ELL780: Minor Test II

October 7, 2016

Maximum Marks: 25

1. Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

   (i) Obtain the 'SVD in compact form' of A.

   (ii) Determine the generalised inverse A^+.

   (iii) Obtain the minimum 2-norm solution $||\bar{x}||_2$ of the system $Ax = b$. **[5]**

2. Let $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$.

   (i) Express the above quadratic form as x^TQx by identifying the vector x and the matrix Q.

   (ii) Evaluate $||Q||_F$, $||Q||_1$, $||Q||_2$, and $||Q||_\infty$.

   (iii) It is claimed that Q is positive definite. Verify this claim. **[5]**

3. Consider the LPP

$$\begin{aligned} \text{Max} \quad & z = 4x_1 + 3x_2 \\ \text{subject to} \quad & \\ & x_1 + x_2 \leq 8 \\ & 2x_1 + x_2 \leq 10 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

   Starting with the b.f.s. corresponding to the corner point $(x_1 = 0, x_2 = 8)$ use the simplex algorithm to determine its optimal solution. (Do NOT use the graphical method.) **[5]**

4. Give an example of each of the following (if no such example is possible, give reasons for the same):

   (i) an inner product space which is not a normed linear space.

   (ii) a norm $|| \cdot ||$ which does not satisfy the parallelogram law.

   (iii) a (3×3) nonsingular matrix A whose at least one eigenvalue is zero.

   (iv) a set of three orthonormal vectors in $\mathbb{R}^3$ which does not constitute its basis.

   (v) the orthogonal complement of the subspace $S = \{(x, y) : 3x + 4y = 0\}$ in $\mathbb{R}^2$. **[5]**

5. (i) Solve the following LPP graphically:

$$
\begin{aligned}
\text{Max} \quad & z = 4x_1 + 3x_2 \\
\text{subject to} \quad & \\
& 2x_1 + x_2 + x_3 = 4 \\
& -x_1 + x_2 - x_4 = 1 \\
& x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.
\end{aligned}
$$

(ii) Let $\{x_n\}$ be a sequence in a Hilbert space H. Let $\{x_n\} \to x$. Let $x, y \in H$. Show that $y \perp x_n$ and $x_n \to x$ together imply that $x \perp y$. **[5]**