ELL780: Minor Test II

October 4, 2017

Maximum Marks: 25

1. Let
$$A = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix}.$$
(i) Find the SVD (in compact form) of A.
(ii) Obtain the generalised inverse A^+ of A.
(iii) Use A^+ to find the minimum 2-norm solution of the system
$$\begin{aligned} -x_1 + x_2 + x_3 &= 1 \\ 2x_1 - x_2 + x_4 &= 2. \end{aligned}$$
[5]

2. Consider the inner product space $(\mathbb{R}^2, <,>)$. Let $S = \{(x, y) : 2x - y = 0\} \subseteq \mathbb{R}^2$.
(i) Show that S is a subspace of $\mathbb{R}^2$.
(ii) Obtain $S^\perp$ and show that $S^\perp$ is also a subspace of $\mathbb{R}^2$.
(iii) Sketch S and $S^\perp$ in $\mathbb{R}^2$.
(iv) Obtain a basis of $S^\perp$.
(Here $S^\perp$ denotes the orthogonal complement of S.) **[5]**

3. Consider normed linear spaces $(\mathbb{R}^2, \|\cdot\|_1)$ and $(\mathbb{R}^2, \|\cdot\|_2)$.
Show that $S \subseteq \mathbb{R}^2$ is open in $(\mathbb{R}^2, \|\cdot\|_1)$ if and only if it is open in $(\mathbb{R}^2, \|\cdot\|_2)$. Is S also open in $(\mathbb{R}^2, \|\cdot\|_\infty)$? Give reasons. **[5]**

4. Is the following a convex optimisation problem? Give reasons for your answer.
$$\max\ 4x_1 + 3x_2 - 2x_1^2 - 4x_2^2$$
Subject to
$$\begin{aligned} x_1^2 + x_2^2 &\leq 1 \\ x_2 &\leq x_1^2 \\ x_2 &\geq 0. \end{aligned}$$
[5]

5. Are the following statements true? Give reasons for your answer.
(i) A and A^{T} have the same SVD (in compact form).
(ii) Let A be a (3×3) real symmetric matrix with eigenvalues -2, 0, and 3. Let $B = I + A^2$. Then B is a positive definite matrix.
(iii) $(\mathbb{R}^2, \|\cdot\|_1)$ is a Hilbert space.
(iv) The function $f(x) = 4e^{2x} + 3e^{-2x}, x \in \mathbb{R}$ is a convex function.
(v) $\max\limits_{(x_1,x_2)} (4x_1 + 3x_2 - 6) = \min\limits_{(x_1,x_2)} (-4x_1 - 3x_2) + 6.$ **[5]**