ELL780: Minor Test II

October 8, 2015

Maximum Marks: 20

1. Consider the LPP

$$\begin{aligned} &\text{Max} \qquad z = 4x_1 + 3x_2 \\ &\text{subject to} \\ &\quad x_1 + x_2 \leq 8 \\ &\quad 2x_1 + x_2 \leq 10 \\ &\quad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

   (i) Identify the b.f.s. ($x_B = B^{-1}b$, $x_R = 0$) corresponding to the corner point ($x_1 = 0, x_2 = 8$). **[2]**

   (ii) Starting with the b.f.s. corresponding to the corner point ($x_1 = 0, x_2 = 8$) and using the simplex algorithm, obtain an optimal solution of the given LPP. **[3]**

2. Use the simplex algorithm to show that the lines $x_1 + x_2 = 1$ and $2x_1 + x_2 = 4$ do not intersect in the first quadrant. **[5]**

3. Give the Lebesgue measure of each of the following sets, with justification:

   (i) $[5, 10[$

   (ii) $\{5, 6, 7, 8, 9, 10\}$

   (iii) $[5, 10] \times]-3, -2[$

   (iv) $\{x : x = y/2z;\ y, z \in \mathbb{N}\}$ **[2]**

4. Check for uniform continuity of $f(x)$ for $x \in S$ in the following instances; in each case, either prove or disprove.

   (i) $f(x) = x^2$, $S = \{x \in \mathbb{R} : 0 < x < 4\}$ **[1]**

   (ii) Given $|f(x_1) - f(x_2)| \leq M|x_1 - x_2|\ \forall x_1, x_2 \in S$, where M is a positive constant **[1.5]**

   (iii) $f(x) = \frac{1}{x}$, $S =]0, \infty[$ **[1.5]**

5. For which real numbers x do the vectors $(x, 1, 1, 1)$, $(1, x, 1, 1)$, $(1, 1, x, 1)$, and $(1, 1, 1, x)$ *not* form a basis of $\mathbb{R}^4$? For each such value of x that you find, what is the dimension of the subspace of $\mathbb{R}^4$ spanned by the given vectors? **[2]**

6. Let $S = \{1, x, x^2\}$ be the standard basis for P_2, and suppose that $T : P_2 \to P_2$ is a linear operator such that $T(1) = 3x - 5$, $T(x) = x^2 + 1$, and $T(x^2) = 3$.

   (i) What is $T(2x^2 + 1)$? **[0.5]**

   (ii) What is $T(a_0 + a_1x + a_2x^2)$? **[0.5]**

   (iii) Find a matrix which induces T. (Hint: Think of 1 as the vector $(1, 0, 0)$, x as $(0, 1, 0)$, etc.) **[1]**