{ML} = \sqrt{\frac{1}{5}\left(+6.2^2 + 24.8^2 + 7.2^2 + 5.2^2 + 6.2^2\right)}$$

$$= 12.4 \text{ min.}$$

(i) $$\log \tilde{L} = -\frac{N}{2}\log(2\pi) - N\log\sigma - \sum_n \frac{(k_n-\lambda)^2}{2\sigma^2}$$

$$-\frac{1}{2}\log(2\pi) - \log\beta - \frac{(\lambda-\alpha)^2}{2\beta^2}$$

$$-\log p(k)$$

$$\frac{\partial \log \tilde{L}}{\partial \lambda} = +\sum_n \frac{\cancel{2}(k_n-\lambda)}{\cancel{2}\sigma^2} - \frac{\cancel{2}(\lambda-\alpha)}{\cancel{2}\beta^2}$$

(j) Set $$\sum_n \frac{k_n-\lambda}{\sigma^2} - \frac{\lambda-\alpha}{\beta^2} = 0$$

$$\frac{\sum_n k_n}{\sigma^2} - \frac{N\lambda}{\sigma^2} - \frac{\lambda}{\beta^2} + \frac{\alpha}{\beta^2} = 0$$

$$\lambda\left(\frac{N}{\sigma^2} + \frac{1}{\beta^2}\right) = \frac{\sum_n k_n}{\sigma^2} + \frac{\alpha}{\beta^2}$$

$$\Rightarrow \hat{\lambda}_{MAP} = \frac{\beta^2 \sum_n k_n + \sigma^2 \alpha}{\beta^2 N + \sigma^2}$$

(l) $$\hat{\lambda}_{MAP} = \frac{\frac{\cancel{\sigma^2}}{9}(6+37+5+7+6) + \cancel{\sigma^2}\cdot 5}{\frac{\cancel{\sigma^2}}{9}\cdot 5 + \cancel{\sigma^2}}$$

$$= \frac{61+45}{5+9} = \frac{106}{14} = 7.6 \text{ min.}$$