ELL780: Major Test

November 22, 2017

Maximum Marks: 50

1. Consider the nonlinear programming problem (NLP):

$$\begin{aligned} &\text{Max } z = \alpha x_1 + \beta x_2 \\ &\quad s.t. \\ &\qquad x_1^2 + x_2^2 \leq 5 \\ &\qquad x_1 - x_2 \leq 2. \end{aligned}$$

(i) Write the KKT conditions for the given (NLP).

(ii) Determine all possible values of α and β for which $(\bar{x}_1 = 1, \bar{x}_2 = 2)$ is optimal to the given (NLP).

(iii) Write the Wolfe dual (NLD) of the given (NLP). **[6]**

2. Let $(\mathbb{R}^3, \|\cdot\|_\infty)$ be the given Normed Linear Space.

(i) Sketch the unit ball (*i.e.*, ball with centre as $(0,0,0)$ and radius 1) in $(\mathbb{R}^3, \|\cdot\|_\infty)$.

(ii) Show that the sequence $\left\{\left(\frac{1}{n}, -1, \frac{2}{n}\right)\right\}$ in $(\mathbb{R}^3, \|\cdot\|_\infty)$ converges to $(0,-1,0) \in \mathbb{R}^3$.

(iii) Show that $(\mathbb{R}^3, \|\cdot\|_\infty)$ is NOT an inner product space. **[6]**

3. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(i) Obtain the SVD of A.

(ii) Write (without doing any computation) the SVD of A^2. **[6]**

4. Consider the LPP:

$$\begin{aligned} &\text{Max } z = 4x_1 + 3x_2 \\ &\quad s.t. \\ &\qquad 3x_1 + 4x_2 = 12 \\ &\qquad 2x_1 - x_2 \leq 12 \\ &\qquad x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

(i) Determine the basic feasible solution (bfs) corresponding to the convex point $(x_1 = 4, x_2 = 0)$.

(ii) Let the given LPP be solved by the simplex algorithm starting with the corner point $(x_1 = 4, x_2 = 0)$. Write the initial simplex tableau. (Do NOT proceed any further even if needed.) **[6]**

5. Consider the problem of minimisation of $f(x_1, x_2) = 4x_1^2 + 6x_2^2 - 8x_1x_2$ over $(x_1, x_2) \in \mathbb{R}^2$.

(i) Solve the given unconstrained optimisation problem (UMP) by Newton's method starting with $\mathbf{x}^{(0)} = (x_1^{(0)} = -5, x_2^{(0)} = 10)$.

(ii) Let the same (UMP) be solved by the Steepest Descent Method, starting with $\mathbf{x}^{(0)} = (x_1^{(0)} = 2, x_2^{(0)} = 6)$. Obtain the next iterate $\mathbf{x}^{(1)}$. (Do NOT proceed any further even if needed.) **[6]**

6. Let X be a discrete random variable.

x	:	0	1	2	3
$p(X = x)$	:	1/8	3/8	3/8	1/8.

(i) Determine the cumulative distribution function (c.d.f.) F of the random variable X.

(ii) Sketch the c.d.f. $F(x)$ of X.

(iii) Evaluate $P(1 \leq X < 3)$.

(iv) Obtain $E(X)$ and $V(X)$. **[6]**

7. Let $S = \{(x_1, x_2) : (x_1 + x_2 \leq 8) \text{ or } (2x_1 + x_2 \leq 10), x_1 \geq 0, x_2 \geq 0\}$. Let $Conv(S)$ denote the convex hull of S. Use the Wolfe restricted entry simplex algorithm to solve

$$\begin{aligned} &\text{Max } 4x_1^2 + 6x_2^2 - 8x_1x_2 \\ &s.t. \\ &\qquad (x_1, x_2) \in Conv(S). \end{aligned}$$

Write only the first tableau. (Do NOT proceed any further even if needed.) **[6]**

8. Are the following statements true? Give reasons for your answer.

(i) For the constraints $x_1 - x_2 + x_3 = 2$, $x_1 \geq 0$, $x_2 \geq 0$; the point $(\bar{x}_1 = 4, \bar{x}_2 = 2, \bar{x}_3 = 0)$ is a basic feasible solution.

(ii) The set $S = \{(x_1, x_2) : x_2 - x_1^2 \leq 0, -1 \leq x_1 \leq 2\}$ can NOT be the feasible region of a convex optimisation problem.

(iii) Let $V(X)$ denote the variance of a random variable X. Then

$$V(aX + bY) = aV(X) + bV(Y) + 2ab\,Cov(X, Y).$$

(iv) The system of equations

$$\begin{aligned} x_1 + x_2 + x_3 &= 8 \\ 2x_1 - x_2 + 2x_3 &= 10 \end{aligned}$$

has infinitely many solutions.

(v) In a metric space (X, d) if $\{x_n\} \to x$ and $\{y_n\} \to y$ then $\{d(x_n, y_n)\} \to d(x, y)$ in $\mathbb{R}$.

(vi) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$. Then the dimension of the null space of A is 2.

(vii) Let V be the set of all (2×2) positive semidefinite matrices. Then V is a convex cone.

(viii) Let $X \sim \mathcal{N}(\mu = 100, \sigma^2 = 16)$. Let $Y = (2X - 3)$. Then $Y \sim \mathcal{N}(\mu = 200, \sigma^2 = 64)$. **[8]**